NOTES AND CORRESPONDENCE

Considerations on Open Boundary Conditions for Regional and Coastal Ocean Models

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ABSTRACT

This paper reviews the usual open boundary conditions (OBCs) for coastal ocean models and proposes a complete set of open boundaries based on stability criteria, on mass and energy conservation arguments, and on the ability to enforce external information. This set includes a radiation condition for barotropic variables, an equation of wave propagation for baroclinic velocities, and an advection equation for tracers. Considerations on mass and energy conservation properties suggest a suitable numerical treatment of the barotropic scheme, which is different from what is commonly used. Restoring terms, when classically added in the Sommerfeld OBCs, are not consistent with external fields. It is shown that this can be avoided if proper numerical schemes are used or if OBCs are applied on differences between the model and forcing rather than on absolute variables. Finally, this paper shows that simplistic advection-type methods for temperature and salinity should not be used in sigma coordinate models because this introduces errors in the computation of the horizontal pressure gradient.

1. Introduction

In coastal ocean modeling, open boundary conditions (OBCs) have a crucial impact on the inner domain solution. This is largely due to the fact that time scales associated with the propagation of waves throughout the coastal area are comparable to the length of the simulation itself, when they are not much shorter. A barotropic wave can, for instance, cross a 100-m-deep and 100-km-long shelf in about 1 h and a 1 m s^{-1} internal wave can cross it in about 1 day. This is much shorter than the length of a classical forecast, which is at least a week long. Moreover, it is well known that OBCs lead to basically ill-posed problems (Oliger and Sundström 1978) and that perfect OBC schemes do not exist. Actually, the OBC problem can be considered as one of the most challenging aspects of coastal modeling.

Schematically, OBCs have a double purpose. They are first of all required to force the inner solution with external fields (obtained from observations or large-extent models) under incoming conditions. At the same time, they should allow waves to radiate out or water masses to leave the modeling domain under outgoing conditions, without any spurious reflections. But it is difficult to satisfy these two objectives simultaneously and modeling systems tend to choose locally between one scheme or the other according to the incoming or outgoing character of the dynamical field. The direction of wave propagation is often obtained by inversion of an equation of "wave propagation" based on model variables in the neighborhood of open boundaries. The incoming or outgoing character of the local dynamics is related to the sign of the computed phase speed in the direction normal to the boundary (Orlanski 1976). This method is of course questionable as a large spectrum of waves can propagate through coastal areas, and their main characteristics (phase speed, direction, or dispersion) are poorly taken into account by usual methods. Such methods are generally optimal for trivial cases such as single nondispersive waves propagating in a direction normal to the boundary. This could prove to be a severe limitation as recent studies (Marchesiello et al. 2001) rather suggest that the ability of modeling systems to distinguish outgoing from incoming regimes (in other words their ability to choose a proper OBC scheme), could be as important as the OBCs themselves.

Most of the OBCs have been largely studied in the

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past 20 years (Palma and Matano 1998, 2000). These studies provide a comparison of several OBCs in well-identified cases (free wave propagation, wind-induced coastal jets, traveling storms, etc.) for which reference solutions exist (either analytical solutions or numerical solutions obtained from the same model but with cyclic boundary conditions or using a larger domain). Briefly, the most popular schemes can be divided into three classes (Palma and Matano 1998): 1) radiative conditions (Blumberg and Kantha 1985), 2) characteristic methods (Hedstrom 1979), 3) relaxation methods (Martinsen and Engedahl 1987). The goal of this paper is not to add another set of tests to the already numerous existing studies dedicated to this topic but to bring up some new considerations on a particular set of OBCs that now seem to be used by a large community of coastal modelers. In practice, we selected the following OBC: the radiation method proposed by Flather (1976) for barotropic variables, radiative methods for baroclinic velocities, and advection schemes for tracers. Mass and energy conservation properties, the reliability of adaptative methods, restoring terms limitations, and potential traps associated with the use of sigma coordinates are examined and, accordingly, suitable numerical treatments are suggested.

This paper is mainly concerned with the widely used, free surface sigma coordinate model, such as the Princeton Ocean Model (POM; Blumberg and Mellor 1987) or the Regional Ocean Modeling System (ROMS; Shchepetkin and McWilliams 2005). Model equations are given in section 2. OBCs for barotropic variables are discussed in section 3. Conservation of mass and energy is examined in sections 4 and 5. OBCs for baroclinic velocities and tracers are discussed in sections 6 and 7. Section 8 is concerned with buffer zones in which restoring conditions toward prescribed fields can be applied. Finally, possible developments to improve present OBCs are discussed in section 9.

2. Model equations

Most numerical coastal ocean circulation studies are based on models solving the Navier–Stokes equations on a vertically staggered C grid using classical finite-difference methods. These models generally have a free surface and sigma terrain-following coordinate, and use the Boussinesq and hydrostatic approximations. In this paper, modeling is carried out with the Symphonie model, a description of which, together with recent examples of applications, can be found in Ulses et al. (2005), Pairaud and Auclair (2005), Petrenko et al. (2005), and Estournel et al. (2005). With the exception of the variables of the turbulence closure scheme (Gaspar et al. 1990), this model is based on six variables, (u, v, w, T, S, η), computed using the following equations.

