Shipborne Polarimetric Weather Radar: Impact of Ship Movement on Polarimetric Variables at C Band

M. THURAI
Colorado State University, Fort Collins, Colorado

P. T. MAY AND A. PROTAT
Centre for Australian Weather and Climate Research, Melbourne, Victoria, Australia

(Manuscript received 15 November 2013, in final form 11 April 2014)

ABSTRACT

The effect of ship motion on shipborne polarimetric radar measurements is considered at C band. Calculations are carried out by (i) varying the “effective” mean canting angle and (ii) separately examining the elevation dependence. Scattering from a single oblate hydrometeor is considered at first. Equations are derived (i) to convert the measured differential reflectivity for nonzero mean canting angles to those for zero mean canting angle and (ii) to do the corresponding corrections for nonzero elevation angles. Scattering calculations are also performed using the T-matrix method with measured drop size distributions as input. Dependence on mean volume diameter is examined as well as variations of the four main polarimetric parameters. The results show that as long as the ship movement is limited to a roll of less than about $10^\circ$–$15^\circ$, the effects are tolerable. Furthermore, the results from the scattering simulations have been used to provide equations for correction factors that can be applied to compensate for the “apparent” nonzero canting angles and nonzero elevation angles, so that drop size distribution parameters and rainfall rates can be estimated without any bias.

1. Introduction

Ground-based polarimetric radars are providing important new information on cloud microphysics and quantitative precipitation measurement for both research and operational applications (e.g., Bringi and Chandrasekar 2001). Recent plans include the operation of polarimetric radar from ships, such as the new Australian research vessel (R/V) under construction, the R/V Investigator (details can be found at http://www.marine.csiro.au/nationalfacility/Investigator/). This, and other such planned ships, will allow cloud studies around the globe in key climate regions ranging from the tropics to the deep Southern Ocean. However, before this can be realized it is necessary to understand the limitations that may be imposed by the deployment of such radars on a moving platform. While Doppler radar usage on ships is well established, the impact of ship motion on polarimetric radar variables has not been explored. This paper considers the effect of the ship motion on polarimetric radar variables using calculations as well as scattering simulations at C band.

2. Scattering simulations

The impact of ship motion is assessed either by varying the “apparent” mean canting angles assuming $0^\circ$ elevation or by setting the mean canting angle to zero and varying the elevation angle. Note that the canting angle is defined as the angle between the projection of the particle symmetry axis onto the plane of polarization of the incident wave and the projection of the local vertical (gravitational) direction onto the same plane.

We consider and compare here two approaches, one where Rayleigh approximation is used along with a single scatterer represented by an oblate spheroid and the other where scattering calculations are used that include non-Rayleigh scattering effects as well as measured rain
drop size distributions (DSD) as input with more realistic drop shapes. For the first case, only the effect on the differential reflectivity $Z_{dr}$ is evaluated, since this is the main polarimetric parameter that is needed for DSD retrievals and rain-rate estimations. We first consider the effect of canting angle variation.

*a. Single scatterers*

For a single scatterer, we consider—as mentioned earlier—a single oblate raindrop and for this we refer to Figs. 2.10(a) and 2.10(b) and Eqs. 2.53(a) and 2.53(c) in Bringi and Chandrasekar (2001). Figure 1 in this paper is adapted from Holt (1984).

The spheroid symmetry axis is oriented along O–N, with angles $\theta_b$ and $\phi_b$. For horizontal incidence (i.e., elevation angle zero) and when the symmetry lies in the Y–Z plane—that is, $\phi_b = 90^\circ$—then $\theta_b$ equals the canting angle $\beta$ and, as a result, Eqs. 2.53(a) and 2.53(c) simplify to

\[
(S_{hh})_{BSA} = \frac{k_0^2}{4\pi\varepsilon_0} \left[ \alpha_{Zb} \sin^2\beta + \alpha \cos^2\beta \right] \quad (1a)
\]

\[
(S_{vv})_{BSA} = \frac{k_0^2}{4\pi\varepsilon_0} \left[ \alpha_{Zb} \cos^2\beta + \alpha \sin^2\beta \right], \quad (1b)
\]

respectively, where $k_0$ is the free space propagation constant; $\varepsilon_0$ is the dielectric constant of the scatterer; $(S_{hh})_{BSA}$ and $(S_{vv})_{BSA}$ are the backscatter amplitudes for the horizontal ($h$) and vertical ($v$) polarizations, respectively; and the term $\beta$ can be considered as the apparent canting angle; and $\alpha_{Zb}$ and $\alpha$ are the polarizability of the spheroid along the symmetry axis and in the plane orthogonal to it, respectively. Note Eqs. (1a) and (1b) assume Rayleigh scattering.

