Statistical Retrieval of Humidity Profiles from Precipitable Water Vapor and Surface Measurements of Humidity and Temperature

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ABSTRACT

A new method is presented of statistical retrieval of humidity profiles based on measurements of surface temperature ξ_1, surface dewpoint ξ_2, and integrated water vapor ξ_3. In this method the retrieved values of humidity depend nonlinearly on predictors $\xi_{1,2,3}$. A self-training algorithm was developed to obtain the values of parameters that enter into the retrieval algorithm. The data from two years of measurements in eight different locations were used for training. The method was applied to an independent dataset (including nonmonotonic profiles) of one month of surface measurements and integrated water vapor obtained from microwave radiometers. Three constraints were imposed: 1) the integrated retrieved humidity profiles had to be equal to the measured values ξ_3, 2) the retrieved surface humidity had to coincide with the measured value, and 3) the retrieved humidity had to be positive. The rms deviations of restored humidity values from measured profiles were approximately two times less than natural variations. A limited comparison with conventional linear statistical inversion showed that the nonlinear method may improve the recovery of vertical structure.

1. Introduction

Before presenting our new nonlinear method of retrieving water vapor profiles from measurements of precipitable water vapor and surface meteorological measurements, a few comments on alternative approaches to the problem are in order. We wish to derive an absolute humidity profile $\rho(h)$ from measurements of V and $\rho(0)$, where

$$V = \int_0^\infty \rho(h)dh. \tag{1}$$

Since humidity profiles tend to decrease, on the average, as exponential functions of height, perhaps the simplest method of deriving a humidity profile is to assume an exponential form, with a scale height H,

$$\rho(h) = \rho(0)\exp\left(-\frac{h}{H}\right). \tag{2}$$

If we integrate (2), we get

$$V = \rho(0)H, \tag{3}$$

from which H, and hence $\rho(h)$, can be determined.

The exponential method has the obvious disadvantage that it cannot describe profiles with inversions, and it also does not easily lend itself to incorporating additional information that may be available about the profile. However, it is stable to errors in V and $\rho(0)$ and produces a strictly positive profile.

A standard method of deriving profiles, and one that incorporates climatological information into retrievals is linear statistical inversion (e.g., Westwater 1993). Here, the estimate of the humidity profile is given by

$$\hat{\rho}(h) = \bar{\rho}(h) + \langle\rho'(h)\mathbf{d}'^{\mathrm{T}}\rangle\langle\mathbf{d}'\mathbf{d}'^{\mathrm{T}}\rangle^{-1}\mathbf{d}', \tag{4}$$

where $\bar{\rho}(h)$ is the average of ρ at height h, $\mathbf{d}'$ is a data vector containing V and surface measurements, angle brackets refer to average over an ensemble of profiles, and the primed quantities refer to departures from the average. This method is an improvement over (2) and (3), since statistical characteristics of $\rho(h)$ constrain the solution. Other variations of the statistical method include decomposition of the original ensemble into subensembles that have a unique measureable characteristic—for example, decomposition of an ensemble of profiles into clear or cloudy ones, or decomposition of those measured during cloudy conditions into those having base heights within a given altitude range (Ham et al. 1994). However, neither the original method nor the variants constrain the profile to be positive. In addition, the integral of the solution

$$\hat{V} = \int_0^\infty \hat{\rho}(h)dh \tag{5}$$

is not identically equal to V, although, in a practical situation, it is close.

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There is a method, using (4), to derive a strictly positive estimate of $\rho(h)$. Since $\rho(h)$ is nonnegative, we can make a nonlinear functional transformation expressing ρ as a nonnegative function, such as $\rho(h) = \exp[y(h)]$, or $\rho(h) = [y(h)]^2$. For purposes of illustration, we choose the exponential transformation. Then $y(h) = \ln[\rho(h)]$ and we can derive an estimate of $y(h)$ from (4) yielding

$$\hat{y}' = \langle \ln[\rho(h)]'\mathbf{d}^{\mathrm{T}\prime}\rangle \langle \mathbf{d}'\mathbf{d}^{\mathrm{T}\prime}\rangle^{-1}\mathbf{d}'. \qquad (6)$$

Thus, we can get

$$\hat{\rho}(h) = \exp[\hat{y}(h)], \qquad (7)$$

which is positive, but it is not an unbiased estimator of $\rho(h)$ and its integral does not equal V. In the following, we introduce our method that yields an estimate that is positive and integrates to V but is not an unbiased statistical estimator.

Many problems in climatology and weather forecasting require information about humidity fields of the atmosphere. There is now an increased availability of instruments to measure integrated moisture content of the atmosphere from the ground—for example, microwave radiometers (Westwater 1993), Global Positioning System (GPS) receivers (Bevis et al. 1992), and optical solar trackers (Reagan et al. 1992). Probably the most mature of these technologies is dual-channel microwave radiometry. In addition to providing temporal resolutions from 10 s to 2 min, the radiometers measure integrated moisture content with an accuracy of the same order of magnitude as that of radiosondes (Martner et al. 1993). Additionally, dual-channel radiometers work in an automatic mode unattended for weeks. Their main disadvantage is cost. GPS techniques also provide the possibility of yielding unattended observations, with temporal resolutions of 20 min and an accuracy equal to that of the radiometers. Finally, data from geostationary satellites also provide accurate measurements of integrated moisture during clear conditions (Birkenheuer 1991; Gao et al. 1992). Thus, remote sensors offer the possibility of obtaining nearly continuous data on precipitable water vapor (PWV) in a vertical column above a fixed location. In contrast, humidity profiles, when measured by radiosondes, belong to changing geographical coordinates, since, during the course of one launch, the radiosonde drifts with the prevailing winds. For mesoscale forecasting it is important to know the initial meteorological information at fixed geographical points for all vertical levels. The grid spacing of contemporary numerical forecasts is 30 km, but future models will have horizontal levels of 10-km spacing, a number approaching that of radiosonde drift.

