NOTES AND CORRESPONDENCE

Determination of the Horizontal Pressure Gradient Force Using Global Positioning System on Board an Instrumented Aircraft

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ABSTRACT

The horizontal pressure gradient force is the single most important dynamical term in the equation of motion that governs the forcing of the atmosphere. It is well known that the slope of an isobaric surface is a measure of the horizontal pressure gradient force. Measurement of this force over mesoscale distances using an airborne platform has been attempted for over two decades in order to understand the dynamics of various wind systems. The most common technique has been to use a radar altimeter to measure the absolute height of an isobaric surface above sea level. Typical values of the horizontal pressure gradient force in the atmosphere are quite small, amounting to an isobaric surface slope of 0.0001 for a 10 m s\(^{-1}\) geostrophic wind at middle latitudes. Detecting the horizontal pressure gradient over irregular terrain using an instrumented aircraft has proven to be especially difficult since correction for the underlying terrain features must be made. Use of the global positioning system (GPS) is proposed here as a means to infer the horizontal pressure gradient force without the need for altimetry and terrain registration over irregular surface topography. Differential kinematic processing of data from dual-frequency, carrier phase tracking receivers on research aircraft with similar static base station receivers enables the heights of an isobaric surface to be determined with an accuracy estimated to be a few decimeters. Comparison of results obtained by conventional altimetry-based methods over the ocean and Lake Michigan with GPS reveals the potential of the GPS method at determining the horizontal pressure gradient force, even over complex terrain.

1. Introduction

The most fundamental forcing term in the equation of motion that governs atmospheric dynamics is the pressure gradient force. In particular, it is the horizontal component of the pressure gradient force that is critical to the understanding of atmospheric motion. Typically, the horizontal component of the pressure gradient force (PGF) is four orders of magnitude smaller than the vertical component, yet is responsible for the forcing of nearly all atmospheric motions. Knowledge of the PGF allows the flows to be distributed in terms of geostrophic and ageostrophic components, thereby providing insight into the dynamical state of the atmosphere. It is well known that the geostrophic component represents an essentially nondivergent, purely rotational component while the ageostrophic component is divergent and irrotational. From continuity arguments, it is the divergent component that is critical to the forcing of vertical motion and hence development of significant weather events. One could argue that it is the small ageostrophic components that are vital to our understanding of the three-dimensional motion field. Measurements of the PGF allow for such ageostrophic components to be directly inferred.

Airborne observational studies of atmospheric motion have traditionally been limited to kinematics since detection of the PGF is often subject to considerable error. From an observational point of view, accurate measurement of the horizontal pressure gradient force and hence the geostrophic wind is difficult. A 10 m s\(^{-1}\) geostrophic wind at 43° latitude corresponds to a horizontal pressure change of slightly more than 1 mb over a distance scale of 100 km. This implies an isobaric...
slope of only about $10^{-4}$, which represents a change of 10 m over a distance of 100 km. Detection of such a signal over baselines less than 100 km is difficult. Conventional rawinsonde-derived horizontal pressure gradients may be used to infer geostrophic winds over synoptic scales in the free atmosphere with an accuracy of approximately 5 m s$^{-1}$ or so. Given the spacing of the radiosonde network, it is not possible to resolve horizontal pressure gradients over mesoscale distances of 100 km or less without significantly greater uncertainties.

In situ measurement using instrumentation on board research aircraft is an alternate means of inferring the PGF over mesoscale distances. Conceptually, the means by which the PGF is determined is quite simple. By flying at constant pressure using the autopilot, the aircraft can map the absolute height of an isobaric surface. The slope of the isobaric surface is a measure of the PGF. The small deviations of the aircraft from the selected pressure level can be corrected using the hydrostatic equation. To find the PGF it is necessary to determine the height of the isobaric surface above some reference level, which is generally taken to be sea level. Until recently, this has implied that both the height of the aircraft over the underlying surface and the exact geographic position must be known with a high degree of precision. The height above ground is measured using a radar altimeter. Previous work has shown that high-resolution radar altimeters enable height measurements above ground level with an accuracy of 0.5 m (see Brown et al. 1981; Shapiro and Kennedy 1981, 1982; Parish et al. 1988; Rodi and Parish 1988; Parish 2000). Onboard navigation provides a measurement of horizontal position which, using digital terrain datasets, allows the height of the underlying terrain to be determined. The radar altitude of the aircraft is added to the terrain height, giving an absolute measure of the height of the isobaric surface, and hence the slope of the surface can be determined.

