An Iterative Filtering Technique for the Analysis of Copolar Differential Phase and Dual-Frequency Radar Measurements

J. Hubbert and V. N. Bringi

Department of Electrical Engineering, Colorado State University, Fort Collins, Colorado

20 June 1994 and 18 November 1994

ABSTRACT

Copolar differential phase is composed of two components, namely, differential propagation phase and differential backscatter phase. To estimate specific differential phase $K_{DP}$, these two phase components must first be separated when significant differential backscatter phase is present. This paper presents an iterative range filtering technique that can separate these phase components under a wider variety of conditions than is possible with a simple range filter. This technique may also be used when estimating hail signals from range profiles of dual-frequency reflectivity ratios.

1. Introduction

The processing and interpretation of copolar differential phase measurements $\Psi_C$ has generated much research interest in recent years. Mueller (1984) first gave a procedure to calculate $\Psi_C$ for radars that transmitted alternately horizontal and vertical pulses, which was further analyzed by Jameson and Mueller (1985) and whose statistical properties were investigated by Sachidananda and Zrnić (1986). Of primary interest was the calculation of the specific differential phase $K_{DP}$ (which is the slope of the mean trend of the copolar differential propagation phase $\phi_{DP}$ with range), since modeling studies have shown that $K_{DP}$ (or $K_{DP}$ combined with $Z_{DR}$) is a superior estimator of rain rate and rainwater content (Sachidananda and Zrnić 1986; Bringi et al. 1990; Jameson 1994; Jameson and Caylor 1994) as compared to estimators based on reflectivity alone. Moreover, modeling and measurements indicate that $K_{DP}$ is definitely superior when raindrops are mixed with hailstones (Balakrishnan and Zrnić 1990a; Aydin et al. 1995; Tan et al. 1991).

If the propagation medium between the radar and the resolution volume is characterized by a diagonal matrix

$$P = \begin{bmatrix} e^{-jk_{h,v}} & 0 \\ 0 & e^{-jk_{h,v}} \end{bmatrix},$$

(1)

where $k_{h,v} = k_{h,v}^- - jk_{h,v}^+$ are the complex propagation constants, then the Mueller algorithm gives $\Psi_C$ as

$$\Psi_C = \arg(\langle S_{vv}S_{hh}^* \rangle) + 2(k_{h,v}^- - k_{h,v}^+)r = \delta + \phi_{DP},$$

(2)

where $S_{vv}$, $S_{hh}$ are the scattering amplitudes; superscripts $r$, $i$ denote real and imaginary parts; $\delta$ is the differential scattering phase; $\phi_{DP}$ is the differential propagation phase; and angle brackets denote ensemble or time averaging. The specific differential phase $K_{DP}$ is estimated by finite differences as

$$K_{DP} = \frac{\phi_{DP}(r_2) - \phi_{DP}(r_1)}{2(r_2 - r_1)}.$$

(3)

If $\delta$ is nonzero, as can occur due to Mie scattering, and if it varies over the range interval $r_2 - r_1$, then it must be accounted for, or the $K_{DP}$ will be in error.

The problem of separating $\delta$ from $\Psi_C$ is similar in principle to separating hail signal (HS) from specific attenuation in dual-frequency power ratios (Tuttle and Rinehart 1983), that is,

$$\frac{\langle |S_{hh}(s)|^2 \rangle}{\langle |S_{hh}(x)|^2 \rangle} + 4[\alpha_{h(x)}]_{10} \log(e)$$

$$= HS + \alpha_{h(x)} (\text{dB}),$$

(4)

where DFR is the dual-frequency reflectivity ratio (ratio of reflectivities at S and X bands) and $\alpha_{h(x)}$ is the X-band attenuation introduced by the propagation medium. The specific attenuation $A_3$ (dB km$^{-1}$) is estimated by finite differences as

$$A_3 = \frac{\alpha_{h(x)}(r_2) - \alpha_{h(x)}(r_1)}{2(r_2 - r_1)}.$$

(6)

Modeling and observations suggest that under certain situations involving partially melting hydrometeors $\delta$ can be significant ($\delta > 5^\circ$) at C band and even at S-
band wavelengths (Meischner et al. 1991; Balakrishnan and Zrnić 1990b; Zrnić et al. 1993a,b; Aydin and Zhao 1990). Large raindrops at both C and X bands can also contribute significantly to $\delta$. Thus, the application of $K_{DP}$ to improve estimates of rainwater content at higher frequencies (such as 13 GHz) hinges on the accuracy of the estimation of $\delta$, especially under those situations where $\delta$ has gradients over the interval $r_1-r_2$ used for estimating $K_{DP}$ (typically 2–4 km to reduce statistical fluctuations) (Jameson 1994; Jameson and Caylor 1994). In addition, the estimation of $\delta$ itself is important as an indicator of partially melting hydrometeors or as an indicator of Mie scattering effects.