Although both radiometers and GPS techniques measure moisture directly overhead, they measure only the integrated moisture content of the atmosphere. The restoration of humidity profiles from either radiometer data or the GPS technique is an inverse problem and one that is ill posed. A recent technique by Kuo et al. (1992) derives humidity profiles from a horizontal network of measurements of PWV, as well as wind and temperature profiles. In the work presented here, a new method is explained for retrieving profiles from only single-station measurements of PWV and surface temperature and humidity. This method uses statistical relations between the integrated moisture content and the ground humidity of the air. Note that this retrieval technique is not the linear regression method of Westwater (1993), because this nonlinear method incorporates additional information about the humidity profile, namely, that the values are strictly positive. As shown in section 4, excellent retrievals are made even when the associated profiles are nonmonotonic.

2. Retrieval algorithm

As discussed in section 1, several measurement systems can yield accurate and temporally frequent measurements of PWV, including radiometric, optical, and GPS techniques. The problem of inferring humidity profiles is considered, given measurements of surface air temperature ξ_1 (°C), the surface dewpoint temperature ξ_2 (°C), and the integrated water vapor ξ_3. In the context of statistical estimation, the values $\xi_{1,2,3}$, known from the measurements, are used as predictors in the retrieval of the humidity profiles. Let $\bar{\xi}_{1,2,3}$ denote the average of these values, calculated for a fixed spatial coordinate, season, and time of day. In later calculations, values averaged over two years are used. We consider an a priori ensemble of humidity profiles and let the index m represent a member of the ensemble. Let us designate the value of the mth realization of absolute humidity (g m^{-3}) at the level h by $a^m(h)$, and let $\bar{a}(h)$ be the average of the same quantity. (Similar notation is used for the surface measurements $\bar{\xi}_j$.) Let $\tilde{a}^m(h)$ be the restored (predicted) value of the absolute humidity of the level h. The usual method of linear regression to determine the restored values uses the linear combination

$$\tilde{a}^m(h_j) = \bar{a}(h_j) + \sum_{k=1}^{3} (\xi_k^m - \bar{\xi}_k)c_k(h_j), \qquad (8)$$

where the coefficients $c_k(h_j)$ are determined by minimizing the mean square of the error

$$F[c_k(\ )] = \frac{1}{N}\sum_m \sum_j [\tilde{a}^m(h_j) - a^m(h_j)]^2 = \min, \qquad (9)$$

where N is the full number of terms in the double sum. The condition (9) leads to a system of linear equations to determine $c_k(h_j)$. However, in general, such a solution will not satisfy the conditions

$$\tilde{a}^m(h_0) = a^m(h_0) \qquad (10)$$

$$\tilde{a}^m(h_j) > 0 \qquad (11)$$

$$\sum \tilde{a}^m(h_j)\Delta h_j = \xi_3^m. \qquad (12)$$

To satisfy the first supplementary condition, we simply define the reconstructed value at the surface to be the measured value

$$\tilde{a}^m(h_0) = a^m(h_0). \tag{13}$$

To satisfy the second supplementary condition, note that the a priori measured values $a^m(h_j)$ are always positive. Therefore, if some retrieved quantity $\tilde{a}^m(h_j)$ is negative, then it can be replaced by its modulus, which decreases the difference in (9). Therefore, the modulus is introduced in the right-hand side of (8) to yield the formula

$$\tilde{a}^m(h_j) = |\bar{a}(h_j) + \sum_{k=1}^{3} (\xi_k^m - \bar{\xi}_k)c_k(h_j)|. \tag{14}$$

Such a method is difficult to use in analytical computations but it is very convenient for any numerical algorithm.

Finally, to satisfy the third condition (coincidence of the restoration of the integrated moisture content with the measured value), we introduce the additional coefficient K^m in (14) for all $h_j > h_0$:

$$\tilde{a}^m(h_j) = K^m|\bar{a}(h_j) + \sum_{k=1}^{3} (\xi_k^m - \bar{\xi}_k)c_k(h_j)|, \quad h_j > h_0, \tag{15}$$

and the coefficient K is determined from the condition

$$\sum_{j=0}^{n} \tilde{a}^m(h_j)\Delta h_j = \sum_{j=0}^{n} a^m(h_j)\Delta h_j = \xi_3^m, \tag{16}$$

where n is the number of levels.

Substitution of (15) into the left-hand side of (16) results in the formula for K^m:

$$K^m = \frac{\xi_3^m - a^m(h_0)\Delta h_0}{\sum_{j=1}^{n} |\bar{a}(h_j) + \sum_{k=1}^{3} (\xi_k^m - \bar{\xi}_k)c_k(h_j)|\Delta h_j}. \tag{17}$$

Taking into account this expression, we find that the formula for determination of $\tilde{a}^m(h_j)$ takes the following form:

$$\tilde{a}^m(h_j) = \begin{cases} a^m(h_0), & j = 0 \\ \dfrac{[\xi_3^m - a^m(h_0)\Delta h_0]|\bar{a}(h_j) + \sum_{k=1}^{3} (\xi_k^m - \overline{\xi_k})c_k(h_j)|}{\sum_{p=1}^{n} |\bar{a}(h_p) + \sum_{k=1}^{3} (\xi_k^m - \bar{\xi}_k^m)c_k(h_p)|\Delta h_p}, & j > 0. \end{cases} \tag{18}$$

From formula (18) it follows that $\tilde{a}^m(h_j)$ nonlinearly depends on the parameters $c_k(h_j)$. (They enter into the numerator and the denominator and also within the modulus.) Therefore if (18) is substituted into (9), then (in the best case, if it is possible to carry out the averaging) a complicated nonlinear system of equations for determining $c_k(h_j)$ will be obtained. To solve this system, we used a combined Gauss–Newton and modified Newton numerical algorithm (Kalitkin 1979) that minimizes a function of many variables $F[c_k(h_j)]$.