Momentum and continuity equations are given by

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial vu}{\partial y}+\frac{\partial wu}{\partial z}-fv=\frac{-1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left[K_v\left(\frac{\partial u}{\partial z}\right)\right]+\frac{\partial}{\partial x}\left[K_h\left(\frac{\partial u}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[K_h\left(\frac{\partial u}{\partial y}\right)\right], \tag{1}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial wv}{\partial z}+fu=\frac{-1}{\rho_0}\frac{\partial p}{\partial y}+\frac{\partial}{\partial z}\left[K_v\left(\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial x}\left[K_h\left(\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[K_h\left(\frac{\partial v}{\partial y}\right)\right], \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{3}$$

The hydrostatic pressure is given by

$$p(z)=\int_z^\eta g\rho\,dz. \tag{4}$$

The free surface elevation anomaly η is deduced from the divergence of the depth-averaged current ($\overline{u}$, $\overline{v}$):

$$\frac{\partial \eta}{\partial t}+\frac{\partial (h+\eta)\overline{u}}{\partial x}+\frac{\partial (h+\eta)\overline{v}}{\partial y}=0, \tag{5}$$

where h is the ocean depth. The density of water ρ is related to the temperature T and the salinity S, which are given by

$$\frac{\partial T}{\partial t}+\frac{\partial uT}{\partial x}+\frac{\partial vT}{\partial y}+\frac{\partial wT}{\partial z}=\frac{\partial}{\partial z}\left[K_v\left(\frac{\partial T}{\partial z}\right)\right]+\frac{1}{\rho C_p}\frac{\partial I_s}{\partial z}, \tag{6}$$

$$\frac{\partial S}{\partial t}+\frac{\partial uS}{\partial x}+\frac{\partial vS}{\partial y}+\frac{\partial wS}{\partial z}=\frac{\partial}{\partial z}\left[K_v\left(\frac{\partial S}{\partial z}\right)\right], \tag{7}$$

where $\partial I_s/\partial z$ stands for the solar radiation forcing. The vertical diffusivities K_v are computed using the turbulence closure scheme proposed by Gaspar et al. (1990). The constant horizontal viscosity K_h is set to 15 $m^2\,s^{-1}$ in the application of section 4. Barotropic ($\overline{u}$, $\overline{v}$) and baroclinic (u', v') components of the current ($u=\overline{u}+u'$, $v=\overline{v}+v'$) are computed separately using the time-splitting technique described by Blumberg and Mellor (1987).

3. Barotropic boundary conditions

We chose the Flather condition (Flather 1976) that, according to the conclusions of the numerous studies previously mentioned, seems to be the best compromise among the usual barotropic OBCs. Several authors (Blayo and Debreu 2005; Shulman and Lewis 1995; Palma and Matano 2001) have recently commented on the interesting properties of mass and energy conservation of this scheme. Our numerical approach to the barotropic OBCs took these considerations into account and led to a new implementation of the Flather condition. We propose to apply this condition to the surface elevation anomaly rather than to the velocity. To illustrate this in a simple way, we consider here the equations for a barotropic linearized case in the x direction only, namely:

$$\frac{\partial \bar{u}}{\partial t} = -g\frac{\partial \eta}{\partial x}, \qquad (8)$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial U}{\partial x}, \qquad (9)$$

with open boundaries at $x = 0$ and $x = L$. Note that our hypothesis of linearization also leads us to neglect the sea surface elevation term in the integrated continuity equation (i.e., $U = h\bar{u}$). Using the same leapfrog time-stepping and centered gradient schemes as in our model, we obtained the following numerical expressions for (8) and (9):

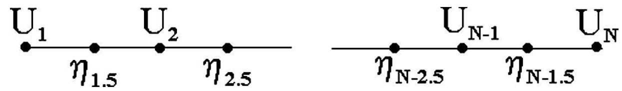

FIG. 1. Sketch of the C grid in the Ox direction.

$$\frac{\bar{u}_i^{t+1} - \bar{u}_i^{t-1}}{2dt} = -g\frac{\eta_{i+1/2}^t - \eta_{i-1/2}^t}{dx}, \qquad (10)$$

$$\frac{\eta_{i+1/2}^{t+1} - \eta_{i+1/2}^{t-1}}{2dt} = -\frac{U_{i+1}^t - U_i^t}{dx}. \qquad (11)$$

Equation (10) was computed from $i = 2$ to $i = N - 1$ with boundary conditions for $\bar{u}$ at $i = 1$ and $i = N$. This computation was carried out over a horizontally staggered C grid where sea surface elevation and current grid points were shifted (Fig. 1). An integrated energy balance, from the initial time and over the whole domain, can be obtained from Eq. (8) once multiplied by U (Gill 1982):

$$\int_0^L [U\bar{u}/2 + g\eta^2/2]_{t_0}^t\, dx = -g\int_{t_0}^t [U\eta]_{x=0}^{x=L}\, dt', \qquad (12)$$

where (9) was used to compute the potential energy term (second term on the left-hand side). Equation (12) means that the global variation of mechanical energy (left-hand side) is equal to the integral over time of the potential energy fluxes through the open boundaries (right-hand side). A numerical expression of (12), based on (10) and (11), is given by

$$\sum_{i=2,N-1}[h_i\bar{u}_i^{t'-1}\bar{u}_i^{t'}/2]_{t_0}^{t+1}dx + \sum_{i=2,N-2}[g\eta_{i+0.5}^{t'-1}\eta_{i+0.5}^{t'}/2]_{t_0}^{t+1}dx = -g\sum_{t'=t_0,t}(U_{N-1}^{t'}\eta_{N-0.5}^{t'} - U_2^{t'}\eta_{1.5}^{t'})dt. \qquad (13)$$

