Wind Measurements from Arc Scans with Doppler Wind Lidar

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(Manuscript received 12 March 2014, in final form 27 April 2015)

ABSTRACT

Defining optimal scanning geometries for scanning lidars for wind energy applications remains an active field of research. This paper evaluates uncertainties associated with arc scan geometries and presents recommendations regarding optimal configurations in the atmospheric boundary layer. The analysis is based on arc scan data from a Doppler wind lidar with one elevation angle and seven azimuth angles spanning 30° and focuses on an estimation of 10-min mean wind speed and direction. When flow is horizontally uniform, this approach can provide accurate wind measurements required for wind resource assessments in part because of its high resampling rate. Retrieved wind velocities at a single range gate exhibit good correlation to data from a sonic anemometer on a nearby meteorological tower, and vertical profiles of horizontal wind speed, though derived from range gates located on a conical surface, match those measured by mast-mounted cup anemometers. Uncertainties in the retrieved wind velocity are related to high turbulent wind fluctuation and an inhomogeneous horizontal wind field. The radial velocity variance is found to be a robust measure of the uncertainty of the retrieved wind speed because of its relationship to turbulence properties. It is further shown that the standard error of wind speed estimates can be minimized by increasing the azimuthal range beyond 30° and using five to seven azimuth angles.

1. Introduction

During the last few decades, both airborne and ground-based lidars have been developed and applied to remotely measure wind velocity (Menzies and Hardesty 1989; Rye and Hardesty 1997; Werner et al. 2001). Among the most versatile and useful of these remote sensing technologies in the atmospheric boundary layer is the coherent Doppler wind lidar (herein called lidar), which uses the coherent heterodyne technique to detect a Doppler frequency shift in the laser light backscattered from aerosols carried by winds and to estimate wind velocity along the line of sight or radial velocity (Hardesty et al. 1997; Werner 2005). Measurements of radial velocity from more than three directions are required to reconstruct the three-dimensional wind vector; hence, a lidar needs to operate with a preconfigured scanning geometry that guides it to scan at different combinations of azimuth and elevation angles. Choosing an appropriate scanning geometry is based on measurement needs (see Fig. 1). Two basic types of scanning geometries are the plan position indicator (PPI) and the range–height indicator (RHI). A PPI scan holds the elevation angle constant but varies the azimuth angle, collecting radial velocities for a vertical slice of the atmosphere. As a result, a PPI scan has large coverage in the horizontal plane and is often used to investigate horizontal flow structures and dynamics over a large spatial domain (Choukulkar et al. 2012). Spatial statistics of turbulence can also be derived from PPI scans using radial velocity structure functions and the von Kármán model (Frehlich and Cornman 2002). An RHI scan keeps the azimuth angle constant but varies the elevation angle, collecting radial velocities for a vertical slice of the atmosphere. RHI
scans can be used to observe vertical profiles of the mean and standard deviations of wind speed as well as momentum fluxes by assuming homogeneous wind fields in multiple elevations (Gal-Chen et al. 1992; Banta et al. 2006). A velocity–azimuth display (VAD) scan uses a full 360° azimuth PPI scan, usually with a large elevation angle, and is suitable for measuring vertical wind shear above a given “point” in space (Werner 2005). Lidars can also operate in a staring mode (nonscanning) by fixing the azimuth and elevation angles. This mode is suitable for deriving turbulence statistics and measuring vertical wind speeds (Frehlich et al. 1998; Lothon et al. 2006).

Retrieval techniques usually employ assumptions of a horizontally homogeneous or linear wind field and use the least squares fit method to estimate wind velocities (Banta et al. 2002; Krishnamurthy et al. 2013) (see section 3a). If the wind field is inhomogeneous, the optimal interpolation may be employed for wind retrieval (Choukulkar et al. 2012). An alternative to the optimal interpolation is four-dimensional variational data assimilation, in which measured radial velocities are assimilated into numerical models to produce a time-evolving, three-dimensional wind field (Chai et al. 2004).

Three-dimensional scanning lidars have become economically feasible for wind energy applications and have been applied in a range of configurations to improve understanding of wind properties within and beyond modern wind turbine rotor areas and across whole wind farms (Emeis et al. 2007; Banta et al. 2013; Hasager et al. 2013). For example, with multiple PPI scans enclosing a large volume of the atmosphere, Krishnamurthy et al. (2013) showed that lidars can facilitate wind farm layout optimization using the derived spatial distribution of wind speed. Banta et al. (2008) applied RHI scans to lidars and revealed characteristics of the Great Plains nocturnal low-level jet that are of value to wind resource assessments and wind farm operations. Lidars operated with VAD scans have become essential instruments for conducting wind resource assessments and wind turbine power performance tests (Wagner et al. 2011; Gottschall et al. 2012). They are used primarily to measure vertical wind shear up to approximately 200 m, thus reducing uncertainty related to wind profile extrapolation over the rotor disk (Wagner et al. 2011). Other applications include measuring inflow conditions for wind turbine control (Simley et al. 2014) and investigating flow dynamics in the wake (Bingöll et al. 2010). Using dual lidars, it is possible to observe the complex flow structure and turbulence spectra in the wake of a wind turbine (Iungo et al. 2013). Although most lidar deployments to date have occurred onshore, the feasibility of using lidars for offshore wind energy applications has been evaluated by Peña et al. (2009) and Pichugina et al. (2012).