3. Results

To use our method of profile retrieval, it is necessary to have available a training set of profiles of humidity and surface temperature. We used an excellent set of research-quality multistation radiosonde data that were obtained during the Winter Icing and Storms Project (WISP) that was held in the Front Range of the Rocky Mountains during 1990–91 (Rasmussen et al. 1992). To obtain homogeneous initial data, we initially classified the meteorological stations by geographical location and by height above sea level. Then using every collection of the preliminary classified data, we calculated the function $F[c_k(h_j)]$, which depends on $3n$ of the arguments $c_k(h_j)$, and we numerically determined the minimum of this function. Usually, when one searches for an extreme of a nonlinear function one can discover many approximately equivalent extreme sets $c_k(h_j)$. In our case different solutions corresponding to the different subsets (from the full archive of data) approximate well the profiles of humidity.

Calculation of the coefficients $c_k(h_j)$ and training of the algorithm used radiosonde data that consisted of about 600 vertical profiles of absolute humidity and surface temperature for several locations at which WISP observations were taken. Two radiosonde systems were available: National Weather Service (NWS) radiosondes and the Cross-chain Loran Atmospheric Sounding System (CLASS) research radiosondes. It is known that the CLASS radiosondes measurements of relative humidity below 20% have more reliability than NWS radiosondes (Wade and Wolfe 1989; Wade 1994). Twice-daily NWS measurements were routinely available in Denver; during special meteorological events, as determined by WISP forecasters, three-hourly CLASS soundings were made near several cities within the WISP domain. The locations and altitudes of these stations are shown in Fig. 1. All these cities are located in the eastern Front Range of the Colorado Rockies and are on the lee side of the Rockies. Because of the geographical similarity of the stations, some of the data were combined when water vapor retrieval coefficients $c_k(h_j)$ were constructed. The groups of data were constructed by taking into account the time of the measurements (three groups) and the height above sea level (four groups). Table 1 shows the groups combined with respect to time and altitude. The combination of the measurements by time was done to take into account the diurnal variability of the humidity

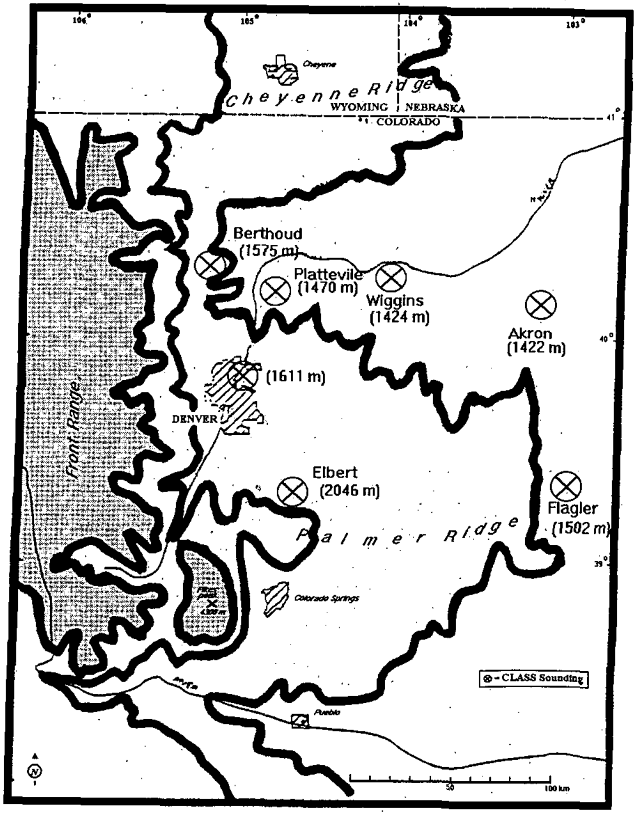

FIG. 1. Class sounding station locations.

and the temperature; the separation by height was done to account for station altitude effects.

Following the groupings shown in Table 1, we determined the coefficients $c_k(h_j)$ and the average profiles for the three groups of stations using 3-h data for March 1990 and 1991 and for the standard radiosonde data in Denver for March 1991 and 1992. These data were used for the dependent sample restoration of the vertical profiles of absolute humidity. In addition, humidity profiles were derived for Denver for March 1993 for an independent dataset, that is, data that were not used in calculating the coefficients $c_k(h_j)$ and the average profiles. However, both dependent and independent sets were analyzed. In addition to radiosonde data, NWS local climatological data and the 3-h synoptic maps of the National Oceanic and Atmospheric Administration (NOAA) for North America were used.

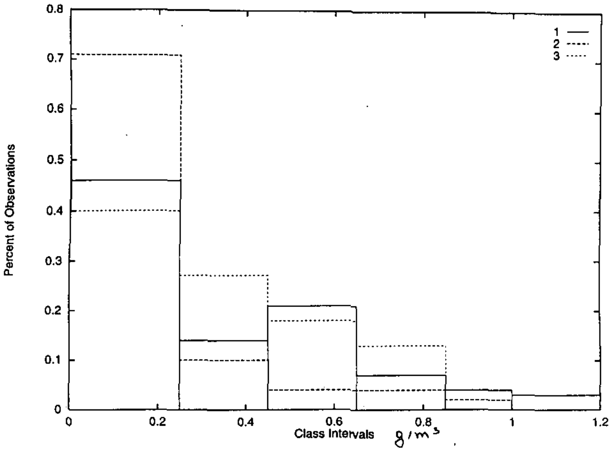

FIG. 2. Histograms of the maximum differences between retrieved and observed humidity profiles grouped into difference categories labeled on the abscissa. Data for group 1 (solid) are for all stations in Table 1 except Elbert and Denver; data for group 2 (dashed) are for Elbert; and data for group 3 (dotted) are for the independent set of Denver.

