A Study of the Wavenumber Spectra of Short Water Waves in the Ocean. Part II: Spectral Model and Mean Square Slope

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(Manuscript received 21 December 1995, in final form 21 February 1997)

ABSTRACT
This paper presents a wavenumber spectral model of the surface water waves in the capillary–gravity regime. The database for the analysis consists of spatial and temporal measurements of surface slopes obtained from a scanning slope sensor buoy operating in free-drift mode in the Atlantic Ocean. These data indicate that the contribution of mean square slopes from capillary–gravity waves (wavelengths from 4 mm to 6 cm) is a significant portion of the total mean square slopes. The resulting mean square slopes derived from the proposed spectral model are in excellent agreement with existing field datasets.

The properties of short waves, including the mean square slopes and wind speed dependence of the spectral intensity of individual wave components, derived from optical and microwave sensors are also compared. Significant differences in terms of the magnitudes of the mean square slopes and the exponents of wind speed dependence are found. Some of the possible explanations of the discrepancies are explored.

1. Introduction
For ocean remote sensing applications, active microwave sensors depend on short surface waves to scatter back the radar waves through Bragg resonance mechanisms (e.g., Wright 1966, 1968; Valenzuela 1978; Plant 1990) and other non-Bragg processes, including specular reflection and contribution of steep surface features (e.g., Holliday et al. 1986; Winebrenner and Hasselmann 1988; Wetzel 1993; Trizna et al. 1991, 1993). Short waves with length scales from a few millimeters to a few decimeters contribute significantly to the overall surface steepness and high curvature events (Hwang et al. 1993; Hwang 1995). These waves are very sensitive to environmental parameters including wind speed, wind gust, near-surface stability conditions, and surface currents induced by various sources, such as internal waves, background gravity waves, and even the bottom topography (Hughes 1978; Beal et al. 1982; Phillips 1984; Stewart 1985; Thompson 1988; Hwang and Shemdin 1990; Hwang 1992). In the past, most of the short-wave spectra have been presented in the frequency domain because the data were obtained through single-point measurements using thin wires or optical slope sensors. However, studies of wave dynamics, such as wind generation, surfactant damping, and surface current modulation, require wavenumber resolution. Attempts to derive wavenumber information from the measured time series have not been very successful because the convection of short waves by surface currents causes a large Doppler frequency shift in the band of capillary–gravity waves. In addition to the kinematic convection, the dynamic interactions between waves and currents, such as current blockage, wave breaking, and the reflection and refraction of short waves in the rather complicated surface current field, further complicate the frequency-to-wavenumber conversion. Direct spatial measurements of wave properties are therefore needed.

With this comprehension, a scanning slope sensor was developed to measure the spatial structure of short wind waves with wavelengths ranging from 0.4 to 10 cm, covering the region where both gravity and capillary effects are important. This range of wavelengths is also of interest to remote sensing applications because the Bragg resonance waves of many microwave radars fall within this region. The scanning slope sensor is evolved from laser slope gauges that measure the refraction of a laser beam to derive the sea surface slope. The critical improvement of the scanning technique is that instead of dwelling the laser beam at a fixed position, the beam is controlled to quickly scan a predetermined pattern to accomplish spatial mapping of the surface (Hwang et al. 1993; Hwang et al. 1996).
The spatial measurements provide directly the wavenumber resolution and eliminate the problem of the Doppler frequency shift described above. The concept was tested first in the laboratory with success (Hwang et al. 1993). The primary results from the laboratory study include the following: 1) From the space–time series, both the wavenumber and frequency spectra can be calculated. 2) The space–time tracking of surface slope motion provides a detailed mapping of the motion of short waves at the water surface. 3) The curvature of the water surface can be derived from spatial differencing of the slope data. 4) The high curvature events were found to correlate very well with the sea spike events measured simultaneously by a time-domain reflectometer (TDR) radar (Trizna et al. 1993).

Following this success, a prototype for field application was built and tested in the Puget Sound and the Anacost River. The first successful sea trial was performed during the HiRes II Field Experiment (Hwang et al. 1996). The scanning speed of the system used in the field experiment is more than 40 m s⁻¹, which is much faster than the phase speed of the wave components being measured, and the surface can be regarded as frozen during the scan. The data obtained represent the first systematic measurements of the wavenumber spectra of capillary–gravity waves in the open ocean (Hwang et al. 1994; Hwang et al. 1996; Hwang 1995).

This article presents the new results from the continuous analysis of the field data. A more detailed discussion of the experimental technique and the field conditions is given in Hwang et al. (1996) and only a brief summary will be given later. Based on these ocean data, a wavenumber spectral model emphasizing the capillary–gravity region is presented in section 2. In the absence of other wavenumber measurement of capillary–gravity wave data, a preliminary verification of the spectral model is provided in terms of the mean square slope calculation. The results are compared with available oceanographic datasets in section 3. Subsequently, a comparison of short-wave properties measured by optical and microwave sensors is given in section 4. Significant discrepancies are found between optical and radar measurements exist, and a few possible explanations contributing to the observed differences are offered. The summary and conclusions are given in section 5. Considerable background information of the spectral properties of short waves is given by Phillips (1977, 1984, 1985, 1988), and the development of this paper follows closely the results presented in Hwang et al. (1996).