We can see that a sea surface elevation OBC such as $\eta_{N-0.5} = U_{N-1}/c$ and $\eta_{1.5} = -U_2/c$ with $c = \sqrt{gH}$ (i.e., a Flather condition on surface elevation anomaly with no external forcing terms) would make the right-hand side of (13) always negative, therefore enhancing the stability of simulations since OBCs could in no way be responsible for an unexpected increase in global energy. We finally retained this scheme but with a slight modification in order to define an additional boundary condition for U_1 and U_N [required when horizontal gradients related to momentum advection and diffusion are restored in Eq. (10)]. At the same time we expect the scheme to satisfy the local mass conservation constraint, namely, Eq. (11). In practice, currents at open boundaries are obtained from Eq. (11) together with the so-called Flather scheme, but applied to sea surface elevation taken at iteration $t + 1$:

$$\begin{aligned}
U_1^t &= U_2^t + \frac{dx}{2dt}\eta_{1.5}^{t+1} - \frac{dx}{2dt}\eta_{1.5}^{t-1} \\
&= U_2^t - \frac{dx}{2dt}\frac{U_2^t}{c} - \frac{dx}{2dt}\eta_{1.5}^{t-1} \\
U_N^t &= U_{N-1}^t - \frac{dx}{2dt}\eta_{N-0.5}^{t+1} + \frac{dx}{2dt}\eta_{N-0.5}^{t-1} \\
&= U_{N-1}^t - \frac{dx}{2dt}\frac{U_{N-1}^t}{c} + \frac{dx}{2dt}\eta_{N-0.5}^{t-1}.
\end{aligned} \qquad (14)$$

Equation (14) is usually more complicated for two reasons. First, the Flather condition must take external forcing into account

$$\eta = \eta_F \pm (U^N - U_F^N)/c, \qquad (15)$$

where U^N is the transport in the direction normal to the boundary and F refers to external forcing terms. Second, the gradient in the direction tangential to the boundary of the tangential component of transport, which we intentionally omitted in (11), in fact reappears in the right-hand side of (14). A boundary condition for the tangential component of the transport, is given by

$$\frac{\partial U^T}{\partial n} = \frac{\partial U_F^T}{\partial n}, \qquad (16)$$

where n refers to the direction normal to the open boundaries. We see that OBCs (15) and (16) result in the radiation of the difference between computed and forcing fields out of the model domain as recommended by Blayo and Debreu (2005) or Perkins et al. (1997). We investigate this point in sections 6 and 7 for three-dimensional OBCs. Although many authors (Marchesiello et al. 2001; Palma and Matano 2001) apply the Flather condition to the normal component of transport and complete their scheme with two other radiative conditions for the tangential component of transport and sea surface elevation, we see that considerations on energy and local mass conservation actually suggest that the Flather condition should be applied to sea surface elevation and the scheme completed with one radiative condition on the tangential component of transport, the normal component then being deduced from the previous pair of variables thanks to the continuity of Eq. (5).

4. Global mass conservation

It is well known that OBCs sometimes do not conserve global mass in a reasonable way. Blumberg and Kantha (1985) reported a mean sea surface elevation drift of several centimeters after a few days of simulation, using the Sommerfeld boundary conditions. That shortcoming was circumvented by adding to the wave equation a restoring term toward some control variable. Some other authors have suggested the constraint that the integral of the normal component of transport over the open boundaries as a whole must vanish—this condition keeping the mean sea surface elevation strictly unchanged (Marchesiello et al. 2001).

Although global mass cannot be exactly conserved with a Flather condition, we believe that this OBC nevertheless has interesting mass conservation properties and could be used without the aforementioned mass correction. To illustrate this point of view, we consider now a mass balance based on the integral of (5) over the numerical area, using the Green–Ostrogradsky property to simplify the divergence of the transport integral:

$$\int\int \frac{\partial \eta}{\partial t}\, dxdy = -\oint_L \mathbf{U}\cdot\mathbf{n}\, dl, \qquad (17)$$

where the right-hand side symbol stands for a closed integral along the whole boundary ($\mathbf{n}$ being a unit outward vector) and the left-hand side integration gives the global variation of sea surface elevation over the domain area S. We can then define the mean sea surface elevation by $\overline{\eta} = (1/S)\int\int \eta\, dx\, dy$ and a sea surface anomaly η' such that $\eta = \overline{\eta} + \eta'$. To make the sea surface elevation appear on the right-hand side of (17) we use a Flather condition with a constant surface elevation forcing term but zero forcing current (i.e., $\eta - \eta_F = \pm\, U/c$, such an external field could for instance correspond to an "inverse barometer" type of response of the ocean to atmospheric pressure). Finally, we suppose that bathymetry is constant. Thus, (17) now reads

$$\frac{\partial \overline{\eta}}{\partial t} + \frac{L}{S} c\overline{\eta} = \frac{L}{S} c(\eta'_B + \eta_F), \qquad (18)$$

where L is the whole length of the open boundary and η'_B is the sea surface anomaly at the open boundary. In general cases (18) should not admit trivial solutions. If the size of our numerical domain is small compared to the barotropic deformation radius (this assumption does not seem unreasonable considering our coastal context), sea surface elevation should be nearly equal to its domain-averaged value. In that case, we can consider that η'_B is small enough to be neglected in the right-hand side of (18). A solution for the mean sea surface elevation is then given by