In comparing the retrieved profiles with the real ones, we computed the maximum (over all levels) of the modulus of their differences $\Delta = \max_z[|a(z) - \tilde{a}(z)|]$. These values were grouped into the following categories: 0–0.25, 0.25–0.45, 0.45–0.65, 0.65–0.85, 0.85–1.0, and 1.0–1.2 g m^{-3}. Figure 2 gives three histograms of Δ for the difference categories. The abscissa gives

TABLE 1. Grouping of sounding stations according to time of soundings and height above sea level of the station, used for determining the vertical profiles of absolute humidity.

	Sites			
Altitude (m MSL)	2103 m	1525–1570 m	1480–1420 m	1611 m
Time (UTC)				
0000–0600	Elbert	Berthoud Loveland Platteville	Flagler Wiggins Akron	
0900–1700	Elbert	Berthoud Loveland Platteville	Flagler Wiggins Akron	
2000–2300	Elbert	Berthoud Loveland Platteville	Flagler Wiggins Akron	
0000 and 1200				Denver

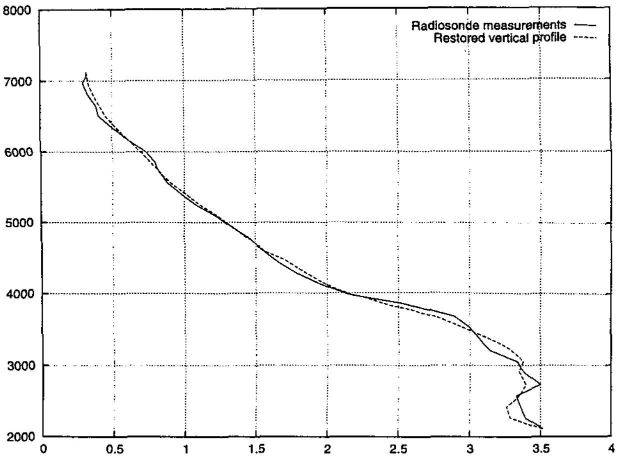

FIG. 3. Retrieved vertical profile of absolute humidity for Elbert, 0325 UTC 23 March 1990.

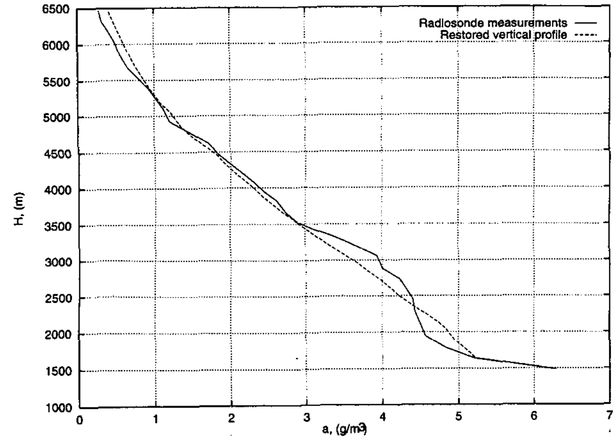

FIG. 5. Retrieved and measured vertical profiles of absolute humidity for Flagler, 0600 UTC 27 March 1991, at a time of low cloudiness that could turn into fog.

the Δ categories, and the ordinate gives the percentage of observations corresponding to a given Δ. Retrieved profiles for all stations with the exception of Elbert and the independent set for Denver were included in group 1 in Fig. 2. Profiles of Elbert (group 2) were separated from this main grouping because its location was greater than 2000 m MSL, and moisture profiles there are similar to the free atmosphere. Group 3 (for Denver) is separate because it shows reconstructions for the independent sets of Denver; furthermore, this location uses NWS radiosondes. Figure 2 illustrates that all three groups have their maximum for the interval 0–0.25 g m^{-3} (40% for independent data, 70% for Elbert, and 47% for the group including all other stations). As seen from these results, the best restorations are for Elbert, where the meteorological conditions are similar to the free atmosphere. For example, in Fig. 3, a nearly perfect, but dependent sample, retrieval of a vertical profile for Elbert is shown. This profile has a maximum difference in the category 0–0.25 g m^{-3}.

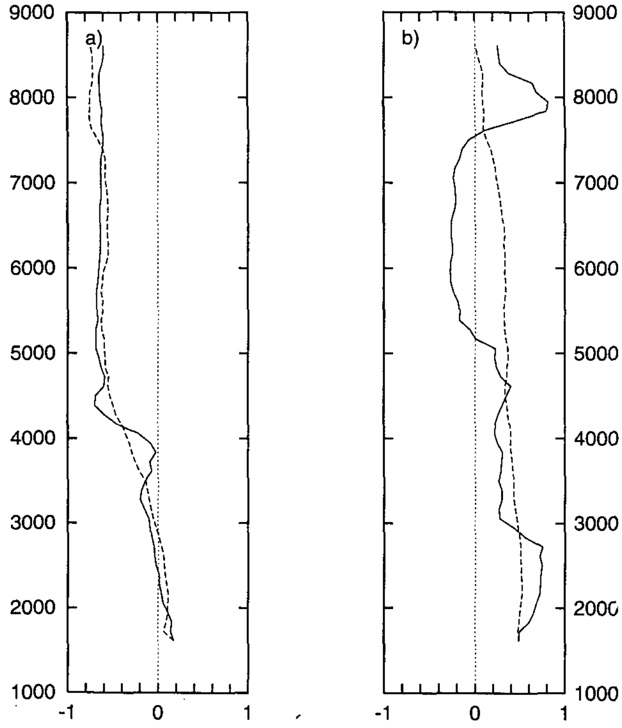

FIG. 4. Relative deviations of radiosonde and retrieval profiles from the mean profile for independent data for Denver: (a) 2311 UTC 11 March 1993, (b) 2315 UTC 16 March 1993. The rms values of humidity deviations from mean profile σ are as follows: 0.89 g m^{-3} for z = 2000 m, 0.57 g m^{-3} for z = 3000 m, 0.47 g m^{-3} for z = 4000 m, 0.35 g m^{-3} for z = 5000 m, 0.18 g m^{-3} for z = 6000 m, 0.09 g m^{-3} for z = 7000 m. Deviations of retrieved profile from radiosonde profile are small in comparison with σ for $z < 5000$ m, where the main vapor content is concentrated.