2. Spectral model of capillary–gravity waves in the ocean

a. Ocean database

Using a scanning slope sensor mounted on a free-drifting buoy to reduce flow distortion to the sensing region, a set of 1D wavenumber spectra of short waves from the Atlantic Ocean was collected (Hwang et al. 1996). The connection between the wavenumber spectrum derived from a linear-scanning slope sensor and the omnidirectional spectrum calculated from the 2D wavenumber spectrum is discussed in earlier literature [e.g., Schule et al. 1971; Phillips 1985. Also, appendix C of Hwang et al. (1996) presents a detailed discussion pertaining to the case of scanning slope sensor measurements]. The range of wavenumbers resolved is between 100 and 1600 rad m⁻¹, corresponding to approximately ½ < k/kₘ < 4, where k is the transect wavenumber and kₘ indicates the wave component that has equal influence from gravity and surface tension. The wind speed ranges from 0.8 to 5.7 m s⁻¹. These results are summarized in Fig. 1 and Table 1. The major observations of these spectra are as follows: 1) A pronounced peak is evident in the curvature spectra, indicating that a change of spectral characteristics occurs within the wavenumber range 100 < k < 1600 rad m⁻¹. The peak wavenumber is found to be relatively constant. 2) The slopes of the curvature spectra are 1 and −1 on the two sides of the spectral peak. 3) The spectral density and properties of mean square roughness increase linearly with wind speed. 4) These observations suggest a function form of the elevation spectrum to be \( \chi(k) = Au_kc^{-2}c_ukk^{-2} \), which is proportional to \( u_kk^{-3} \) in the short gravity wave region and \( u_kk^{-3} \) in the capillary wave region, where \( c \) is the phase speed, \( u_k \) is the wind friction velocity, and \( c_\ast \) is the minimum phase velocity of surface waves (of wave component \( k_\ast \)). The dimensionless spectral coefficient \( A \) reaches an asymptotic constant at each end of the capillary–gravity wave spectra, with a magnitude of 0.0023 in the gravity region and 0.021 in the capillary region. In other words, the wavenumber spectrum of the surface displacement can be expressed as (Hwang et al. 1996)

\[
\chi(k) = A\left(\frac{u_k}{c}\right)\left(\frac{c_\ast}{c}\right)k_kk^{-4},
\]

or written in separate functions for the gravity and capillary regions,

\[
\chi(k) = \begin{cases} 
A\cdot g^{-1}u_kc_\ast k_kk^{-3}, & \text{gravity waves,} \\
A\cdot \tau^{-1}u_kc_\ast k_kk^{-3}, & \text{capillary waves,}
\end{cases}
\]

where \( g \) is the gravitational acceleration, \( \tau \) is the ratio of the surface tension and the fluid density, and the subscripts \( g \) and \( c \) refer to the gravity and capillary regimes, respectively. Based on the analysis of the scanning slope sensor data, the spectral coefficient can be written as
Fig. 1. The measured 1D wavenumber spectra of short ocean waves at six wind velocities. (a) Curvature spectrum $\chi_c(k)$. (b) Slope spectrum $\chi_s(k)$. (c) Displacement spectrum $\chi_d(k)$. (d) The wind speed dependence of the surface roughness statistics ($100 < k < 1600$ rad m$^{-1}$), represented by the mean square curvature ($\square$), slope ($\times$), and displacement ($+$). The averaging time for each case is the total data length ($>1000$ s), as listed in Table 1 (only the results from the last four cases are presented in this figure). (e) The wind speed dependence of mean square curvature of short waves ($100 < k < 1600$ rad m$^{-1}$). The averaging data length is 200 s. (f) The curvature spectral density of individual wavenumber components plotted as a function of wind friction velocity, showing the linear relationship for cases with wind speed exceeding a threshold condition. The spectral densities at $k = 100, 300, 700, 900, 1200,$ and $1600$ rad m$^{-1}$ are shown with connecting circles (Hwang et al. 1996).
TABLE 1. Experimental conditions arranged in the order of ascending surface roughness measured by the mean square curvature (Hwang et al. 1996). Key to symbols: Case number convention, identified with four digits, ddhh, where dd represents the date in June 1993 and hh represents the hour (UTC) when the buoy was deployed (subtract 4 from UTC to get local time); Length, total data length in seconds; $s_{so}$, the mean square curvature; $U_{10}$, the wind velocity at 1.33 m above the mean water level (relative to the moving water surface) measured by the wind sensor on the buoy; $u_{rms}$, the rms wind speed fluctuation; $T_a$, the water temperature; Ambient, the ambient light intensity normalized by the signal intensity; $C_{dyn}$, the average current derived from the coordinates of buoy deployment and retrieval; $U_{10}^*$, the equivalent neutral wind speed at 10 m high; $u_{eq}$, the surface wind friction velocity; and $z_0$, the dynamic roughness height.