The term $z_{dr}$ in linear units (as ratio) then becomes

\[
z_{dr} = \frac{|S_{hh}|^2}{|S_{vv}|^2} = \frac{\alpha_{Zb}^2 \sin^4\beta + \alpha^2 \cos^4\beta + 2\alpha \alpha_{Zb} \sin^2\beta \cos^2\beta}{\alpha_{Zb}^2 \cos^4\beta + \alpha^2 \sin^4\beta + 2\alpha \alpha_{Zb} \sin^2\beta \cos^2\beta} \quad (2)
\]

Note that since $z_{dr} = \frac{z_{dr}^i}{\alpha_{Zb}}$ when $\beta = 0$, Eq. (2) can be written as

\[
z_{dr} = \frac{\sin^4\beta + z_{dr}^i \cos^4\beta + \sqrt{z_{dr}^i} 2\sin^2\beta \cos^2\beta}{\cos^4\beta + z_{dr}^i \sin^4\beta + \sqrt{z_{dr}^i} 2\sin^2\beta \cos^2\beta} \quad (4)
\]

In Fig. 2, we show the variation of $Z_{dr}$ (dB) with the mean canting angle. The various curves (other than the black circles that will be mentioned later) correspond to $Z_{dr}$ of 5, 4, 3, 2, 1, 0.5, and 0 dB for a mean canting angle of 0°. Note, at 45°, $Z_{dr}$ goes to 0 dB as expected, and beyond that it becomes negative, once again as expected. In fact, the curves will have symmetry around $\beta = 45^\circ$ with $Z_{dr}(45 + \beta) = -Z_{dr}(45 - \beta)$ and $Z_{dr}(90) = -Z_{dr}(0)$.

*b. Using DSD*

Our second approach—as mentioned earlier—uses a distribution of scatterers along with the numerical T-matrix method [derived by Waterman (1971), and later developed further by Mishchenko et al. (1996)] that can be used to simulate the canting angle variation. Ryzhkov et al. (2011) have considered an analytic approach to take into account the distribution of particle orientations, but this approach assumes Rayleigh scattering (which may not be applicable at C band, particularly in...
the presence of large drops) and further they assume oblate spheroids for the hydrometeor shapes. On the other hand, the T-matrix method enables us to consider non-Rayleigh effects and provides more “flexibility” in terms of hydrometeor shapes and additionally allows measured DSDs to be used as input.

For our scattering calculations, data from a 2D video disdrometer located in southeast Queensland have been used. Several hundreds of 1-min DSD, with the median volume diameter ranging from 0.5 to 3 mm, were used. The mean canting angles were varied as in Fig. 2, and a narrow Gaussian canting angle distribution is also assumed with a standard deviation of 5° (Huang et al. 2008). Additionally, the “most probable” shapes given in Eqs. (1) and (2) in Thurai et al. (2007) have been used in our calculations. Note these shapes deviate from oblate spheroids for large drops (as a result of an increasingly flattened base) but nevertheless possess a (rotational) symmetry axis.

The resulting variations for C band are superimposed as black circles in Fig. 2. Each point (black circles) represents the resulting $Z_{dr}$ for each of the 1-min DSD and an assumed mean canting angle. The single scatter curves cut through the DSD-based simulations—as expected—and at a tilt of 45°, the $Z_{dr}$ information is lost.

To relate the resulting $Z_{dr}$ to the microphysical parameters, we show in Fig. 3 a, the variation of $Z_{dr}$ with the median volume diameter $D_0$ for mean canting angles varying from 0° to 30°. The light yellow/orange points represent the $Z_{dr}$ for a 0° mean canting angle. As expected, $D_0$ increases with increasing $Z_{dr}$. The rise in $Z_{dr}$ is particularly sharp for $D_0$ values from ~2 to ~2.5 mm, largely due to non-Rayleigh (resonance) scattering effects becoming more significant for larger drops. Also note that the decrease in $Z_{dr}$ with canting angle is higher for DSDs with higher $D_0$, which are often associated with higher rainfall rates. The corresponding changes in the linear depolarization ratio (LDR) are shown in Fig. 3b. An increase in the cross-polar backscatter is obtained (as expected), with the increase getting higher for larger canting angles, particularly for $D_0$ values from 2 to 2.5 mm.

Figure 4 shows the differences in four polarimetric parameters as a result of changing the mean canting angle. The four parameters are $Z_{dr}$, LDR, $K_{dp}$, and $\rho_{hv}$. In all cases with the exception of $\rho_{hv}$, the variation is linear. This is to be expected, since the canting angle term in the scattering calculations decouples from the rest of the scattering matrix computations, except for $\rho_{hv}$.