The problem of separating $\delta$ and $\phi_{DP}$ was addressed by Hubbert et al. (1993), who used finite-impulse response (FIR) and infinite-impulse response (IIR) filters to smooth $\Psi_C$-range profiles. Large deviations of the raw $\Psi_C$ range profile from the filtered range profile were considered to be potentially due to $\delta$. The suggested method will have problems detecting $\delta$ when it is nonzero over a number of contiguous range resolution volumes. This paper gives an iterative range-filtering technique, which will allow for the detection of $\delta$ in such situations and is, therefore, a more robust detector of $\delta$ and $\phi_{DP}$. The iterative filtering technique is applied to sample data gathered by POLDIRAD, the German Aerospace Research Institute's C-band radar. The same technique is applied to dual-frequency reflectivity data gathered by the NCAR CP-2 radar for estimating specific attenuation after correcting for hail signal.

2. Iterative filter

a. Background

Copolar differential phase estimates $\Psi_C$ consist of two phase components, namely, differential propagation phase $\phi_{DP}$ and differential backscatter phase $\delta$. The $\phi_{DP}$ is range cumulative, whereas $\delta$ is only dependent on the arg($S_{vh}, S_{hb}$) in each range resolution volume.

Since $\Psi_C$ is an integrated effect and even in heavy rain increases by less than $8^\circ$–$10^\circ$ km$^{-1}$ (depending on frequency), range profiles of $\phi_{DP}$ are smooth functions of range, which are relatively slowly varying. The "jitter" of a $\Psi_C$ range profile consisting of only $\phi_{DP}$ is caused by statistical fluctuations, and these fluctuations for the Mueller (1984) estimator have typical standard deviations of 2$^\circ$–4$^\circ$ for 128 $H-V$ sample pairs (Sachidananda and Zrnić 1986; Bringi et al. 1990). Specific differential phase $K_{DP}$, which is the slope of the mean trend of $\phi_{DP}$ with range, can be estimated from such a $\Psi_C$ range profile by first smoothing it with IIR or FIR filters (Hubbert et al. 1993). If significant $\delta$ is present, it will appear as a large deviation from the mean trend of the $\Psi_C$ range profiles. To be detectable, these jumps in phase usually need to be greater than the measurement standard deviation of $\phi_{DP}$. This technique will work under two assumptions: 1) the area of consistently positive (negative) $\delta$ extends over only a few range resolution volumes and 2) the differential propagation phase can be measured beyond the range where $\delta$ was nonzero. The first assumption is now addressed, and it is shown how an iterative filtering technique can more robustly detect these extended ranges of nonzero $\delta$.

If significant $\delta$ extends over a great enough range, a single filtering will not suppress this $\delta$ variation, and the resulting $K_{DP}$ estimate will be biased. For example, if a large positive $\delta$ is present over a long enough range, manifested by a large "bump" in the $\Psi_C$ range profile, then this range variation can fall outside the stop band of the selected filter. [For a discussion of pass bands for range filters see Hubbert et al. (1993).] This is best illustrated by an example range profile. Figure 1 shows the raw $\Psi_C$ range profile (dashed curve) with a large bump in the range 48.0–50.5 km due most likely to $\delta$, while the solid curve shows the filtered profile. The filter sharply attenuates spatial fluctuations less than or equal to 1.5 km. See the appendix for filter details. The large bump in this filtered range profile will cause
erroneous $K_{DP}$ estimates (including negative estimates) if used as is. Figure 2 shows the $\delta$ corrected $\Psi_C$ range profile (dashed curve) that results after the iterative filter algorithm has been applied to the raw $\Psi_C$ range profile of Fig. 1. The solid curve of Fig. 2 results after the $\delta$ corrected $\Psi_C$ is filtered one more time. The bump of Fig. 1 has been removed, and a better estimate of $K_{DP}$ can be made from the solid curve of Fig. 2. Note how only the large phase excursions of the raw profile have been eliminated while the remainder of the profile (52–57 km) has been left unchanged, thus, preserving the more subtle mean variations of the unfiltered profile in that area. This would not be the case if a filter had been used such that the large bump fell within the stopband; that is, the large bump could have been eliminated by simply selecting a filter with a low enough cutoff frequency. The large bump would have been eliminated; however, the other more subtle mean variations, which the iterative filter preserves, would have also been eliminated.