<table>
<thead>
<tr>
<th>Case</th>
<th>Length (s)</th>
<th>$s_{so}$ (cm$^2$)</th>
<th>$U_{10}$ (m s$^{-1}$)</th>
<th>$u_{rms}$ (m s$^{-1}$)</th>
<th>$T_a$ (°C)</th>
<th>Ambient</th>
<th>$U_{10}^*$ (m s$^{-1}$)</th>
<th>$u_{eq}$ (cm s$^{-1}$)</th>
<th>$z_0$ (mm)</th>
<th>$U_{10}^{**}$ (m s$^{-1}$)</th>
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<td>1.22</td>
<td>0.40</td>
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</table>

$$A = \begin{cases} 
2.3 \times 10^{-3} = A_o, & k \leq 100 \text{ rad m}^{-1}, \\
8.6 \times 10^{-3}k^m_k, & 100 < k \leq 900 \text{ rad m}^{-1}, \\
2.1 \times 10^{-2} = A_c, & 900 \leq k < 1600 \text{ rad m}^{-1}, 
\end{cases}$$

(3)

Combining with the results of gravity wave observations (Phillips 1985), the following spectral model is proposed:

$$\chi_i(k) = \begin{cases} 
I \left( p + \frac{3}{2} \right) \beta g^{1/2}u_{eq}k^{-5/2}, & \text{gravity waves,} \\
A \left( \frac{u_{eq}}{c} \right) \left( \frac{c_m}{c} \right) k_m k^{-2}, & \text{capillary–gravity waves}, 
\end{cases}$$

(4)

where $\beta$ is the spectral constant, $p$ is the parameter of directional distribution, and $I$ is the integration function defined in Phillips (1985). (Note that the emphasis of this paper is on the short-wave components. For more elaborate spectral models in the long gravity wave region, the readers are referred to, e.g., Donelan and Pierson 1987; Apel 1994; Liu and Yan 1995.) Equation (4) can be further represented by four branches in the following wavenumber ranges (in mks units): $(k_o, k)$, $(k, 100)$, $(100, 900)$, and $(900, 1600)$, where $k_o$ is the wavenumber at the spectral peak and can be related to the reference wind speed $(U_{10})$ by $k_o = g/U_{10}^*$ (Pierson and Moskowitz 1964; Phillips 1977), and $k$, is the wavenumber where the gravity wave model and the first branch of the capillary–gravity wave function match. The constant wavenumbers, 100 and 1600 rad $m^{-1}$, are the resolution range of the scanning slope sensor, and 900 rad $m^{-1}$ is the peak of the curvature spectrum, which is found to be relatively invariant with respect to wind conditions (Hwang et al. 1996). The coefficients $\beta$ and $I(p + \frac{3}{2})$ have been discussed in great detail in Phillips (1988) and $A$ in Hwang et al. (1996). The ocean data (both long and short waves) indicate that the magnitudes of these parameters are determinable to within a factor of 2 at best (to be further discussed later). Substituting the mean values from these studies, the proposed spectral model of the surface slope is

$$\chi_i(k) = \begin{cases} 
5.02 \times 10^{-3} u_{eq}k^{-0.5}, & 0 < k \leq k_o, \\
\frac{g}{U_{10}^2} & k_o < k \leq 100 \text{ rad m}^{-1}, \\
2.01 \times 10^{-2} u_{eq}k^{-1}, & 1.97 \times 10^{-3} u_{eq}(g + \pi k^2)^{-1}, \\
100 < k \leq 900 \text{ rad m}^{-1}, \\
2.57 \times 10^{4} u_{eq}k^{-3}, & 900 < k \leq 1600 \text{ rad m}^{-1}, 
\end{cases}$$

(5)

The coefficients in (5) are dimensional and mks units are used. The matching wavenumber $k$, is calculated to be 16 rad $m^{-1}$. It is noted that different choices of the numerical values of $p$ and $\beta$ in (4) will change this matching wavenumber, but the overall conclusion to be presented is not significantly modified. The corresponding spectra of the displacement $\chi(k)$ and curvature $\chi_1(k)$ can be derived from the above equation by dividing and multiplying, respectively, by $k^2$. Figures 2a–c display $\chi(k)$, $\chi_i(k)$, and $\chi_1(k)$ for wind speeds ranging from 1 to 10 m $s^{-1}$.

The contribution to the total mean slope from the four branches in (5) can be calculated by integration:

$$s_1 = 4.02 \times 10^{-2} u_{eq} - 1.09 \times 10^{-3},$$

$$s_2 = 3.68 \times 10^{-2} u_{eq},$$

$$s_3 = 6.88 \times 10^{-2} u_{eq},$$

$$s_4 = 1.08 \times 10^{-2} u_{eq}.$$
if the peak spectral level is extended to \( k = 0 \), thus representing a conservative maximum of the spectral intensity in the long-wave region, the integrated mean square slopes in the range \( 0 < k < k_0 \) is calculated to be \( 5.45 \times 10^{-4} \), a constant value independent of wind speed. Also, if the slope spectrum for \( k > 1600 \) rad m\(^{-1} \) continues in the same \( k^{-3} \) trend, the contribution to the mean square slope of these short capillary waves integrated from \( k = 1600 \) to \( \infty \) is \( 5.02 \times 10^{-3} u_a \). Both values are very small compared to the sum [Eq. (8) below] of the four components listed in (6). In terms of the mean square slopes, the simple four-branch model of (5) therefore captures the dominant contributing components.