A schematic of the process and an example from the 1983 low-level jet (LLJ) study in Oklahoma described by Parish et al. (1988) are illustrated in Fig. 1. Several difficulties must be overcome to ensure accurate detection of the isobaric slopes. A summary of potential instrumentation errors is found in Rodi and Parish (1988). Foremost among instrumentation considerations was the navigation problem that has been addressed using various techniques before the general use of global positioning system (GPS) (e.g., Rodi et al. 1991). A prime concern has been the correct matching of the radar altimeter signal with the height trace of the underlying terrain. This “terrain registration” problem has been a limiting factor since the early studies of Shapiro and Kennedy (1982) and Parish et al. (1988). Even with GPS measurements and high-resolution digital terrain maps, it is necessary to correctly match two
signals that contain large deviations of the opposite sign. The final isobaric surface height is thus the sum of two large terms that are opposed to one another. Further, the relevant measurement is the slope of that surface. Inspection of the scales in Fig. 1 shows that both radar altimeter and underlying terrain height traces vary in excess of 70 m over the nearly 100-km leg length in the relatively flat Oklahoma region. Obviously, altimetry studies over irregular terrain such as near the Rocky Mountains present a far more daunting task. Accurate navigational positioning is essential to infer the height above sea level of the underlying terrain. Small navigation errors render isobaric slope detection meaningless since errors quickly overwhelm the small isobaric slope signal.

Because of problems arising from available instrumentation and the small isobaric slope signal, previous attempts to determine the PGF over irregular terrain have been met with mixed success. Brown et al. (1981) first discussed the meteorological application of radar altimetry and the determination of the PGF. Shapiro and Kennedy (1981, 1982) first showed that airborne radar altimetry can be used in studies of midlatitude jet stream dynamics. In the 1981 study, the authors used radar altitude measurements from the National Centers for Atmospheric Research (NCAR) Sabreliner to measure the isobaric heights associated with a baroclinic wave feature at 285 hPa to infer the forcing of the attendant jet streak. Since measurements were taken over the ocean, there was no need to determine the height of the underlying terrain, and hence the radar altitudes were a direct measure of the height of the isobaric surfaces above sea level. Their second study concerned determination of the PGF over irregular topography and required accurate navigational positioning to infer the underlying terrain height. This was a significantly more complex task than their first study, especially considering the lack of precise navigational aids such as GPS that exist today. Nevertheless, Shapiro and Kennedy (1982) were successful at determining the ageostrophic winds associated with an upper-level wave feature.

Others have explored the use of altimetry in studies of atmospheric dynamics. Zemba and Friehe (1987) used radar altimetry to infer the PGF associated with the LLJ off the California coast as part of the Coastal Ocean Dynamics Experiment. Parish et al. (1988) continued the work of Shapiro and Kennedy (1982) by conducting a series of flights over the Oklahoma region to directly measure the forcing mechanisms of the Great Plains LLJ during July 1983. Results from that work showed that the isallobaric components of motion were small and could not explain the large diurnal variation of the LLJ, emphasizing that the Great Plains summertime LLJ is primarily the result of the inertial oscillation of the ageostrophic wind vector as suggested by Blackadar (1957). Prater (1994) has extensively evaluated the utility of airborne determination of ageostrophic motions using instrumented aircraft. More recently, Parish (2000) examined the forcing of the California summertime LLJ by radar altimetry to show that the jet is a quasigeostrophic feature of the marine atmospheric boundary layer.