The simulation results in Fig. 4 show that for an expected ship motion of less than about ±15°, the effects are fairly tolerable. Furthermore, the results from the scattering simulations can potentially be used to determine approximate correction factors to be applied to compensate for the apparent nonzero canting angles. To this end, we have fitted the variations of all four parameters with respect to the apparent canting angle to the following set of equations:
\[ p(0) = [c_1(\beta)p(\beta)] + c_2(\beta) \quad \text{with} \]
\[ c_1(\beta) = c_{10} + [c_{11}\beta] + [c_{12}\beta^2] + [c_{13}\beta^3] \]
\[ c_2(\beta) = c_{20} + [c_{21}\beta] + [c_{22}\beta^2] + [c_{23}\beta^3], \]
\[(5)\]

where \( p(\beta) \) represents the measured \( Z_{dr}, K_{dp}, \text{LDR}, \) or \( \rho_{hv} \) when the apparent canting angle is \( \beta \) (°), and \( p(0) \) represents the corresponding parameter when \( \beta = 0 \). Values of the fitted coefficients \( c_{10} - c_{13} \) and \( c_{20} - c_{23} \) for each of the four parameters are given in Table 1. The fitted equations, represented in terms of \( \delta(p, \beta) = p(\beta) - p(0) \) versus \( p(0) \) for the various \( \beta \) angles, are superimposed as dashed lines in Fig. 4 for each of the four panels. With the exception of LDR for \( \beta = 25^\circ \), the fitted equations can be seen to closely represent the simulation results. Hence, for operations-related purposes, Eq. (5), together with the coefficients given in Table 1, can be used to determine the four parameters for the 0° apparent canting angle, which in turn can be used to determine the DSD parameters as well as rainfall rates (e.g., using the algorithms given in Bringi et al. 2009).

**Table 1. Fitted coefficients for Eq. (5) for the four parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( Z_{dr} )</th>
<th>( K_{dp} )</th>
<th>LDR</th>
<th>( \rho_{hv} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{10} )</td>
<td>1</td>
<td>1</td>
<td>—</td>
<td>1.6080 × 10^{-2}</td>
</tr>
<tr>
<td>( c_{11} )</td>
<td>1.1756 × 10^{-2}</td>
<td>1.1958 × 10^{-2}</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( c_{12} )</td>
<td>-8.6695 × 10^{-4}</td>
<td>-8.6856 × 10^{-4}</td>
<td>—</td>
<td>-7.4594 × 10^{-4}</td>
</tr>
<tr>
<td>( c_{13} )</td>
<td>5.5340 × 10^{-5}</td>
<td>5.5903 × 10^{-5}</td>
<td>—</td>
<td>7.6936 × 10^{-5}</td>
</tr>
<tr>
<td>( c_{20} )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( c_{21} )</td>
<td>—</td>
<td>—</td>
<td>-4.6329 × 10^{-1}</td>
<td>-1.6086 × 10^{-2}</td>
</tr>
<tr>
<td>( c_{22} )</td>
<td>—</td>
<td>—</td>
<td>1.6664 × 10^{-3}</td>
<td>7.4674 × 10^{-4}</td>
</tr>
<tr>
<td>( c_{23} )</td>
<td>—</td>
<td>—</td>
<td>7.2111 × 10^{-5}</td>
<td>-7.6963 × 10^{-5}</td>
</tr>
</tbody>
</table>
3. Elevation dependence

The other consideration for shipborne radar is the variation in the apparent elevation angle. Once again, we consider the two approaches mentioned earlier.

For the single scatterer case, it is possible to correct $Z_{dr}$ for a given radar elevation angle under Rayleigh approximation. Using Eqs. (2.53a) and (2.53c) given in Bringi and Chandrasekar (2001), and referring once again to Fig. 1 given in this paper, we can set $\theta_b = 0$, which then leads to

$$S_{hh} = \frac{k^2}{4\pi \epsilon_0} \alpha$$

$$S_{vv} = \frac{k^2}{4\pi \epsilon_0} \left\{ \alpha + [(\alpha_z - \alpha) \sin^2 \theta] \right\},$$

where $\theta_z = 90 - \theta$, with $\theta$ being the elevation angle. The differential reflectivity $z_{dr}$ in linear units for a given elevation angle $\theta$ then becomes

$$z_{dr}(\theta) = \frac{|S_{hh}|^2}{|S_{vv}|^2} = \frac{\alpha}{\alpha + [(\alpha_z - \alpha) \sin^2 \theta]}^2$$

$$= \left( \frac{1}{1 + \left( \frac{1}{\sqrt{z_{dr}(0)^2}} \right) \cos^2 \theta} \right)^2.$$  \hspace{1cm} (7)