The sum of the last two components in (6) is the contribution from capillary–gravity waves and is measured by the scanning slope sensor. Writing \( s_{CG} = s_1 + s_4 \), and \( s = s_1 + s_2 + s_3 + s_4 \), the ratio of \( s_{CG}/s \) is shown in Fig. 2d. The ratios for the six cases (Table 1) analyzed in Hwang et al. (1996) and that served as the database of this study are shown with the circle symbol; the ratios range from 0.52 to 0.68. The large contribution to the mean square slope from the capillary–gravity (CG) waves in the wavenumber range \( \frac{1}{4} < \frac{k}{k_m} < 4 \) is clearly quite significant, explicitly,

\[
s_{CG} = 7.96 \times 10^{-2} u_a, \tag{7}
\]

and the total mean square slope is

\[
s = 1.57 \times 10^{-1} u_a - 1.09 \times 10^{-3}, \tag{8}
\]

The ratio of the two wind speed coefficients in (7) and (8) is 0.51, which is the asymptotic (and the minimal) ratio of the contribution from capillary–gravity wave components to the total mean square slopes at high wind conditions.

Substituting \( u_a = C_D U_{10} \) with \( C_D = 0.0012 \) (Large and Pond 1981; Blanc 1985), (7) and (8) can be expressed in wind speed rather than wind friction velocity:

\[
s_{CG} = 2.76 \times 10^{-3} U_{10}, \tag{9}
\]

and

\[
s = 5.44 \times 10^{-3} U_{10} - 1.09 \times 10^{-3}. \tag{10}
\]

b. Discussion of the wavenumber spectral models

The demand for a functional wavenumber spectrum in the capillary–gravity wave region dates back to at
least four decades when Bragg resonance scattering was identified (e.g., Croomie 1955) and applications of microwave radars for ocean remote sensing were contemplated (e.g., Wright 1966, 1968). This is much earlier than any observations of wavenumber spectra in the capillary–gravity wave region were reported. Pierson and Stacy (1973) provided one of the most widely used spectral models for remote sensing applications. The model was based on a mix of datasets obtained in the field using capacitance gauges (Sutherland 1968) and in the laboratory using an optical slope sensor (Cox 1958). Shemdin and Hwang (1988) showed that the model spectra are considerably different from the field measurements of high-frequency waves derived from a laser slope gauge.

Donelan and Pierson (1987) presented another composite wave spectral model that combines the field measurements of gravity waves from Lake Ontario (Donelan et al. 1985) and short capillary–gravity wave information interpreted from the normalized radar backscattering cross sections from L-, K-, and K-band radars. Comparison of these model spectra with field observations of high-frequency waves was also not favorable (Shemdin and Hwang 1988). It is emphasized that there is considerable difficulty in retrieving the short-wave data from microwave measurements. This will be further discussed in section 4, where significant discrepancies between optical and radar measurements of short-wave properties are illustrated.

With respect to the practice of combining laboratory and field measurements in the database for constructing wave models, scaling is always a factor for concern. Following the fundamental principles of fluid dynamic simulations, surface gravity waves follow the Froude number similarity; that is,

\[ \text{Fr} = \frac{V}{gL_p} = \frac{V}{gL_m}, \]  

(11)

where \( V \) and \( L \) are the velocity and length scales, and the subscripts \( p \) and \( m \) denote prototype and model, respectively. On the other hand, fluid force (wind forcing) requires Reynolds number similarity; that is,

\[ \text{Re} = \frac{V_L}{\nu_p} = \frac{V_L}{\nu_m}, \]  

(12)

where \( \nu \) is the kinematic viscosity of fluids. Clearly, these two similarity laws cannot be satisfied simultaneously—if the wind forcing is simulated correctly, the resulting waves will not be similar, and if the wave conditions are simulated correctly, the wind forcings must be different. It is therefore not reasonable to assume that the datasets from ocean measurements and laboratory experiments can be combined directly. For capillary–gravity waves, there is a third similarity parameter, the Weber number, arising from the influence of surface tension;

\[ \text{We} = \frac{V^2 L_p}{\tau_p} = \frac{V^2 L_m}{\tau_m}, \]  

(13)

To fulfill all three similarity laws is a very difficult, if not impossible, task. It is realized, however, that laboratory simulation is a very effective tool for tasks such as proof of concept, identification of specific mechanisms, and performance of pilot experiments. The products from laboratory simulations, however, are most likely very different from the field results. In fact, a detailed comparison of the wavenumber spectra derived in laboratory wind–wave facilities (Jahne and Riemer 1990; Hwang et al. 1993) with ocean data was presented in Hwang et al. (1996). Many significant differences in the short-wave properties from the two environments were found. The differences include nearly all the major features of wave spectra, including the spectral density level, the threshold wind condition, the wind dependence of mean square roughness, the wind dependence of spectral densities of individual wavenumber components, the apparent spectral slope, and the variation of spectral slopes in the gravity and capillary regions (see Fig. 7 in Hwang et al. 1996 and the related discussion).