Horizontal positioning improved dramatically during the last decade with the widespread availability of GPS technology, which was introduced on the University of Wyoming King Air (UWKA) in 1992. Navigational errors were reduced routinely to a fraction of those occurring during the previous decade. The navigational positioning problem, which haunted previous attempts at airborne-derived geostrophic wind determination, became much less of an issue. Despite having refined horizontal positioning, problems were still encountered with the correct terrain registration. It was found that regardless of the digital terrain dataset used, matching of the radar altimeter trace and underlying terrain height continued to yield noisy isobaric height traces. Prater (1994) noted that this problem was most evident over terrain features with sharp gradients. Significant postprocessing was still required before an adequate PGF determination could be made. It was thought that three other potential error sources could also be adding to the problem. Radar altitude measurements contain what has been referred to as “artifact errors” due to the numerous obstacles at the surface such as trees, houses, etc. This results in a one-way bias such that the radar altimeter measurements are always less than they should be. There has also been some concern regarding the “footprint” of the radar altimeter signal, especially for flights at higher elevations due to the way the radar altimeter integrates the return signal over a finite region. Finally, Digital terrain maps contain some degree of smoothing. Although matching of the radar altimeter signal to the terrain heights obtained using GPS-derived position yields an overall excellent mirror image fit, the slope of the isobaric surface becomes degraded by the aforementioned problems.

2. GPS determination of the PGF

GPS receivers have become an increasingly common research tool for geosciences in the last decade or so, and applications for their use abound. During that period, the accuracy of the position estimates provided by GPS units has taken a quantum leap as new and more capable receivers became available to the general pub-
lic. Prior to May 2000, the Department of Defense limited real-time accuracy to about 100 m by intentionally introducing errors into the satellite clock and broadcast ephemeris through a process called selective availability (SA). While affecting real-time performance, these errors could be eliminated in postprocessing using differential GPS techniques as described below. After SA was turned off, refinements in dual-frequency single-receiver real-time solution techniques have been shown to reduce the uncertainty of three-dimensional position estimates to less than 3 m. This is still insufficient for determination of the PGF.

Receivers are available that track and record carrier phase on two GPS frequencies (labeled L1 and L2) in addition to the coarse acquisition (C/A) code broadcast on L1, and such a receiver (an Ashtech Z-Sensor) was installed on the UWKA in 2002. Postprocessing software is available that can utilize the carrier phase on the two frequencies, providing a position solution that is more precise than is possible from simply the C/A code.

Differential positioning with GPS (DGPS) requires data from two or more receivers. In our application, one receiver is static with known coordinates (the reference station) and the other receiver is on the aircraft with its location to be corrected (rover). In real time, range and range-rate corrections for each satellite can be transmitted to the aircraft. Or, as in our application, the DGPS is done including carrier phase corrections in postprocessing. In either case, the objective is to reduce errors caused by ionospheric and tropospheric propagation delays and receiver clock errors, which are the principal unknowns in the solution (Hofmann-Wellenhof et al. 2001). Under optimal conditions (good satellite geometry with at least six satellites and minimal multipath and antenna blockage), DGPS corrections using range and range-rate measurements can produce 1–5-m accuracy in the roving platform. The principal obstacle to the correction is a large separation, or baseline, between the moving platform and the static base station, where the assumption that both receivers experience the same errors becomes less valid.

Differential processing of single- or dual-frequency carrier phase adds additional accuracy. The biggest obstacle in the processing of carrier phase is the resolution of ambiguities in phase of the centimeter-long waves as the aircraft moves at relatively high speed. This becomes increasingly difficult when baselines become larger than 50–100 km, the differences in tropospheric propagation between the base and the rover become significant, or when the signal becomes noisy because of multipath or antenna blockage caused by aircraft maneuvering or poor satellite geometry. Dual-frequency carrier phase processing can eliminate the ionospheric delay errors entirely, and resulting accuracy can be as small as 0.025 m for 5-km baselines or 0.3 m for 35 + km baselines, but can be much larger if carrier phase lock is intermittent.