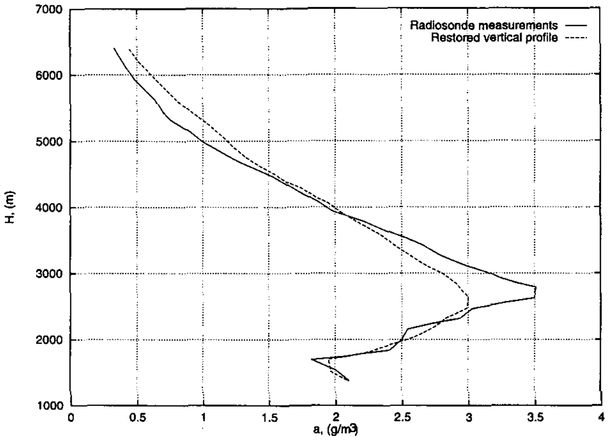

FIG. 6. Retrieved and measured vertical profiles of absolute humidity for Wiggins, 1500 UTC 2 March 1991. (The center of an intense upper-low system was near Wiggins.)

Note that the maximum difference between the restored profile and radiosonde profile was always observed in the layer between the surface and 1.5 km AGL. The absolute humidity in this layer is of the order of 3–4 g m^{-3} for the period of observation. Thus, the relative errors of restorations were less than 8% with the probability about 40%–70% and they were more than 30% with the probability less than 15%. Of course, the maximum relative errors were observed in the upper layers, where the absolute humidity is small. In Fig. 4 the relative deviations of retrieved and radiosonde profiles from the mean profile (Denver, March 1991–93) are presented.

The quality of the retrievals can be also estimated by calculating the mean square of the deviation F, a quantity that is calculated in the process of minimization. Changes in the value of F within the limit 0.1–0.2 g^2 m^{-6} corresponds to an rms of deviations between the measurement and the restoration profiles of 0.3–0.45 g m^{-3}. To evaluate the quality of the retrieved humidity profiles as an aid to weather forecasts, we compared our data with synoptic information. From NOAA synoptic maps of the United States, data that were examined included total cloud cover, cloud type, temperature, and humidity. From NOAA tables of local climatological data, data were examined on the horizontal visibility, as well as for type and quantity of precipitation. These data were necessary for the qualitative evaluation of the correspondence between the retrieved profiles and the type of the weather. Owing to this analysis, it was determined that the maximum differences between the measurement and the restoration profiles (0–0.25 g m^{-3}) in the majority of cases were insignificant and that the restored profiles completely correspond with the observed weather. (The relative differences were about 7%–10%.) Figure 5 gives an example for 27 March 1991, when an intense upper-level low pressure center was located near Flagler, Colorado. Rain and nimbostratus clouds completely covered the sky, and the absolute humidity on the surface was very high ($\approx$6 g m^{-3}). Typical values of humidity for that season are approximately 3.0–3.5 g m^{-3}. Figure 5 shows a good retrieval of the absolute humidity profile; the maximum difference between measured and retrieved profiles is 0.3 g m^{-3}. In addition, qualitative analyses show that sometimes even larger differences did not make the restored profile unreasonable. Figure 6 illustrates one of these cases. On 2 March 1991, an upper-level trough passed Colorado, and it was accompanied by heavy showers in Denver and Elbert. In northern Colorado (Wiggins), there was no rain, but cloudiness and heavy fog were present. This type of weather usually is accompanied by a sharp increase in humidity throughout the region of fog. Although the maximum difference between the retrieved and measured values was 0.5 g m^{-3}, from the qualitative point of view this profile correctly represents the vertical distribution of the humidity.

Figure 7 illustrates another profile retrieval from a time when a large anticyclone from the northwest to the southeast passed through Colorado on 9 March 1993. All the vertical profiles correctly reflected the distribution of humidity for this type of weather, and even Akron, which had a maximum difference of 0.25 g m^{-3}, gave an adequate characterization. Figure 8 shows retrieved profiles that were obtained during an intense upper-level circulation. At that time, in Colorado rain and strong cloudiness were observed. During such weather, a strong increase of absolute humidity at altitudes of 2.5–3.0 km AGL is typical. The profiles in Fig. 8 are for 1500 or 1800 UTC 2 March 1991. All profiles were accurately retrieved, and the inversions were calculated almost correctly.

Examples showing the retrieval of the humidity profiles from the independent data for Denver are presented in Figs. 9 and 10. Figure 9 illustrates an accurate retrieval of the humidity profile for clear weather on 23 March 1993. Figure 10 shows the retrieved profile on 11 March 1993. On this day, a gale with very heavy snow passed through Colorado and, in particular, through Denver. But an intense low pressure system passed very quickly, and during these rapid processes, peculiarities (for example, an inversion) in the distribution in the vertical did not usually appear. Analysis of similar situations shows these cases are retrieved very well. Accordingly, we made an analysis to correlate the accuracy of retrieved profiles to the type of the weather.