3. Mean square slopes of ocean waves

Since the milestone paper of Cox and Munk (1954) on the sea surface statistics from sun glitter measurements, there have been continuous efforts to acquire more detailed information in addition to the mean square slopes of the sea surface. These subsequent measurements can be grouped into two major categories: point measurements and area imaging. Only a few of these ocean measurements reported mean square statistics. Altogether, these datasets with mean square slopes reported include the measurements derived from aerial photographs of sun glitter near the island of Maui (Cox and Munk 1954, 1956); surface slope measurements by a laser slope gauge mounted 10 m ahead of a ship’s bow and collected at the Butte Inlet with the wind fetch ranging from 4 to 77 km (Hughes et al. 1977); surface slope measurements by a laser slope gauge mounted on a mechanical wave-follower, carried out on the North Sea tower during the MARSEN (Marine Remote Sensing) Experiment (Tang and Shemdin 1983) and during the TOWARD (Tower Ocean Wave and Radar Dependence) Experiment offshore of Mission Bay, California (Hwang and Shemdin 1988); and the data from the scanning slope sensor buoy (Hwang et al. 1996) described above. A short description of these datasets is given in Table 2. Due to the large quantity of the scanning slope sensor data, only the windward slope component has been processed. This component of the mean square slope is also reported in the other studies cited above. The average ratio of the windward component to the total mean square slope is found to be 0.57 ± 0.043 in Cox and Munk (1954, 1956), 0.59 ± 0.035 in Hughes
et al. (1977), 0.48 ± 0.071 in Tang and Shemdin (1983), and 0.64 ± 0.078 in Hwang and Shemdin (1983). The larger data scatter of the ratios from the wave-follower measurements may be partially attributed to the flow disturbance by the instrument platform (Dobson 1985), which is carried by a servo motor to compensate for the motion of large waves in the ocean. Although some of the disturbances may be removed by the high-pass procedure performed during the data processing stage, a quantitative estimate of the removal of contamination is not available.

In this section, the ocean data of the windward component and the total mean square slopes are discussed. To obtain the total mean square slope for the scanning slope sensor dataset, the windward component is divided by 0.57. Also, the scanning slope sensor measures the mean square slope in the capillary–gravity band. To recover the total mean square slope, the computed ratios of $s_w/s$ as discussed in section 2a [(7) and (8)] and Fig. 2d are applied. As illustrated in Fig. 3, most of the data points obtained from sun glitter and laser beam refraction are in agreement within a factor of 2 (i.e., within 3 dB). Two groups of data show conspicuous enhancement of the mean square slopes with respect to the mean trend and clearly deviate from the linear wind speed dependence, as discussed below.

a. The lower wind cases of Cox and Munk (1954, 1956).

A closer examination shows that this is due to the greater contribution by longer waves to the mean square slope at low wind conditions. Because these long waves are not locally generated, the wind dependence of the total mean square slope becomes less meaningful at lower wind velocities unless the long-wave component can be filtered out. To illustrate this point, the reported significant wave periods $T_s$ of the four low wind cases in the Cox and Munk (1954, 1956) dataset are compared with the theoretical calculation of the saturation wave periods at these wind speeds (Pierson and Moskowitz 1964; Phillips 1977). The results are listed in Table 3. The large discrepancy in the measured and predicted magnitudes of the significant wave periods strongly supports the suggestion that little correlation with the local wind condition can be expected for these low wind cases, as reflected from the data plotted in Figs. 3a and 3c. Assuming the mean square slope of the long swells is $(KH)^{5/8}$, where $K$ and $H$ are the wavelength and wave height of the long waves, the contribution of swells can be removed. The swell period and wave height (except for one case at $U_{10} = 0.84$ m s$^{-1}$) were tabulated in the original paper (Table 1 of Cox and Munk 1954) to facilitate the computation. The residual mean square slopes of these low wind cases fall within the expected factor-of-2 envelope (Figs. 3b and 3d).

The measurements from the scanning slope sensor show considerably less data scatter with wind speed over the range from 0.7 to 6 m s$^{-1}$. Because detrending of the scanning slope data was performed in the space domain over the 6.2-cm scanning distance, the contribution from longer waves was largely removed. The results reveal that the linear wind dependence extends to wind speeds as low as 0.7 m s$^{-1}$.

b. A small group of MARSEN (Tang and Shemdin 1983) and TOWARD data (Hwang and Shemdin 1988) near $U_{10} = 5$ m s$^{-1}$.

This is the “mixed-sea” case defined in Tang and Shemdin (1983), but the exact mechanism for the enhanced surface slopes is unknown. It is noticed that the wind dependence of the mean square slope derived from wave-follower measurements resembles the response curve of a mechanical system with a resonance frequency near 0.3 Hz, which is approximately the peak frequency of ocean waves at 5 m s$^{-1}$. It is possible that mechanical resonance of the instrument platform may have contributed to the observed anomaly in the wave-follower datasets.