In this project, we used dual-frequency carrier phase base station data from the National Geodetic Survey Continuously Operating Reference Stations (CORS) network, and precise SP3 satellite ephemeris data for satellite locations and clock errors. Datasets used in this study were obtained from straight and level flight legs with good satellite coverage and baselines shorter than 100 km, which should provide an overall decimeter accuracy in the derived position with dual-frequency, carrier phase differential processing (DGPS).

3. Examples of PGF measurements

Concurrent independent measurements must be made to validate GPS-derived PGF observations. Parish (2000) has shown that high-fidelity PGF measurements are possible over the ocean with the UWKA using radar altimetry. Flying over a body of water eliminates the need for terrain registration in the PGF assessment. The UWKA has two radar altimeters, a Bendix-King KRA 405b radar altimeter (RALT1) that is capable of operating to a height of 760 m and a Stewart-Warner APN-159 (RALT2) that can operate to heights in excess of 10 km. Results shown here are taken from flights conducted during 2004 field programs conducted over Lake Michigan (ROLLS04) in January, off the California coast (GPS04) in June, and over the Gulf of Alaska (GOA04) during October. CORS data were available from the Milwaukee, Wisconsin, and Grand Rapids, Michigan, stations for ROLL04; the Cape Mendocino, California, station for GPS04; and the Juneau, Alaska, station for GOA04.

Prior to PGF determination, it was necessary to evaluate the integrity of the height measurements from the two altimeters and compare those heights with heights derived from the DGPS measurements. During each of the three field programs, the APN-159 suffered from a number of problems. Data obtained from the KRA-405b were considerably more complete and are used in following comparisons. Nine flights were selected from the aforementioned field experiments for comparison between radar altitude and heights derived from DGPS. Most of the flight data for these cases were obtained from altitudes less than 760 m, thereby allowing a comparison between RALT1 and the DGPS. The 1-Hz data were used to compare both heights, and a least squares fit was conducted to determine the best agreement. Table 1 shows the results of the compari-
sons for each flight. A consistent trend can be seen between the two measurements throughout the nearly 12 h of data. RALT1 consistently reports heights approximately 2% greater than those obtained through DGPS. It should be pointed out that the flight strategy for these nine flights varied considerably from one another. Several of the flights (21 June, 4 and 5 October) consisted of extensive vertical profiling between 100 and 700 m. Data from these legs were sorted into height classes of 100-m increments and again the RALT1 and DGPS heights were subjected to a least squares fit. Results from the various height classes (not shown) show only minor variations from those shown in Table 1.

Given these differences, it is necessary to determine which, if either, of the measurements is correct. It was thought that the DGPS heights are very accurate and that the 2% differences shown in Table 1 are indicative of a bias in the RALT1 measurements. Differences in heights can be evaluated by integrating the hydrostatic equation from some arbitrary point in a sounding and comparing with heights from RALT1 or the DGPS. Slow-rise vertical profiling maneuvers are typically conducted to take soundings. Most of the profiles cover a vertical distance of 600 m for the three cases alluded to above. A 2% error over the course of the individual profile should result in a difference between RALT1 and the hydrostatic calculations of approximately 12 m. [Relevant instrumentation on the UWKA includes a Rosemount 1501 HADS for static pressure (accuracy of 0.5 hPa, resolution of 0.006 hPa), reverse flow temperature (accuracy of 0.5 K, resolution 0.006 K), and Cambridge Model 137 C3 for dewpoint temperature (accuracy of 1 K, resolution 0.006 K).] As an example, Fig. 2a illustrates the vertical profiles from the 5 October 2004 flight at 1-Hz intervals. Starting at a point near the beginning of the profiling maneuvers, the hydrostatic equation was integrated throughout the entire set of seven legs. During the profiling maneuvers, the aircraft travels a horizontal distance of approximately 15 km. If horizontal pressure gradients are present, it can be expected that the agreement between an exact height measurement and that calculated from the hydrostatic equation will show increasing departures with time. A 10 m s\(^{-1}\) geostrophic wind component corresponds to a height change of an isobaric surface of 1 m (10 km)\(^{-1}\) or approximately 1.5 m per individual leg as shown in Fig. 3a. This is still considerably less than those associated with the 2% error in RALT1. If the DGPS measurements are correct, no detectable differences should be seen between the hydrostatic calculated and actual heights aside from the trends in the horizontal pressure field.