$$\overline{\eta} = \eta_F + \eta_0 e^{-(Lct/S)}. \qquad (19)$$

Equation (19) means that if our run begins with an initial mismatch η_0 on sea surface elevation, the mean sea surface elevation will quickly approach the external value, the initial difference being divided by 10 after a time roughly equal to $2S/Lc$ (or approximately half an hour for $S = 10\,000$ km^2, $L = 400$ km, $H = 100$ m). We have verified on different academic cases that (19) is in reasonable agreement with computed surface elevation. Figure 2 shows that (19) is still relevant for the realistic and complex bathymetry of the northwestern Mediterranean. In this case, the time scale for mass conservation adjustment S/Lc is about 0.7 h.

5. Global energy conservation

It is generally considered that models perform better when global conservation properties are respected. OBCs have a significant impact on the global mecanical energy budget through the boundary potential energy

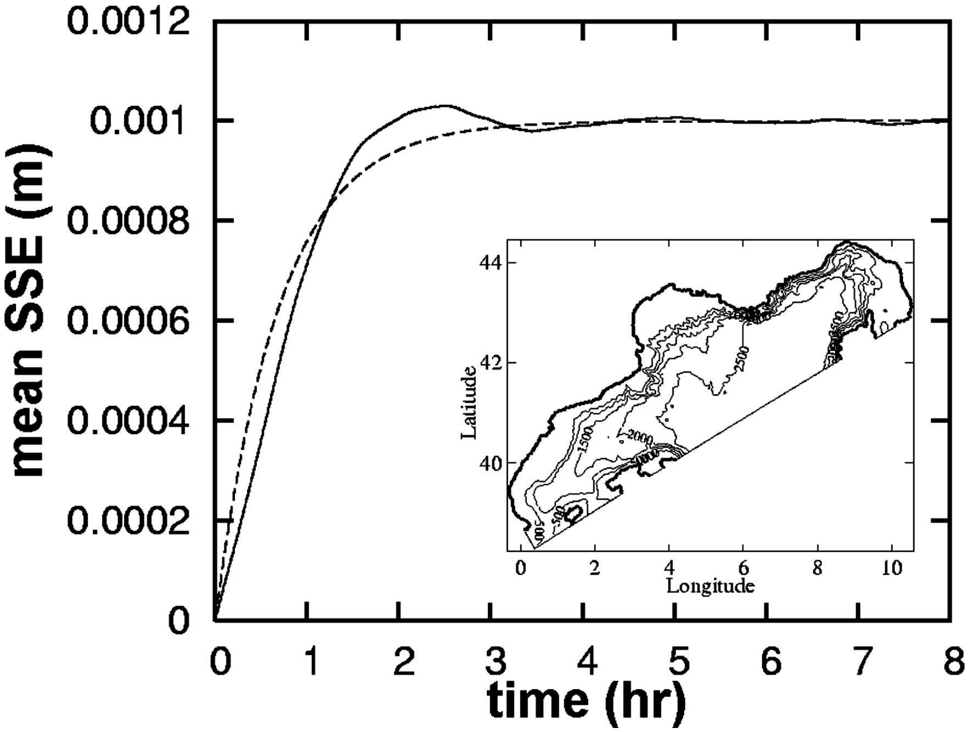

FIG. 2. Mean sea surface elevation computed during an 8-h barotropic run initialized at rest and forced at open boundaries by a sea surface elevation of 0.001 m and zero current (solid line), and corresponding analytical solution computed from Eq. (19) using a domain-averaged value of the barotropic wave celerity (dotted line). The insert presents the domain modeled in the northwestern Mediterranean.

flux. Shulman et al. (1998) and Shulman (1997) showed for instance that simulations of tidal- and wind-driven coastal circulations were clearly improved when a constraint on the boundary energy flux was applied.

If a general form of the Flather condition is used (i.e., a form including external forcing terms) potential energy fluxes at open boundaries are given by

$$-gU\eta = -gU_F\eta_F - gc\eta'^2 - g\eta'(U_F + c\eta_F), \tag{20}$$

where $\eta' = \eta - \eta_F$ is the difference between the computed sea surface elevation and its external counterpart. Note that for sake of simplicity only one open boundary (at $x = L$) is considered. In an idealized case where external field is constant and where η' consists of periodic waves, the last term on the right-hand side of (20) becomes negligible when integrated over a long time:

$$-g\int U\eta\, dt \approx -g\int U_F\eta_F\, dt - gc\int \eta'^2\, dt. \tag{21}$$

As the second term on the right-hand side of (21) is always negative, the cumulated energy flux never exceeds its external counterpart, suggesting that the Flather condition should prevent, in this simple case, the spurious increase of global energy.