There will be a dramatic difference between the two $K_{DP}$ signatures, and the iterative filter obviously gives a much better estimate of $K_{DP}$ in this instance, especially within the range of 49–51 km. The $\delta$ is estimated by a simple differencing of the filtered (solid) curve of Fig. 2 and the original raw $\Psi_C$ range profile (dashed curve) of Fig. 1. The resulting $\delta$ range profile is shown in Fig. 3. The large 20° phase excursions around 49–50.5 km are well above the standard measurement error for $\phi_{DP}$ for this dataset (estimated to be about 4°), and therefore, we believe this phase can be attributed to $\delta$. Further support for this conclusion rests in the accompanying range profiles of reflectivity and $Z_{DR}$ (not shown here), which indicate that this area is within the storm core where larger melting ice particles that would cause such signatures are likely to be found.

The iterative filtering technique is extremely useful since it provides the means for the automatic detection of $\delta$ and can be applied under a wider variety of conditions as compared to the simple filtering technique presented earlier in Hubbert et al. (1993). For example, this method was used to estimate the spatial variations of $\delta$ in the hailstorm case analyzed by Zrnić et al. (1993b). The iterative algorithm is described next.

b. The iterative algorithm

To correct for $\delta$ a new iterated range profile, called $\tilde{\Psi}_C$, is constructed by selecting data points from either the raw data profile or its filtered version as determined by a threshold. This threshold is set according to the expected standard deviation $\sigma$ of $\Psi_C$. Thresholds of 1.25 to 2 times $\sigma$ were found to give good results, and the threshold used to construct the plots in this paper was 5°. The filtered range profile and the raw $\Psi_C$ range profile are differentiated at each range resolution volume. If the absolute value of the difference is less than the threshold, the raw $\Psi_C$ value is selected for the iterated $\Psi_C$ range profile, otherwise the filtered $\Psi_C$ value is selected for the iterated $\Psi_C$ range profile. In this way, the iterated range profile $\tilde{\Psi}_C$ is created. This process is then repeated by filtering $\tilde{\Psi}_C$, and this newly filtered range profile is again differenced with the original raw $\Psi_C$ range profile. Convergence can be determined in the usual way by requiring that the change in $\Psi_C$ from one iteration to the next be within some tolerance. It was found that repeating the filtering process 10 times produced good results. The effects of the iterative process can be clearly seen in Figs. 4 and 5, which show $\Psi_C$ and filtered $\Psi_C$ range profiles, respectively, after several iterations. Curves A–G result after 1, 3, 5, 7, 9, 11, and 13 iterations of the above-described filtering process. (The original raw profile is in Fig. 1.) To obtain the successive $\Psi_C$ range profiles, difference the given filtered $\tilde{\Psi}_C$ of Fig. 5 with the original raw $\Psi_C$ of Fig. 1. If the difference is greater than 5°, use the filtered $\tilde{\Psi}_C$ data point for the next $\tilde{\Psi}_C$ curve, otherwise, use the data

![Fig. 3](image-url)  
**Fig. 3.** The estimate of $\delta$ present in the $\Psi_C$ range profile shown in Fig. 1.