Figure 3b illustrates the differences between the hydrostatic calculations and the RALT1 and DGPS
It can be seen that as suspected RALT1 contains a systematic bias that mirrors the legs. The magnitudes of the differences are between 10–15 m and are expected given the 2% bias for the heights shown in Fig. 3a. Note that the differences from the DGPS heights and the hydrostatically calculated heights show no mirroring of the profiles themselves, an indication of the quality of the height measurements provided by the DGPS. In particular, the first 30 km of profiling show excellent agreement between DGPS heights and hydrostatic heights. The differences from 30 to 90 km are thought to be in large part due to the ambient horizontal component of the pressure gradient. Wind speeds normal to the leg are between 8 and 15 m s$^{-1}$ for this case; trends in both the difference curves are consistent with the large-scale pressure field. Results from the other flights with extensive vertical profiling provide similar results to those shown in Fig. 3. From these measurements it can be concluded that the DGPS heights are accurate and RALT1 heights are consistently 2% too high. It should be added that the 2% error is within the manufacturer’s specifications for the radar altimeter and that similar results have been obtained from other field projects with this altimeter.

To calculate the PGF, isobaric legs less than 750 m above the surface were identified for comparison of the altimetry and DGPS techniques for each of the nine flights. All legs were conducted over water to eliminate any problems with terrain registration. Legs were typically 50–70 km in length and heights were corrected for deviations of the aircraft off the mean isobaric surface using the hydrostatic equation. Postprocessing was used to refine original GPS position estimates for each leg using CORS data as discussed previously. Heights from the DGPS are determined with respect to the 1984 World Geodetic System (WGS84) datum to which GPS satellites are referenced, and then converted to orthometric heights using undulation values from GEOID99. The final DGPS heights shown are thus with respect to an equipotential surface. For this comparison, the RALT1 raw heights have been multiplied by 0.979 based on results discussed above. This correction has little effect on PGF calculations since isobaric height deviations along a leg are on the order of 10 m for a 100-km leg (for a 10 m s$^{-1}$ geostrophic wind), implying that the geostrophic wind correction amounts to only 0.2 m s$^{-1}$ in this case.

Figure 3 illustrates isobaric heights from RALT1 and those determined from DGPS for several legs from ROLLS04 taken on 22 January 2004. Solutions shown in Fig. 3 were based on the Milwaukee CORS station data. Processing was also conducted using the Grand Rapids CORS station; nearly identical results were obtained as that from the Milwaukee CORS station and hence are not shown. Legs for this flight were flown in a stair-step pattern, beginning at a level of about 685 m above Lake Michigan and dropping between 150 and 200 m for subsequent legs. Flight paths were approximately 50 km in length and alternated between south-southwest (Figs. 3a,c) and north-northeast headings (Figs. 3b,d). Winds were from the north-northwest at...
10–15 m s$^{-1}$ and so the flight track was oriented roughly normal to the wind. From Fig. 3 it is apparent that both RALT1 and DGPS heights track the isobaric surface with extremely high fidelity for each of the four legs and are representative of all legs flown. The trend in the DGPS signal matches well with that from RALT1 for all legs. Isobaric slope calculations for each leg show close agreement between the RALT1 and DGPS measurements with maximum differences in the equivalent geostrophic wind of less than 0.8 m s$^{-1}$.

Comparison of the heights from either RALT1 or DGPS shows that the deviations about the mean are much smaller than those seen in Fig. 1. Results from Fig. 3 suggest that DGPS techniques to infer the PGF can be successfully applied over irregular terrain, provided that a source of differential corrections is available within a reasonable baseline from flight paths. RALT1 heights in Fig. 3 are measured with respect to Lake Michigan as the base level while the DGPS heights are with respect to a geoid that is near sea level, which is the reason for the offset in absolute height.