From least squares fitting of the data points in Figs. 3b and 3d, which excluded the wave-follower datasets and corrected the swell effect in the Cox and Munk dataset, the total mean square slope and the windward component can be represented by the following two equations:

$$s = 5.12 \times 10^{-3} U_{10} + 1.25 \times 10^{-3},$$  \hspace{1cm} (14)

and

$$s_w = 3.04 \times 10^{-3} U_{10} + 3.41 \times 10^{-4},$$  \hspace{1cm} (15)

Both equations are accurate to within 1.5 dB (i.e., $\sqrt{2}$), based on comparison with available ocean data. Also plotted in these figures are the predicted mean square sea surface slopes (solid curves) using the wave-number spectral model presented in section 2 [(9) and

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<th>Source</th>
<th>Location</th>
<th>Method</th>
<th>Wind range (m s$^{-1}$)</th>
</tr>
</thead>
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<td>Maui, Hawaii</td>
<td>Aerial photography of sun glitter</td>
<td>0.68–13.5</td>
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<tr>
<td>Hughes et al. (1977)</td>
<td>Bute Inlet, Canada</td>
<td>Bow-mounted laser slope gauge</td>
<td>3.5–7.7</td>
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<td>Hwang and Shemdin (1988)</td>
<td>Mission Bay Tower, California</td>
<td>Wave-follower laser slope gauge</td>
<td>2.5–6.6</td>
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<td>Hwang et al. (1996)</td>
<td>Atlantic Ocean near Gulf Stream</td>
<td>Scanning slope sensor buoy</td>
<td>0.47–6.0</td>
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</tbody>
</table>
FIG. 3. Field measurements of the windward component of the mean square slope (a, b) and total mean square slope (c, d) as functions of wind speed. The symbols used in all the plots are ○: Cox and Munk (1954, 1956); *: Hughes et al. (1977); ⊕ or ⊙: Tang and Shemdin (1983); ×: Hwang and Shemdin (1988); +: the scanning slope sensor data (Hwang et al. 1996) from the current processing. The factor-of-2 envelope is shown with dashed curves. The solid curve represents the integrated mean square slopes of the proposed spectral model (9). In (b) and (d) the swell effect in the Cox and Munk (1954) dataset is removed; also, the measurements of the wave-follower experiments are excluded from these two plots due to the concern of possible contamination from mechanical resonance of the instrument frame, as described in the text.

TABLE 3. Comparison of the measured and predicted significant wave periods of the four low wind cases (clean water) of Cox and Munk (1954, 1956).

<table>
<thead>
<tr>
<th>$U_{10}$ (m s$^{-1}$)</th>
<th>$U_{19}$ (m s$^{-1}$)</th>
<th>Measured $T_s$ (s)</th>
<th>Predicted $T_s$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>0.68</td>
<td>3</td>
<td>0.44</td>
</tr>
<tr>
<td>0.89</td>
<td>0.84</td>
<td>not reported</td>
<td>0.53</td>
</tr>
<tr>
<td>1.33</td>
<td>1.77</td>
<td>4</td>
<td>1.1</td>
</tr>
<tr>
<td>1.39</td>
<td>1.27</td>
<td>4</td>
<td>0.81</td>
</tr>
</tbody>
</table>

* $U_{10}$ is the wind speed measured at a height of 12.5 m (41 ft), as reported in Cox and Munk (1954, 1956).

These curves are in excellent agreement with the ocean data.

4. Comparison of optical and radar measurements

When reviewing literature on the measurements of short ocean waves a disturbing pattern emerges, which reveals that the short waves interpreted by radars (altimeter and scatterometer) are very different from the short waves measured by in situ optical sensors (aerial photographs, laser slope gauge, and scanning slope sensor). Two examples are presented below.

a. Mean square slope

A detailed review of the derivation of mean square slope from the altimeter output is given in Brown (1990). It is shown that the specular scattering cross section is related to the filtered surface slopes. Theoretically, the bandwidth of surface waves contributing to the optical measurements of Cox and Munk (1954, 1956) is broader than that contributing to radar altimetry. Measurements from GEOS-3, Seasat, and Geosat, however, show that the radar-derived mean square slope is considerably higher than the optical sensor results (Brown 1990). Jackson et al. (1992) analyzed the data...
from a K_u-band airborne radar (ROWS) operated in the nadir-looking altimeter mode. The mean square slope was derived from the “gross-fit” of return-pulse waveforms. Jackson et al. (1992) also compared ROWS’ altimeter results with airborne scatterometer data (Wentz 1977) and the linear equation given by Cox and Munk (1954). Figures 4a, 4b, and 4c plot the original data of the optical and radar measurements in separate panels to illustrate the coherence of optical measurements and the contrasting discrepancies of radar data. The spaceborne altimeter data of Brown (1990) were digitized from his Fig. 7, and the airborne altimeter and scatterometer data reported in Jackson et al. (1992) were digitized from his Fig. 4a. It is reemphasized that all the mean square slope datasets collected in the ocean, using techniques that are well understood and with wind speed and other environmental parameters simultaneously measured with the slope data, are in good agreement. The optical data points are mostly confined within the factor-of-2 envelope. In contrast, radar-derived results may have large discrepancies. For example, the magnitudes of the resulting mean square slopes derived from the airborne altimeter are found to be about half of those obtained from the spaceborne altimeters. The discrepancies between the airborne and spaceborne operations is partially attributed to the diffraction effect that was considered in Jackson et al. (1992) but not in Brown (1990).