More realistic situations should be less favorable. Under outgoing conditions, the circulation in the vicinity of open boundaries is more influenced by interior dynamics than boundary forcing terms and thus the model can drift from the external field. The surface elevation anomaly η' eventually contains a part of the general circulation and the last term on the right-hand side of (20) is no longer negligible when integrated over time. This means that the Flather condition in (15) is likely to lead to erroneous potential energy fluxes in cases of outward propagation. Actually, adaptative versions of characteristic propagation methods, omitting external forcing terms when the latter correspond to an outgoing regime (i.e., when we have $g\eta_F\mathbf{U}_F \cdot \mathbf{n} > 0$), are eventually chosen, especially in the case of tidal modeling since the propagation properties of the tidal forcing can be rather simple and reliable (Ruddick et al. 1994). Generally, the greatest difficulty comes from the impossibility of distinguishing between active and passive OBCs. Indeed, realistic situations are rather a complex mixture of incoming and outgoing waves and, in most cases in (15), ends up being applied, just as it is, whatever the boundary grid node. Nevertheless, potential energy fluxes can be balanced if necessary, by using methods similar to those controlling the mass budget for instance. A first estimate of the sea surface elevation η^* provided by (15) can indeed be adjusted in order to keep the global boundary potential energy flux (GBPEF) under a threshold value, namely, the GBPEF calculated with the corresponding external variables:

$$\eta = \eta^* + \gamma \mathbf{U}\cdot\mathbf{n}$$
$$\gamma = -\frac{\oint_L (\eta_F \mathbf{U}_F - \eta^* \mathbf{U})\cdot\mathbf{n}\,dl}{\oint_L (\mathbf{U}\cdot\mathbf{n})^2 dl} \quad \text{if} \quad -g\oint_L \eta^* \mathbf{U}\cdot\mathbf{n}dl > -g\oint_L \eta_F \mathbf{U}_F\cdot\mathbf{n}\,dl \tag{22}$$
$$\gamma = 0 \quad \text{if} \quad -g\oint_L \eta^* \mathbf{U}\cdot\mathbf{n}\,dl \le -g\oint_L \eta_F \mathbf{U}_F\cdot\mathbf{n}\,dl,$$

where γ can be seen as a Lagrangian multiplier (Arfken 1985) providing optimal perturbations of sea surface elevation such that the computed GBPEF never exceeds the corresponding external GBPEF.

6. Boundary conditions on baroclinic velocities

As far as baroclinic velocities are concerned, radiative conditions based on the wave propagation equation are often used. However, the methods used to deal with propagation or to introduce external forcing terms differ somewhat in the literature. We show in the present section that the external solution needs to be specified with care.

Propagation is most often assumed to be unidirectional (Palma and Matano 2000):

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial n} = 0, \tag{23}$$

where ϕ is any of the two horizontal components of the baroclinic velocities, c is now the phase speed of internal waves, and n refers to the outward direction normal to the boundary. Propagation can, however, be treated in a multidirectional way (Raymond and Kuo 1984), but some cases of numerical instabilities have been reported and possible compromises like the normal projection of oblique (NPO) radiation scheme are therefore proposed (Marchesiello et al. 2001; Barnier et al. 1998). Phase speed is often deduced from the inner solution itself by using (23) over grid nodes located next to the boundary points considered (Orlanski 1976), that is,

$$c = -\frac{\partial\phi/\partial t}{\partial\phi/\partial n}. \tag{24}$$

Some authors consider a constant phase speed instead of Eq. (24) (Blumberg and Kantha 1985; Kourafalou et al. 1996). Although their choice may seem rough, it could also be considered as a rational way of avoiding obvious shortcomings of (24). Indeed Eq. (24) is not well suited to multiwave propagation patterns nor to the dispersion effects that are to be expected in 3D realistic situations. Moreover (24) clearly shows a singularity when $\partial\phi/\partial n = 0$. The latter, generally avoided by some Courant–Friedrichs–Levy (CFL)-type constraints (Orlanski 1976), should be regarded as a possible source of errors or numerical instabilities. Unfortunately, this unfavorable case is highly probable since it occurs every time (24) is computed on wave crests. Nevertheless, the sign of phase speed computed from (24) may be helpful to determine whether boundaries are passive (outgoing waves) or active (incoming waves). In the latter case, it is usually assumed that open boundaries should be largely driven by external incoming waves and thus a restoring term toward a forcing variable may be added to OBC (23); that is,

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial n} = -\frac{\phi_F - \phi}{\tau_R}, \tag{25}$$

with τ_R a restoring time scale. The forcing term ϕ_F is generally set equal to ϕ_{ext}, the corresponding value in the external field. This choice is somewhat questionable. Indeed, since model solutions should be close to external fields in the vicinity of active boundaries, external fields should also balance OBC equations. This means that ϕ_F should in fact be related to ϕ_{ext} according to

$$\phi_F = \phi_{\mathrm{ext}} - \tau_R\left(\frac{\partial \phi_{\mathrm{ext}}}{\partial t} + c\frac{\partial \phi_{\mathrm{ext}}}{\partial n}\right), \tag{26}$$

or, since most Orlanski-type schemes (Orlanski 1976) set $c = 0$ in case of incoming waves:

$$\phi_F = \phi_{\text{ext}} - \tau_R \frac{\partial \phi_{\text{ext}}}{\partial t}. \quad (27)$$

According to (27), the assumption $\phi_F = \phi_{\text{ext}}$ is not correct unless the temporal variability of the external fields is weak enough for the second term of the right-hand side of (27) to be neglected. Actually, a judicious way to ensure the compatibility of the external fields with OBCs, as stressed by Blayo and Debreu (2005), is to apply OBCs to perturbations of variables from the external field rather than to absolute variables. Indeed, ϕ_{ext} is a trivial solution of $B\phi = B\phi_{\text{ext}}$, where B is the following boundary operator:

$$B = \frac{\partial .}{\partial t} + c \frac{\partial .}{\partial n} - \frac{.}{\tau_R}. \quad (28)$$

7. Boundary conditions for temperature and salinity

Radiative conditions for both temperature and salinity remain popular even though they can lead to serious discrepancies as far as the horizontal pressure gradient is concerned.