4. Qualitative aspects of retrievals

In addition to possible numerical applications of water vapor retrievals, forecasters frequently use plots of profiles to assess the current meteorological situation. In such an application, if the general shape of a profile corresponds well to reality, the information may help a forecaster keep up with a changing meteorological situation. Thus, we tried to come up with at least a rough measure of how well a retrieval corresponded to reality. We considered restorations as good according to the following criteria: (a) for profiles without inversions, well-constructed profiles correspond to those profiles having maximum deviations between the restored and measured profiles not more than 0.25 g m^{-3}; (b) for profiles with inversions, restorations are counted as good, irrespective of their maximum deviations, if they preserve the shape of the profile and have the humidity maximum within 200 m of the true profile. If a profile has a poorly retrieved shape—for example, with an improperly placed maximum—then it is considered a poor retrieval, irrespective of its maximum deviation.

On the basis of such a classification, we constructed Table 2, which gives the results of the classification analysis of all well-retrieved profiles. In Table 2, all observations are divided according to two types of weather: "cloudy" and "clear." All cases when the

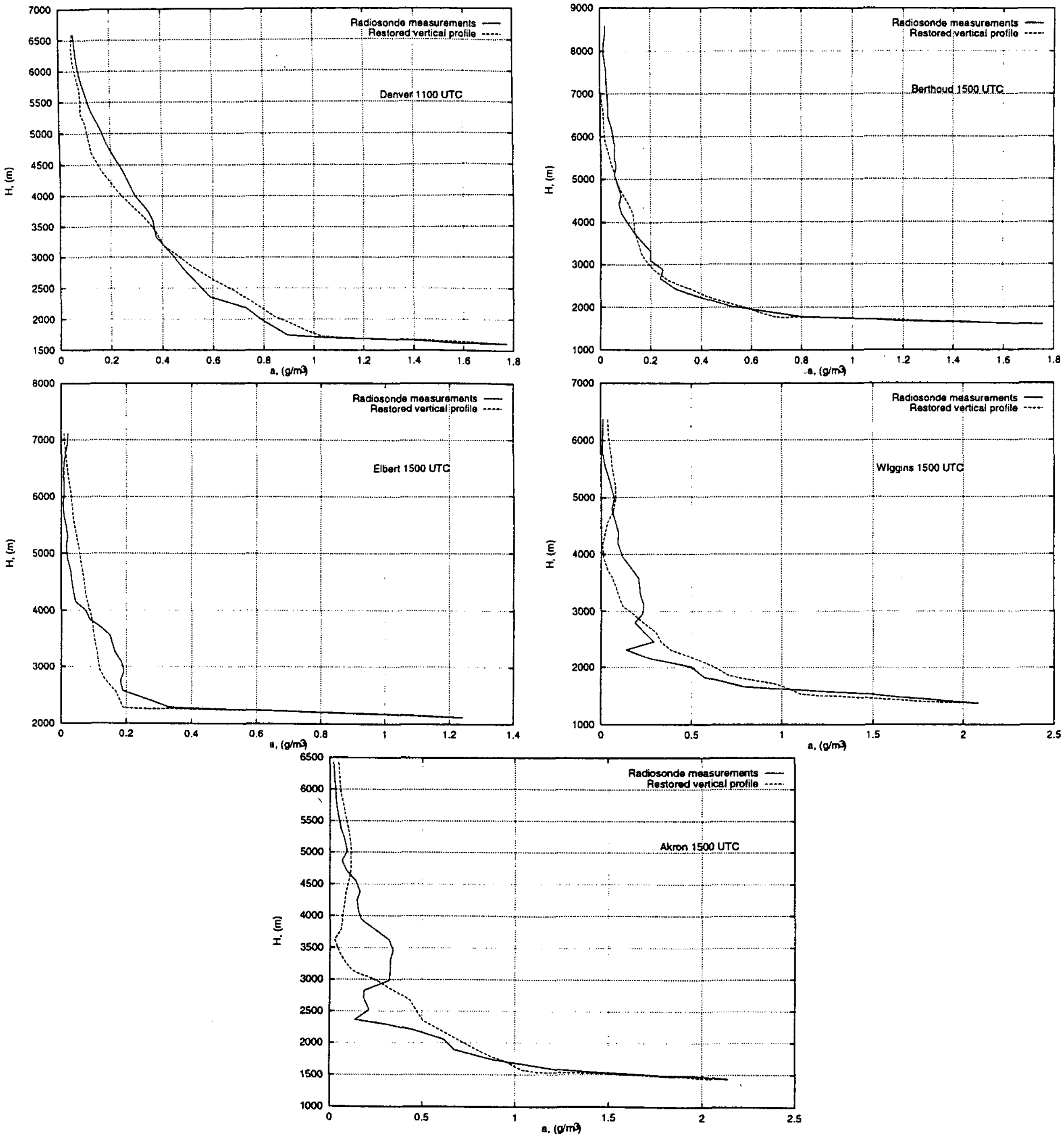

FIG. 7. Retrieved and measured vertical profile of absolute humidity at the time of a large anticyclone passed through Colorado, 1500 UTC 9 March 1991.

cloud cover was 0.5 or more were included in the clouds category; the remainder of the cases were included in the clear category. Retrievals for Denver using independent data were classified separately.

This table shows that for all locations, the percentages of well-retrieved profiles for cloudy weather were significantly less than the percentages for clear weather. Apparently, if it were possible to take into account the altitude of the clouds, as predictors, then a better retrieval of humidity profiles for cloudy weather would result. For example, in several cases the humidity calculated for Berthoud in the altitude range of 2 km was more than 100%. In principle, such oversaturated humidity can appear during retrieval of the profiles. This situation occurs in