![Fig. 4](image-url)  
**Fig. 4.** An example of iterative filtering technique showing $\Psi_C$ after 1, 3, 5, 7, 9, 11, and 13 iterations, corresponding to curves A–G, respectively, of the filtering process. The original unfiltered curve is seen in Fig. 1 (dashed curve).
that the medium where bad data was present produced no additional $\phi_{DP}$. The latter is typically a very good assumption since low SNR typically means very little precipitation was present. Since the filter is a 20th-order FIR filter, the $\Psi_C$ range profiles need to be extended by 10 data points on each end. The initial extension values are determined by averaging the first three $\Psi_C$ data points, while the end extension values are determined by averaging the last three $\Psi_C$ data points. The filtering technique is summarized by the block diagram shown in Fig. 6. The implications are that weather radars with advanced signal processors could potentially display $K_{DP}$ and $\delta$ (or, HS and $A_3$) in real time. The method works well for range profiles, which extend through the entire precipitation core unless the signal is attenuated to below noise level (unusual at S and C bands but not at X band). We have found that this filtering process worked well in the many convective storm cases, which we have examined (see, e.g., Zrnić et al. 1993b; Bringi et al. 1994).

3. Application to dual-frequency data

The iterative algorithm has also been applied to dual-frequency data from the NCAR CP-2 radar to estimate hail signal (HS) and X-band specific attenuation. The range profile example shown here was taken on 24 June 1992 in an intense storm described by Bringi et al. (1994). Figure 7 shows the range profiles of several

```plaintext
Begin
Determine Start and End Points
Set Initial and Final Conditions
$\Psi_C = \Psi_C$
Filter $\Psi_C$; Output = $\Theta$
Is $|\Psi_C - \Theta_i| >$ Thresh. ?
No $\Psi_C_i = \Psi_C_{i+1}$
Yes $\Psi_C_i = \Theta_i$
Convergence ?
No
Yes Stop

Fig. 6. Block diagram of the iterative filtering algorithm.
```

point from the original $\Psi_C$ range profile. Between 49 and 50.5 km, the successive curves tend to approximate the mean $\phi_{DP}$ trend within the high-$\delta$ region. This is only an approximation given the available data, but based on the previous physical arguments, curve G should provide a reasonably good estimate of $\phi_{DP}$. In effect, curve G connects the original raw range profile from the value at 48 km to the value at 52 km, where $\delta$ is very small and $\phi_{DP}$ dominates. Curve G is filtered once again, and the slope of the resulting curve is the final estimate of $K_{DP}$. The overall effect is that the filter applied to the original $\Psi_C$ range profile from 48 to 52 km has a lower cutoff frequency as compared to the filter applied to the data from 52 to 57 km in range.

Useful, though not necessary, is knowledge of the initial phase offset of $\Psi_C$ due to the electronics of the radar. Knowing this, initial conditions for the filter can be set, or alternately, they can be set using the average of the first few data points to be filtered. This assumes that these first few data points of $\Psi_C$ contain insignificant $\delta$.

For any given $\Psi_C$ range profile there may be points where the SNR (signal-to-noise ratio) is low enough so that the standard deviation of $\Psi_C$ is increased to a high level. The result is poor phase estimates that have the potential to bias the $K_{DP}$ estimates. These poor phase estimates should be eliminated by using some SNR threshold as a criterion for accepting the phase estimate. The threshold used here was SNR = 10 dB, which had to be exceeded for three consecutive resolution volumes to qualify as good phase estimates. The end of the good data was determined by three consecutive SNR values less than 10 dB. After the end of good data has been found, the remaining data points of the range profile may be checked for further good data, for example, two separate storm cores could be present. The missing bad data points can be filled in by the average of the previous last three good data points. The assumption here is that these last three good data points reflect a good estimate of $\phi_{DP}$ and
parameters in a three-panel presentation. Figure 7a shows a “light”-filtered DFR profile where DFR is the ratio of reflectivities at S and X bands. The light filter sharply attenuates spatial fluctuations less than or equal to 0.3 km. Figure 7a also shows the iterated version, called \( \alpha_{i(x)} \), after two iterations. Figure 7b shows a “heavy” filter applied to the iterated \( \alpha_{i(x)} \) from which the specific X-band attenuation \( A_3 \) (dB km\(^{-1}\)) is obtained by finite differences. Finally, in Fig. 7c, both reflectivity at S band \( (Z_b) \) and hail signal (HS, dB) are shown. The HS is obtained as the difference between the light-filtered DFR curve (solid curve in Fig. 7a) and the final \( \alpha_{i(x)} \) (solid curve in Fig. 7b). The peak HS is 8 dB near the 74-km range. The “noise” in the HS away from the peak is around ±2 dB, which is acceptable. The threshold was set at 3 dB for this case.