The fidelity at which the DGPS can track an isobaric surface is revealed in a number of legs in which small-scale irregularities are present. Examples of this are shown in Fig. 4. Figure 4a illustrates both RALT1 and DGPS heights of the 966-hPa surface for a 130-km isobaric leg over the Gulf of Alaska on 5 October 2004. The height field exhibits a number of small-scale variations that are thought to reflect perturbations in the horizontal pressure field. Heights obtained from the two independent measurements show nearly identical height structure throughout the leg. The slope of the isobaric surface corresponds to a geostrophic wind slightly in excess of 16 m s$^{-1}$ in each case. A similar situation is seen in Fig. 4b, which shows isobaric heights obtained from RALT1 and DGPS measurements over the Pacific Ocean off California on 22 June 2004. As in Fig. 4a, isobaric heights from these independent measurements show a high degree of coherence. There seems no doubt that heights derived from DGPS are at least as accurate as those obtained from altimetry methods. The detailed tracking of the DGPS heights with those obtained from RALT1 shows that, without question, the application of DGPS to determine the PGF is possible.

A comparison summary of the equivalent geostrophic wind calculations based on isobaric slopes obtained from RALT1 and DGPS measurements for 28 legs flown during 2004 is shown in Fig. 5. Inspection shows that excellent agreement in equivalent geostrophic wind calculations exists between the two measurement techniques. The average absolute geostrophic wind difference for the cases shown is 0.6 m s$^{-1}$. It can be concluded from this summary that determination of the PGF using DGPS techniques is achievable to an accuracy of 1 m s$^{-1}$. It can be concluded that the limiting factor in understanding atmospheric dynamics using an airborne platform will be limited to the non-steady behavior of the PGF and issues associated with the time–space transformation discussed earlier. The
ability of an airborne measurement platform to map the height of an isobaric surface is evident.

Close inspection of the individual points in Fig. 5 show several cases in which differences are greater than 1 m s\(^{-1}\). These cases all represent legs flown over the open ocean and work in progress to understand the nature of these small differences. A central theme concerns sea level in the open ocean. When comparing isobaric heights, it is assumed that the radar altimeter measurement reflects true sea level and that a direct comparison with respect to the reference geoid used in DGPS can be made. This is, of course, a simplification. The ocean surface is subject to wind-induced waves, larger-scale swells, and tidal motions that complicate sea level measurements using a radar altimeter. There are no doubt height differences in the ocean surface owing to tides. It is well known that, directly near the shore, tidal displacement can be measured on the order of meters in certain locations; ocean levels are subject to far smaller deviations past the continental shelf.

4. Summary

An understanding of the horizontal component of the pressure gradient force is critical in understanding the dynamics of atmospheric motions. Research over the past two decades has shown that measurement of the PGF in the atmosphere using an airborne platform is difficult. This is especially true over complex terrain since the fundamental requirement is precise matching of the radar altimeter–derived heights with the underlying terrain heights. The PGF thus represents the slope of the sum of two terms that are relatively large and of opposite sign. Previous work has shown that it is possible to measure the PGF over the ocean using radar altimeter measurements while flying along an isobaric surface since no terrain registration process is involved.

The use of DGPS techniques described in this paper provides an alternate means of inferring the height profile of an isobaric surface, thereby allowing the PGF to be determined. The technique involves flying along a constant pressure surface and recording dual-frequency carrier phase data from GPS satellites. Postprocessing that incorporates data from a similar GPS receiver at a precisely surveyed location enables position of the kinematic platform to be determined with decimeter accuracy during periods of straight and level flight when six or more satellites are optimally positioned. Since the DGPS technique uses only data from satellites, PGF calculations can be made over mesoscale distances at any site regardless of the underlying terrain. The only requirement is that data from a reference station such as a CORS site are available within 100 km or so of the flight leg. The current CORS network of nearly 670 stations allows for operational locations throughout most of the United States. Comparison of the DGPS method with altimetry-based methods shows that PGF calculations can be made that are at least as accurate as those made by conventional means over the ocean. This opens the possibility of conducting detailed studies into the dynamics of fundamental atmospheric motions and interpretation of observed wind features using airborne platforms over complex terrain.

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