Because wind measurements from satellite data are generally much less accurate (with large spatial and temporal lags) than those available from optical experiments, it is more difficult to judge the accuracy of the physical mechanisms used in the derivation of the sea surface slope information from satellite data. For example, for the spaceborne data discussed above, the spatial lag (with one standard deviation) between the satellite data and surface wind measurements is 30.1 ± 23.2 km, and the temporal lag is 0.56 ± 0.42 h (Brown 1981, 1990). For small-scale phenomena such as mean square slopes and other short-wave characteristics, such spatial and temporal lags are too large, and a high correlation between the satellite data and surface wind measurements cannot be expected. The correlation problem is more serious under low wind conditions, in which...
the effects of spatial and temporal fluctuations of the wind conditions become even more pronounced. This is reflected in the larger scatter of satellite data for \( U_{10} \) less than approximately 4 m s\(^{-1}\), as shown in Fig. 4b. Also, as already mentioned in the optical data discussion (section 3), the contribution of long swells to the total mean square slopes becomes more important at lower wind velocities. Because these long swells are not local-wind generated, the correlation between the measured mean square slope and the local wind velocity at low wind speed is not meaningful, unless the long-wave contributions are removed.

For airborne remote sensing experiments, high-quality measurements of the environmental parameters, including wind, wave, humidity, and temperatures of air and water, are generally available. As reported in Jackson et al. (1992), the mean square slopes derived from airborne altimeters and scatterometers under high wind conditions (\( U_{10} > 8 \) m s\(^{-1}\)) show a linear wind dependence and are in very good agreement with optical measurements; at lower wind speeds (\( U_{10} < 8 \) m s\(^{-1}\)), airborne (scatterometer) data display a very different wind dependence (Fig. 4c). The change of wind speed dependence in the ranges \( U_{10} < 8 \) m s\(^{-1}\) and \( U_{10} > 8 \) m s\(^{-1}\) is a common feature of radar measurements, as illustrated in Fig. 4d, where both satellite and airborne data are combined in the same graph. For reference, the envelopes of optical observations (Fig. 4a) are shown with dashed lines in the same plot. Note the linear wind dependence of the optical measurements and the obvious deviation from the linear trend in both the airborne and spaceborne radar data in the low wind speed range.

Jackson et al. (1992) compared the airborne mean square slope data with the filtered mean square slope computation using Wu’s formula (Wu 1972). The two-segment equation proposed by Wu (1972) was based on the argument that the wave properties in a smooth flow or a transitional flow regime are distinctly different from those in a rough turbulence regime. It was suggested that the data of Cox and Munk (1954), when displayed in a semilogarithmic plot, show a two-segment structure in support of the flow regime argument. As has been discussed in section 2, large swells may distort the correlation of mean square slope and local wind conditions, especially at low wind conditions. Also, when the swell contribution is removed, the linear wind dependence governs the mean square slope over the entire range of wind speeds reported (Figs. 3b, 3d) without any hint of a two-segment structure. The filtered mean square slope based on the spectral model described in section 2 is also linear in the wind speed dependence. For example, from (5) or (6), the filtered mean square slope for \( k < 100 \) rad m\(^{-1}\) [the sum of \( s_1 \) and \( s_2 \) in Eq. (6)] is

\[
s_f = 2.67 \times 10^{-3} U_{10} - 1.91 \times 10^{-3}, \quad (16)
\]

or

\[
s_f = 7.74 \times 10^{-2} u_w - 1.09 \times 10^{-3}. \quad (17)
\]

These results are very different from the mean square slope derived from radar measurements. Although the data in low wind conditions may be affected by long swells, it is somewhat surprising to see that the effect may reach to \( U_{10} \) as high as 8 m s\(^{-1}\) (Fig. 4d). For optical measurements, the swell effect appears to diminish for \( U_{10} > 2 \) m s\(^{-1}\), based on examining the available field data (Fig. 3). Hwang et al. (1997) analyze the correlation of TOPEX/Poseidon altimeter backscattering cross sections and simultaneous buoy wind measurements and suggest that the effect of long surface waves tilting short scattering waves may contribute to an attenuation of the received radar backscatter from the ocean surface. They show that the level of attenuation is consistent with the observed increase of the apparent surface roughness derived from radar cross sections. This is a more likely explanation of the discrepancy between radar and optical measurement of the ocean surface roughness.

b. Wind speed dependence

Jones and Schroeder (1978) summarize the observed dependence of the normalized scattering cross section \( \sigma_0 \) on the wind speed \( U \). They note that the data generally follow the power-law relationship

\[
\sigma_0 = a U^b. \quad (18)
\]