The OBCs for temperature and salinity are often given by

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial n} = 0, \quad (29)$$

where u is the current in the direction normal to the boundary. In some cases, u is also chosen as a combination of this current with a wave phase speed (Palma and Matano 2000). In such cases, an upstream scheme is used allowing external temperature and salinity to enter under inflowing conditions. In sigma coordinate models, it could be tempting to replace horizontal gradient operators by their sigma counterpart (Mellor and Blumberg 1985). The reduction of computing costs or C-grid numerical convenience may motivate this approximation. In the case of (29), this gives

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial n^*} = 0, \quad (30)$$

where $\partial./\partial n^*$ is the counterpart of $\partial./\partial n$ in the sigma coordinate system. The case of the stationary regime (implying $\partial\phi/\partial n^* = 0$ since u has no special reasons to be zero) deserves particular here attention since it should have severe repercussions on the computation of the pressure gradient. This is eventually given by

$$\frac{\partial p}{\partial n} = \frac{\partial p}{\partial n^*} + \rho g \frac{\partial z}{\partial n^*}. \quad (31)$$

Without going into detail (see, e.g., Haney 1991 for further details) we recall here that a weak horizontal pressure gradient may possibly correspond to the balance of two large terms in the right-hand side of (31). Using (30) and at the same time considering a stationary state is somehow equivalent to strongly modifying the first term of the right-hand side of (31) without considering the second. This breaks the equilibrium and thus generates errors on the pressure gradient. A tangential geostrophic current is likely to occur at open boundaries and, in extreme cases [i.e., when Eq. (31) is strongly unbalanced], the model can blow up. We can see nevertheless that a well-adjusted equilibrium on the right-hand side of (31) can be recovered if $\partial z/\partial n^* = 0$ is imposed locally. In sigma coordinate models this is equivalent to setting a zero gradient condition on the bathymetry at the open boundaries. This condition is known to improve OBC robustness (Marchesiello et al. 2001) but misrepresents the major effects of bathymetry on local vorticity balance. A second solution to restore equilibrium in (31) is to apply (30) to perturbations rather than to absolute variables, namely, $B\phi = B\phi_{\text{ext}}$ with $B = \partial./\partial t + u\partial./\partial n^*$. This recalls the removal of a reference state from the density field before computing pressure gradients in order to reduce the horizontal pressure gradient truncation errors (Mellor et al. 1994). If neither of these two methods is chosen, (29) should clearly use a rigorous horizontal gradient operator, practically

$$\frac{\partial \phi}{\partial t} + u\left(\frac{\partial \phi}{\partial n^*} - \frac{\partial z}{\partial n^*}\frac{\partial \phi}{\partial z}\right) = 0, \quad (32)$$

but it should be kept in mind that the staggered C grid, mostly used in coastal modeling, is not optimal for the finite-difference form of the last term of (32), notably because the vertical gradient operator cannot have a centered form on the vertical levels contiguous to surface or bottom boundaries.

It is, however, tempting to use genuine tracer Eqs. (6) and (7), instead of (32), as OBCs, since, on the one hand, the finite-difference form of a fully 3D advection scheme is well suited to the staggered C grid and, on the other hand, vertical fluxes of temperature and salinity induced by internal wave propagation are explicitly represented by vertical advection. The horizontal advection scheme must eventually be locally adapted to OBC requirements; namely, a finite-difference upstream form is likely to be preferred at open boundaries in order to make external water masses enter the domain under inflow conditions. Obviously, a condition for (6) and (7) to remain reliable is that OBC on velocities themselves perform well since, because of the C-grid geometry, boundary currents will be involved in the computation of tracer fluxes. It is also important that the vertical velocity, related to boundary horizontal current through continuity Eq. (3), be relevant. Surprisingly, Orlanski conditions are rarely applied to vertical currents despite their reliable representation of internal waves, the horizontal currents indeed being eventually dominated by geostrophic currents and/or wind-driven motions. Such a condition could be easily completed by some radiative conditions on the tangential velocities and by normal velocities deduced from the inversion of continuity equation (3) in order to respect conservation properties of advection in (6) and (7). Finally, another advantage associated with the use of Eqs. (6) and (7) as OBCs is to include turbulence mixing effects, which may be dominant in shallow coastal areas subjected to intense atmospheric forcing (Estournel et al. 2003).

8. Nudging boundary layers

Although applying OBCs on the difference between model variables and their forcing counterparts possibly avoids some incompatibilities, it does not ensure that model solutions will be correctly constrained by external information. If we had chosen $B\phi = B\phi_{\text{ext}}$, where B is, for instance, the simple zero gradient operator (16), it is obvious that an infinity of model solutions ϕ can satisfy the boundary conditions without being close to ϕ_{ext}. The drift that is likely to grow between modeled and forcing fields can nevertheless be corrected by adding a nudging layer in the vicinity of open boundaries. In this layer, a restoring term $-(\phi_F - \phi)/\tau_R$ is added to the right-hand side of the model equations. The restoring time scale τ_R is such that this additional term progressively vanishes with the distance to the open boundaries. Ideally, the nudging term should act only in situations of inward propagation. Under outward propagation, a frontal zone of temperature and salinity can be formed if the restored field is significantly different from the advected one. A consequence may be an erroneous tangential geostrophic current. However, determining whether propagations are outward or inward is all the more difficult when the nudging layer is large and dynamical patterns inside are complex. A possible compromise, proposed by Marchesiello et al. (2001), is to apply the restoring term in the whole nudging layer but with τ_R large enough so that incompatibilities with local circulation patterns remain limited. But doing so somehow counteracts the primary purpose of a nudging layer, which is to control the modeled solution using external information.