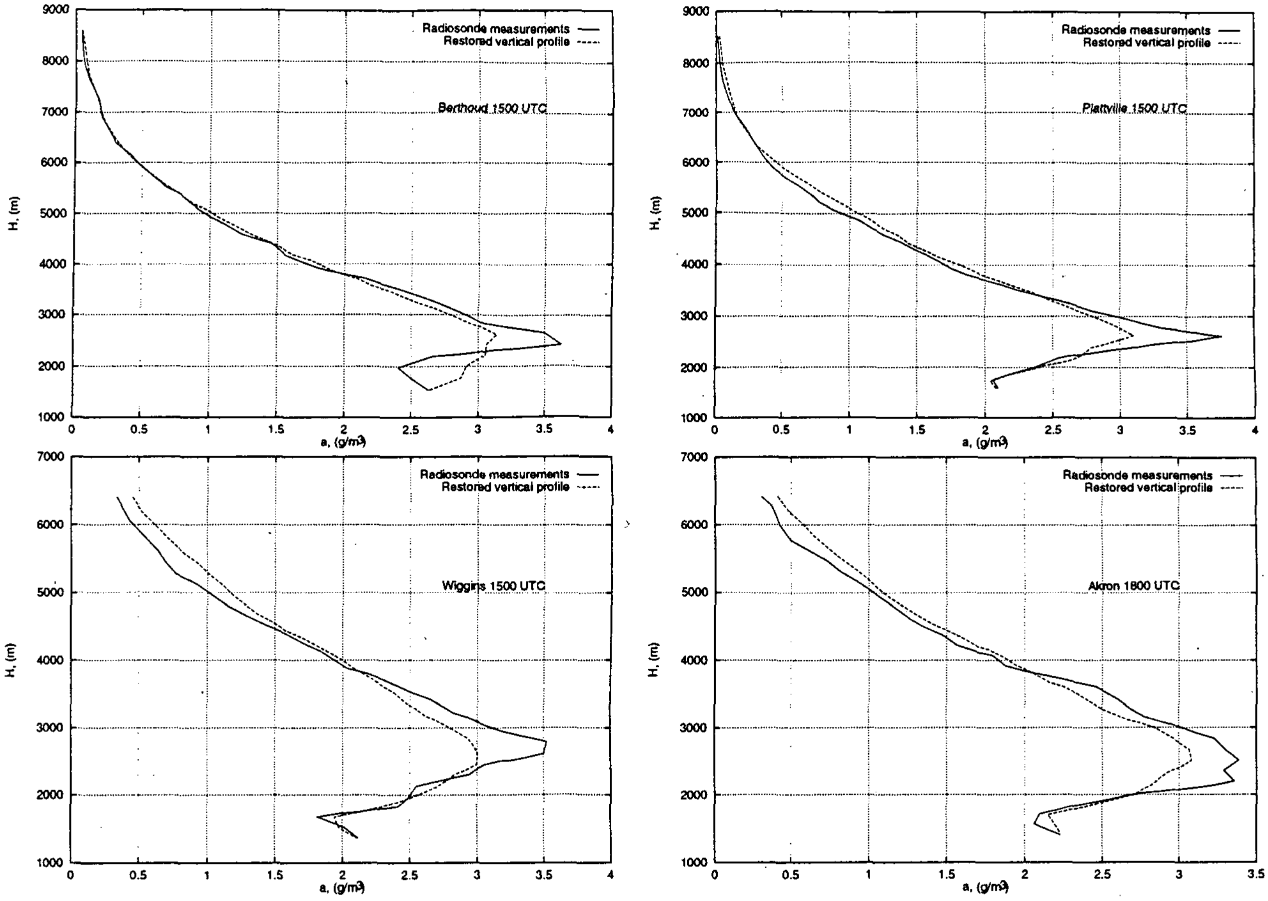

FIG. 8. The retrieved and measured vertical profiles of absolute humidity at a time of rain showers and frontal cloudiness, 2 March 1991.

the case of heavy fog or during the transition from low stratus clouds to fog. To correct retrievals during such situations, it is necessary to know the vertical profiles of the temperature, which can be obtained by a radio acoustic sounding system (RASS) and the cloud height and thickness. The extended availability of Automated Sur-

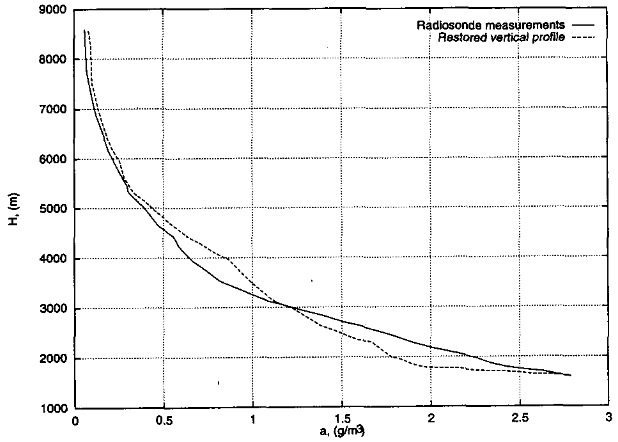

FIG. 9. Retrieved and measured vertical profiles of absolute humidity from the independent data for Denver at a time of clear weather, 1100 UTC 23 March 1993.

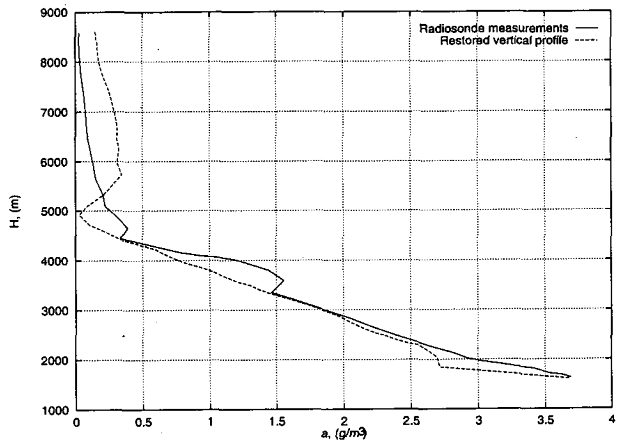

FIG. 10. Retrieved and measured vertical profiles of absolute humidity from the independent data for Denver at a time of heavy snowfall, 2300 UTC 11 March 1993.