The above-mentioned equation is equivalent to Eq. (28) in Ryzhkov et al. (2005), but because it is $z_{dr}(\theta)$ (or the decibel equivalent) that the shipborne radar would measure, we give below the inverted formula to derive $Z_{dr}(0)$, which in turn can be used more readily to perform the DSD retrievals and rain-rate estimates:

$$z_{dr}(0) = \frac{\cos^4(\theta)}{\left[ z_{dr}(\theta)^{-0.5} - \sin^2(\theta) \right]^2}. \hspace{1cm} (8)$$

The correction is less than 0.2 dB for elevation angles less than 10°; hence, under most circumstances, the DSD parameters or rainfall rates from the radar measurements can be retrieved without having to do any correction for $Z_{dr}$. Note also that—as before in Eqs. (3) and (6)—the above-mentioned elevation dependence equations are only valid for oblate (or rotationally symmetric) raindrops under Rayleigh scattering assumptions.

As with the canting angle dependence, scattering matrix calculations were performed using the T-matrix method for various elevation angles, with the same 1-min DSD data. The same assumptions regarding drop shapes and orientation angles were used as before. Figure 5 shows the resulting variations for the same four parameters for four elevation angles, namely, 10°, 20°, 30°, and 45°. Below 10°, the variation was found to be negligible [as was also predicted by Eqs. (7) and (8) earlier] and hence have not been included. Even for 10° elevation angle, the differences are hardly noticeable for all four parameters. The differences only become significant...
when the elevation angle becomes close to or exceeds 20°. Equation (8) can be applied to obtain an approximate correction factor for $Z_{dr}$ or, alternatively, Fig. 5 can be used for more accurate corrections for all four parameters. The comparison between the two is shown in Fig. 6a for 30° elevation angle. The agreement is close, particularly for $Z_{dr}(0°$ elevation) $< 3$ dB. Finally, in Fig. 6b we show the effect of combining the canting angle and elevation angle dependence. The differences in $Z_{dr}$ are plotted as a function of $Z_{dr}$ for 0° mean canting angle and 0° elevation angle for two cases—case 1: 30° mean canting angle and 0° elevation, and case 2: 30° mean canting angle and 30° elevation angle. Also shown is the effect of “adding” the two, assuming that the effects of the two (mean canting angle variation and the elevation angle variation) are decoupled from each other, which is applicable for Rayleigh scattering. There are some differences between the dashed line, representing the Rayleigh scattering scenario, and the curve (“star” signs), representing T-matrix calculations, for case 2. The differences increase with $Z_{dr}$ at 0° elevation angle and 0° mean canting angle. However, the differences do not exceed 0.3 dB—for example, for the maximum $Z_{dr}$ of 5 dB, case 2 gives $\delta(Z_{dr})$ of $-3.44$ dB compared with $-3.75$ dB, assuming the effects are additive. Hence, one can use the correction factors given in Figs. 4 and 5 to determine their combined effects to a reasonable approximation.

4. Summary

The planned deployment of dual-polarization radar on research vessels offers the capability of measuring cloud and precipitation characteristics in areas of key climate and weather importance that are presently poorly sampled. However, before this can be realized, it is necessary to understand the limitations that may be imposed by a deployment on a moving platform. To this end, we have performed scattering calculations for single drops as well as for drop size distribution in rain. Our results show that such deployments are feasible with similar results to land-based systems as long as the ship movement is limited to a roll of less than about 10°–15°. The design specification for the R/V Investigator is less than 5° of roll with ship stabilization, so that good measurements will be obtained, noting the measurements have the same limitations as ground-based systems for other effects regarding simultaneous transmission of horizontal and vertical polarizations that arise with nonzero canting angles and large $\Phi_{dp}$ (Hubbert et al. 2010). We have provided equations to correct for the apparent nonzero mean canting angles as well as for nonzero elevation angles, so that these can be used to determine the DSD parameters as well as rainfall rates using suitable algorithms. In many cases, the equations derived for a single scatterer can be applied, and the joint effect of mean canting angle variation and the elevation dependence can be estimated by adding their individual effects.

Acknowledgments. The authors thank Prof. V. N. Bringi for useful discussions and Dr. G.-J. Huang for his help with the T-matrix computations. MT acknowledges support from NASA Grant NNX10AJ12G as part of the Global Precipitation Mission, Ground Validation program.

REFERENCES

Huang, G.-J., V. N. Bringi, and M. Thurai, 2008: Orientation angle distributions of drops after an 80-m fall using a 2D video