9. Conclusions and perspectives

The present paper revisits a set of OBCs that is now widely used in sigma coordinate free surface models using finite-difference methods on a staggered C grid. The aforementioned OBC consists of a Flather condition for barotropic variables, an Orlanski-type OBC for baroclinic velocities, and an upstream advection OBC for tracers. The OBCs are generally constrained by external forcing terms. Control of the inner solution may be reinforced by means of a nudging layer.

Concerning the Flather condition, the proposed form is not exactly the same as what is apparently used in most studies. Actually, considerations on potential energy fluxes at open boundaries strongly suggest that the Flather condition should be applied to sea surface elevation rather than to the normal current. In the particular case of no external forcing, our scheme leads the boundary potential energy fluxes to be systematically negative, a property increasing numerical stability. In general, sea surface elevation is a powerful variable for controlling the global energy balance, a first OBC guess being easily adjusted by a Lagrangian multiplier method. We propose to complete our scheme with one radiative condition on the tangential transport (using a perturbation form rather than absolute variables) and a mass conservation condition (i.e., inversion of the continuity equation) to obtain the normal transport. Finally, we have discussed the global mass conservation properties of the Flather condition and shown that the constant S/Lc (where S, L, and c are the area of the domain, the length of the open boundaries, and the mean barotropic phase speed, respectively) is a timescale representative of mass adjustment transient processes.

Concerning the baroclinic velocities, we suggested that the usual restoring term added to right-hand side of the Orlanski-type OBC be handled with great care. Indeed it is not correct to assign the forcing variable directly to its external counterpart when short timescale variability dominates the external signal. A better compatibility between modeled and forcing fields is obtained if these OBC are applied to the "modeled-external" difference rather than to absolute variables.

For tracers, we showed that a horizontal advection scheme should avoid oversimplifications on sigma gradient operators and so avoid errors on the pressure gradient term. We recommend the use of the actual temperature and salinity equations, but under the condition that boundary baroclinic velocities are themselves reliable.

Nudging boundary layers can help to control model with external fields but, under outward propagation, frontal conflicts may appear if tracers are strongly restored toward external information.

All these recommendations were used in our recent studies of the Mediterranean Sea (Estournel et al. 2005; Ulses et al. 2005; Petrenko et al. 2005; Dufau-Julliand et al. 2004) and helped us to build OBCs for an operational model of the northwestern Mediterranean Sea (details given in the acknowledgments section).

Open boundary conditions are likely to remain a challenging question for coastal modelers. OBCs should make progress based on adaptativity methods (Marchesiello et al. 2001), that is, when OBCs can adapt themselves to local dynamics. Although widely used, methods based on the inversion of the wave propagation equations (e.g., the Orlanski-type OBC) nevertheless lead to an overly simple classification: grid nodes are under either the inward or outward regime. More complex situations, namely, wave packets propagating in all directions, are not taken into account. Projection of the current on the principal barotropic and baroclinic propagation modes is a possible answer to this limitation. It is to be noted that Blayo and Debreu's (2005) conclusions mention the development of the baroclinic Flather OBC based on a spectral method.

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REFERENCES

Arfken, G., 1985: *Mathematical Methods for Physicists*. Academic Press, 985 pp.

Barnier, B., P. Marchesiello, A. Pimenta de Miranda, M. Coulibaly, and J. M. Molines, 1998: A sigma-coordinate primitive equation model for studying the circulation in the South-Atlantic. Part I: Model configuration with error estimates. *Deep-Sea Res. I,* **45,** 543–572.

Blayo, E., and L. Debreu, 2005: Revisiting open boundary conditions from the point of view of characteristic variables. *Ocean Modell.,* **9,** 231–252.

Blumberg, A. F., and L. H. Kantha, 1985: Open boundary conditions for circulation models. *J. Hydraul. Eng.,* **11,** 237–255.

——, and G. L. Mellor, 1987: A description of a three-dimensional coastal circulation model. *Three-Dimensional Coastal Ocean Models,* N. Heaps, Ed., Coastal Estuarine Science, Vol. 4, Amer. Geophys. Union, 1–16.

Dufau-Julliand, C., P. Marsaleix, A. Petrenko, and I. Dekeyser, 2004: Three-dimensional modeling of the Gulf of Lion's hydrodynamics (northwest Mediterranean) during January 1999 (MOOGLI3 Experiment) and late winter 1999: Western Mediterranean Intermediate Water's (WIW's) formation and its cascading over the shelf break. *J. Geophys. Res.,* **109,** C11002, doi:10.1029/2003JC002019.