TABLE 2. Values of well-restored profiles (numerator), total number of restored profiles (denominator), and the percentage of the well-restored profiles by March at the different locations of the WISP data.

	Weather type	
Cities	Cloudiness (0.5–1.0)	Clear (0–0.5)
Elbert (1990, 1991)	43/65 = 66%	12/15 = 80%
Berthoud		
Platteville		
Loveland (1990, 1991)	66/123 = 54%	26/33 = 79%
Flagler		
Wiggins		
Akron (1990, 1991)	82/174 = 47%	17/26 = 65%
Denver (1991, 1992)	27/66 = 41%	25/48 = 52%
Denver 1993 Independent data	16/36 = 44%	5/9 = 56%
Total	234/464 = 50%	85/131 = 65%

face Observing System ceilometers and RASS (May et al. 1988) make such a possibility feasible.

To evaluate the preference of the nonlinear retrieval method developed, we performed a comparison with a conventional linear method. Typical results are shown in Fig. 11. In cases of monotonic profiles both methods lead to approximately similar results, although the results indicated in Figs. 11b,c showed an approximately 20%–30% improvement in the nonlinear over the linear method. But the linear method was unable to retrieve nonmonotonic profiles, whereas the method developed here does. (e.g., Fig. 11a). This difference is significant because nonmonotonic profiles are frequently related to special weather conditions.

5. Summary and discussion

We have presented here a nonlinear method of retrieving water vapor profiles from measurements of precipitable water vapor and surface meteorological measurements, subject to a constraint that the profiles be positive. The method gave promising results when applied to data from a recent WISP experiment on the front range of eastern Colorado. Comparisons of the nonlinear method with conventional linear statistical inversion gave slightly better results for profiles without structure, but profiles with structure were recovered significantly better.

The nonlinear method was based on a technique of imposing a positive inequality constraint; that is, the profile must be strictly greater than zero. If the temperature profile were known, then the constraint that the relative humidity be less than or equal to 100% could also be imposed. Furthermore, if, in addition to temperature, the boundaries of a liquid cloud were known, then the equality constraint of saturation within the cloud could also be applied. The method presented here could easily be generalized to handle these constraints. The imposition of inequality constraints to linear statistical methods is rather difficult.

In the application of our method, we have been very careful in obtaining a database of high quality

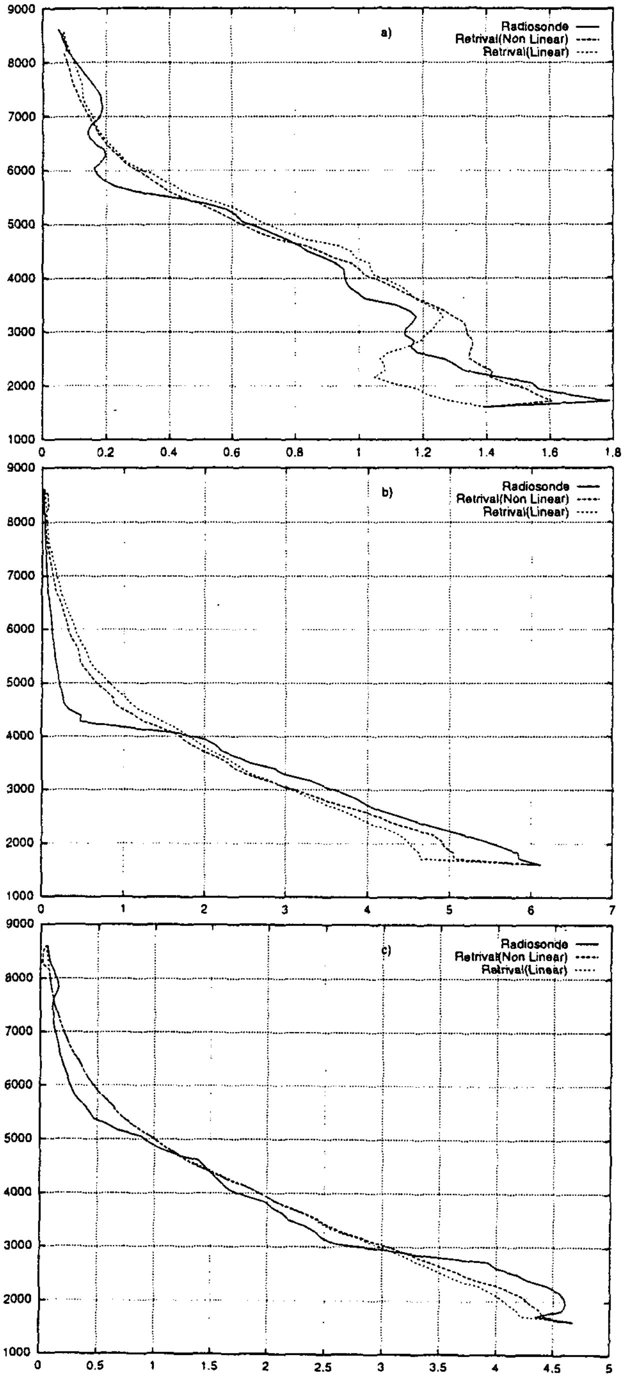

FIG. 11. Comparison of linear and nonlinear retrievals of the vertical humidity profile for a case of nonmonotonic vertical dependence for Denver, by statistically independent data: (a) 2305 UTC 17 March 1991, (b) 0205 UTC 5 March 1992, and (c) 2315 UTC 16 March 1993.

and representativeness. If similar high quality radiosonde data could be obtained, this method could be applied, say, to satellite moisture sounding over the ocean.

In addition, we did use surface moisture measurements as predictors, and these might not always be available for satellite applications. However, the positivity constraint is of general applicability to concentration profiling such as water vapor, cloud liquid, and ozone.

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