The observed \( b \) varies from 0.5 for long Bragg waves (\( \lambda_B \approx 0.7 \) m) to 2 for short Bragg waves (\( \lambda_B \approx 0.02 \) m). This was sometimes interpreted as being due to the fact that the short waves in the ocean are more sensitive to wind than are the longer waves (e.g., Jones and Schroeder 1978; Stewart 1985). Subsequent investigations on the \( b-\lambda_B \) relationship (Masuko et al. 1986; Phillips 1988; Weissman et al. 1994; Colton et al. 1995) reconfirm that the radar-measured \( b \) indeed increases rapidly toward higher Bragg wavenumbers (Fig. 5). It is worthwhile to point out that both the magnitude and the trend of these wind speed exponents are very different from those observed from direct ocean wave measurements. It is well established that the total energy in the wave field is proportional to the fourth-power of the wind speed (e.g., Hasselmann et al. 1973; Phillips 1977). Because the wave spectrum is narrowbanded, this suggests that the spectral energy near the peak frequency increases with the fourth power of the wind speed, that is, \( b = 4 \) for long waves at the spectral peak. On the forward face of the wave spectrum within the gravity wave regime, the spectral energy is found to increase linearly with wind (Phillips 1985), that is, \( b = 1 \) for short gravity waves. There is a strong indication that the linear wind dependence extends into the short capillary wave region as suggested by the measurements of mean square slopes (mainly contributed by short capillary–gravity waves, see Fig. 2d), which consistently show a linear wind dependence (Cox and Munk 1954; Hughes et al. 1977; Tang and Shemdin 1983; Hwang and Shemdin 1988), as summarized in Fig. 2. The spatial
measurements from the scanning slope sensor reported earlier ultimately confirm that the spectral densities of individual water wave components indeed increase linearly with wind speed (Hwang et al. 1996). This statement is valid at least for the wavenumber range resolved by the scanning slope sensor, \(100 < k < 1600 \text{ rad m}^{-1}\), and within the measured wind speed conditions, \(0.7 < U_{10} < 6 \text{ m s}^{-1}\). It is concluded that the interpretation that the short waves are more sensitive (in terms of the wind speed exponent) to wind than the longer waves, as apparent from the radar cross-sectional measurements, is inaccurate. Phillips (1988) presents a detailed analysis of the wind dependence of the radar backscatter cross section and concludes that non-Bragg scattering mechanisms make significant contributions to the radar return from the ocean surface. These non-Bragg mechanisms are closely related to surface wave breaking events. The wind dependence of breaking intensity and breaking frequency are observed to be approximately cubic and much stronger than linear (e.g., Hwang et al. 1989; Jessup et al. 1990), which may explain the larger value of \(b\) in the radar return data.

As a final comment, the data in Fig. 5 show very large scatters. For example, the range of the \(K_a\)-band exponents (circles with plus sign) is from 0.8 to 2.9, extending throughout the full range of the wind exponents observed from all frequency bands. Also, a fair fraction of the data points have exponent values less than one. These “sub-Bragg” data points spread over the full range of Bragg wavenumbers reported, and they cannot be reconciled by the addition of a breaking contribution to the Bragg scattering mechanism. Obviously, clarification of the fundamental mechanisms of radar backscatter from the ocean surface is still not satisfactory.

5. Summary and conclusions

Spatial measurements of ocean surface slopes were acquired from the ocean using a scanning slope sensing buoy. Excellent agreement of the mean square slope with earlier results, including the measurements of Cox and Munk (1954, 1956) and Hughes et al. (1977), reconfirms that the mean square slopes of small-scale waves increase linearly with wind speed. The spatial filtering inherent in the scanning slope sensing technique permits removal of the long-wave contribution to the mean square slopes, which is particularly significant in the low wind conditions (Fig. 3), as found in the measurements of Cox and Munk (1954, 1956). When the swell effect is removed, all the field results show remarkable agreement following a linear wind speed dependence, and most data points fall within the factor-of-2 envelope.

The wavenumber spectra were processed from these direct spatial measurements of short waves in the ocean. Using these results as the database for the capillary–gravity waves and the results of gravity wave research summarized in Phillips (1985), a wavenumber spectral model was constructed, covering the wavenumber range from active wind generation, with wavelengths on the order of 100 m, to capillary waves, with wavelengths on the order of 4 mm. The contribution to the total mean square slope from capillary–gravity wave components
with $100 < k < 1600 \text{ rad m}^{-1}$ was found to be between 0.51 and 0.68. The above wavenumber range corresponds to approximately $\frac{1}{4} < k/k_o < 4$.

The short-wave measurements by optical sensors were then compared with those derived from microwave sensors. Significant differences from the two sensing techniques were found in the mean square slopes and the wind dependence of the spectral density of individual wave components. Although some of the discrepancies can be attributed to the significant time lag or space lag between space data and surface wind measurements, the disagreements between airborne radar measurements and in situ optical observations indicate that the sources of discrepancies are at a more fundamental level. Improvements are needed to clarify the backscatter mechanisms and the statistical properties of short water waves that serve as the boundary condition for the radar wave scattering problem.

Acknowledgments. This work is sponsored by the Office of Naval Research under Contract N000-2496WX3044, NRL Job Order 73-7075-06 and 73-6800-07. The development of the scanning slope sensor buoy was sponsored by the Naval Research Laboratory under Contract N00173-91-M-7602.

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