4. Summary

A new processing algorithm based on previous work by Hubbert et al. (1993) is described for estimating \( K_{DP} \) (or \( A_3 \)) in the presence of nonzero \( \delta \) (or HS). The algorithm has two primary selectable parameters: 1) the cutoff frequency of the filter and 2) the decision threshold value used to construct the iterated \( \Psi_C \) (DFR) range profile. The cutoff frequency is selected based on the accumulative slow varying nature of \( \Phi_{DP} \) \( \left( \alpha_{i(x)} \right) \), the expected range length of significant \( \delta \) (HS), and the sampling period. The threshold is based on statistical fluctuations in the estimator for \( \Psi_C \) or DFR. The range filters can be selected from a family of low-pass infinite-impulse response (IIR) or finite-impulse response (FIR) filters with various low-pass magnitude responses. The number of iterations is also a selectable parameter, but it was found that the algorithm converges quickly with 10 iterations being sufficient for the datasets we have analyzed. While the method works particularly well for a single well-defined precipitation core, it has been adapted for multiple precipitation cores as well and can be extended for real-time computation and display of \( K_{DP} \), \( \delta \) (or \( A_3 \), HS).

Acknowledgments. This research was supported by the National Science Foundation via ATM-9214864. The POLDIRAD radar is operated by the DLR, and the authors acknowledge the assistance and support rendered by the DLR radar group. The NCAR CP-2 radar data were provided by Drs. E. Brandes and J. Vivekanandan of the Research Applications Division.

APPENDIX

Finite Impulse Response Filter

The coefficients of the FIR used for implementing the iterative filter are given in Table A1. The 20th-order filter is symmetric, and the coefficients are given for the complex variable \( Z \) (Proakis and Manolakis 1988). The filter is applied twice in order to obtain a lower cutoff frequency. The frequency response characteristics of the equivalent composite single filter are obtained by simply squaring the frequency response of the filter given in Table A1. The resulting frequency response is shown in Fig. A1. The horizontal axis is labeled in kilometers, based on the 0.15-km spatial sampling period of the radar data used, and can be loosely interpreted as spatial variation (i.e., a \( \delta \) bump) that occurs over that range interval. Figure A1 shows that the filter attenuates sharply (more than −26 dB)

<table>
<thead>
<tr>
<th>Table A1. FIR filter coefficients.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z order</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>( Z^0 )</td>
</tr>
<tr>
<td>( Z^1 )</td>
</tr>
<tr>
<td>( Z^2 )</td>
</tr>
<tr>
<td>( Z^3 )</td>
</tr>
<tr>
<td>( Z^4 )</td>
</tr>
<tr>
<td>( Z^5 )</td>
</tr>
<tr>
<td>( Z^6 )</td>
</tr>
<tr>
<td>( Z^7 )</td>
</tr>
<tr>
<td>( Z^8 )</td>
</tr>
<tr>
<td>( Z^9 )</td>
</tr>
<tr>
<td>( Z^{10} )</td>
</tr>
</tbody>
</table>
spatial variations on the order of approximately 1.5 km and less. The −3-dB point is approximately 4 km. Practically, this filter was chosen because it performed well. Physically, one wishes to preserve the typically increasing phase trend due to the propagation medium, while attenuating statistical fluctuations and fluctuation caused by δ. Differential propagation phase is range accumulative and is, therefore, a smooth function that only increases a few degrees per kilometer. When using data with a 0.15-km sampling period, ϕDP estimates are typically averaged over 1–2-km intervals to reduce statistical fluctuations so that meaningful KDP estimates can be made. Thus, choosing a filter that sharply attenuates fluctuations that occur over 1.5 km or less is appropriate. Differential backscatter phase is nonaccumulative and causes bumps in $\Psi_C$ range profiles. The duration of these bumps can be as long as the core of an intense storm. The FIR filter chosen was able to detect (i.e., the variation was far enough outside the filter passband) δ existing over about 3 km. Again, the advantage of the iterative filtering technique is that filters with lower cutoff frequencies, which would entirely eliminate the bump of Fig. 1, do not need to be used. Using such a narrow band filter would cause the order of the filter to increase substantially, would decrease the range sensitivity of the KDP estimates, and would yield poor δ estimates.

REFERENCES