Estournel, C., X. Durrieu de Madron, P. Marsaleix, F. Auclair, C. Julliand, and R. Vehil, 2003: Observation and modelisation of the winter coastal oceanic circulation in the Gulf of Lions under wind conditions influenced by the continental orography (FETCH experiment). *J. Geophys. Res.,* **108,** 8059, doi:10.1029/2001JC000825.

——, V. Zervakis, P. Marsaleix, A. Papadopoulos, F. Auclair, L. Perivoliotis, and E. Tragou, 2005: Dense water formation and cascading in the Gulf of Thermaikos (North Aegean) from observations and modelling. *Cont. Shelf Res.,* **25,** 2366–2386.

Flather, R. A., 1976: A tidal model of the northwest European continental shelf. *Memo. Soc. Roy. Sci. Liege,* **6** (10), 141–164.

Gaspar, P., Y. Grégoris, and J. M. Lefevre, 1990: A simple eddy kinetic energy model for simulations of the oceanic vertical mixing: Tests at station Papa and long-term upper ocean study site. *J. Geophys. Res.,* **95,** 179–193.

Gill, A. E., 1982: *Atmosphere Ocean Dynamics*. Academic Press, 662 pp.

Haney, R. L., 1991: On the pressure gradient force over steep topography in sigma coordinate ocean models. *J. Phys. Oceanogr.,* **21,** 610–619.

Hedstrom, G. W., 1979: Nonreflecting boundary conditions for nonlinear hyperbolic systems. *J. Comput. Phys.,* **30,** 222–237.

Kourafalou, V. H., L.-Y. Oey, J. D. Wang, and T. N. Lee, 1996: The fate of river discharge on the continental shelf, 1, Modelling the river plume and the inner shelf current. *J. Geophys. Res.,* **101,** 3415–3434.

Marchesiello, P., J. C. McWilliams, and A. Shchepetkin, 2001: Open boundary conditions for long-term integration of regional oceanic models. *Ocean Modell.,* **3,** 1–20.

Martinsen, E. A., and H. Engedahl, 1987: Implementation and testing of lateral boundary schemes as an open boundary condition in a barotropic ocean model. *Coastal Eng.,* **11,** 603–627.

Mellor, G. L., and A. F. Blumberg, 1985: Modeling vertical and horizontal diffusivities with the sigma coordinate system. *Mon. Wea. Rev.,* **113,** 1380–1383.

——, T. Ezer, and L. Y. Oey, 1994: The pressure gradient conundrum of sigma coordinate ocean models. *J. Atmos. Oceanic Technol.,* **11,** 1126–1134.

Oliger, J., and A. Sundström, 1978: Theorical and practical aspects of some initial boundary value problems in fluid dynamics. *SIAM J. Appl. Math.,* **35,** 419–446.

Orlanski, I., 1976: A simple boundary condition for unbounded hyperbolic flows. *J. Comput. Phys.,* **21,** 251–269.

Pairaud, I., and F. Auclair, 2005: Combined wavelet and principal component analysis (WEof) of a scale oriented model of coastal ocean gravity waves. *Dyn. Atmos. Oceans,* **40,** 254–282.

Palma, E. D., and R. P. Matano, 1998: On the implementation of passive open boundary conditions for a general circulation model: The barotropic mode. *J. Geophys. Res.,* **103,** 1319–1341.

——, and ——, 2000: On the implementation of open boundary conditions for a general circulation model: The three-dimensional case. *J. Geophys. Res.,* **105,** 8605–8627.

——, and ——, 2001: Dynamical impacts associated with radiation boundary conditions. *J. Sea Res.,* **46,** 117–132.

Perkins, A. L., L. F. Smedstat, D. W. Blake, G. W. Heburn, and A. J. Wallcraft, 1997: A new nested boundary condition for a primitive-equation ocean model. *J. Geophys. Res.,* **102,** 3483–3500.

Petrenko, A., Y. Leredde, and P. Marsaleix, 2005: Circulation in a stratified and wind-forced Gulf of Lions, NW Mediterranean Sea: In-situ and modeling data. *Cont. Shelf Res.,* **25,** 7–27.

Raymond, W. H., and H. L. Kuo, 1984: A radiation boundary condition for multidimensional flows. *Quart. J. Roy. Meteor. Soc.,* **110,** 535–551.

Ruddick, K. G., E. Deleersnijder, T. De Mulder, and P. J. Luyten, 1994: A model study of the Rhone discharge front and downwelling circulation. *Tellus,* **46A,** 149–159.

Shchepetkin, A. F., and J. C. McWilliams, 2005: Regional Ocean Model System: A split-explicit ocean model with a free-surface and topography-following vertical coordinate. *Ocean Modell.,* **9,** 347–404.

Shulman, I., 1997: Local data assimilation in specification of open boundary conditions. *J. Atmos. Oceanic Technol.,* **14,** 1409–1419.

——, and J. K. Lewis, 1995: Optimization approach to the treatment of open boundary conditions. *J. Phys. Oceanogr.,* **25,** 1006–1011.

——, ——, A. F. Blumberg, and B. Nicholas Kim, 1998: Optimized boundary conditions and data assimilation with application to the M2 tide in the Yellow Sea. *J. Atmos. Oceanic Technol.,* **15,** 1066–1071.

Ulses, C., C. Grenz, P. Marsaleix, E. Schaaff, C. Estournel, S. Meulé, and C. Pinazo, 2005: Circulation in a semi enclosed bay under the influence of strong fresh water input. *J. Mar. Syst.,* **56,** 113–132.