To meet the requirements of wind resource assessment, lidars need to provide accurate measurements of the 10-min mean wind speed profile and turbulence intensity across the rotor disk (IEC 2005a). This can be achieved in several ways. The VAD technique can be applied to measure mean wind speed profiles above the lidar location. A suitable elevation angle is required to ensure that flow is horizontally homogenous within the area scanned by the lidar. Volumetric scans formed by multiple stack PPI scans can provide coverage of wind velocity over the entire wind farm, but their low temporal resolution can lead to high uncertainty in wind speed and consequently wind power prediction (Stawiarzki et al. 2013).

Arc scans, such as those presented here, are another option that have some advantages over VAD scans and volumetric scans. In this configuration, lidars scan with a fixed elevation angle and cover a small azimuthal range (i.e., a sector scan), so that large-scale horizontal inhomogeneity has less impact on wind velocity retrievals than it does in a 360° VAD scan (Schwiesow et al. 1985). Use of a low number of azimuth angles over a small sector can also increase the temporal sampling rate and therefore reduce uncertainty in 10-min wind speed estimates (Banakh et al. 1995), and the scanning azimuth angle can be systematically varied to measure wind velocity in any direction from the lidar location. Last, for low elevation angles arc scans can be used to measure vertical wind shear with a high resolution by assuming horizontal homogeneity and averaging it over a volume with a small horizontal footprint.

Uncertainty in the retrieved wind velocity from arc scans is related to wind turbulence properties and scanning geometry (Banakh et al. 1995). Understanding this relationship is critical to quantifying and reducing uncertainty by optimizing scanning geometry and improving data processing techniques to enhance utility to wind energy applications. The objective of this paper is to use lidar observations from a field experiment to present a synthesis of sources of uncertainty in wind velocity retrievals from arc scans, to quantify the uncertainty, and to...
make recommendations regarding best practices for arc scan geometries. The experiment setup and instrument specifications are described in section 2. The wind retrieval method and data quality control techniques are presented in section 3. Section 4 presents an evaluation of uncertainties in the retrieved wind speed and direction. A brief discussion is given in section 5 regarding the best practices for the arc scan method and data processing.

2. Experiment setup

a. Site

This experiment was conducted at the National Wind Technology Center (NWTC) in Colorado from 15 to 25 February 2013. The site in the immediate vicinity of the NWTC is flat, with only a 20-m surface elevation change throughout a 2 km × 2 km square (Fig. 2), but it is surrounded by complex terrain. The foothills of the Front Range are located approximately 5 km to the west, and as a result westerly winds during the experiment period were often associated with drainage flows from Eldorado Canyon (Clifton et al. 2013). This complex terrain can cause inhomogeneous flow conditions (Wyngaard 2010) and introduce errors to the wind speed estimated from the arc scan. On the northern edge of the site there is a building complex, and on the eastern edge there is a row of installed wind turbines. Measurements on a 134-m meteorological tower (denoted as M5) installed approximately 150 m west of the wind turbines were used in this paper to evaluate lidar performance.

To examine the wind speed spatial variability at the site because of terrain and surface roughness changes, Wind Atlas Analysis and Application Program (WAsP) Engineering (Mann et al. 2002) simulations were run in the range and scale of the proposed observations for 16 wind directions covering the directional sector 210°–360° that is used in the analysis presented herein. Note that the snow-covered surface during part of the experiment period may have produced a more homogeneous surface with lower roughness and therefore resulted in flow–surface interactions that are more reproducible by the linearized model of WAsP Engineering. Figure 2 shows an example of the simulation output for an incident wind speed of 9 m s⁻¹ and a wind direction of 272° at a height of 74 m at M5. Simulation results indicated a relatively homogeneous flow field in the area scanned by the lidar. Wind speed varied no more than 0.2 m s⁻¹ throughout the area scanned by the lidar at a height of 74 m. However, it should be noted that output from WAsP Engineering
TABLE 1. Specification of Galion G4000 three-dimensional scanning Doppler wind lidar (provided by SgurrEnergy).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (μm)</td>
<td>1.56</td>
</tr>
<tr>
<td>Pulse energy (μJ)</td>
<td>30</td>
</tr>
<tr>
<td>Pulse duration (ns)</td>
<td>200</td>
</tr>
<tr>
<td>Pulse repetition frequency (kHz)</td>
<td>20</td>
</tr>
<tr>
<td>Aperture diameter (mm)</td>
<td>75</td>
</tr>
<tr>
<td>Focal length (m)</td>
<td>300</td>
</tr>
<tr>
<td>Maximum range (m)</td>
<td>4000</td>
</tr>
<tr>
<td>Radial velocity accuracy (m s⁻¹)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

provided a simplified representation of flow conditions at the site and, as discussed below, the real flow field may exhibit larger spatial variability than was manifested in these simulations because of mesoscale forcing.

b. Pulsed coherent Doppler lidar

The Galion G-4000 pulsed Doppler lidar, distributed by SgurrEnergy, has three-dimensional scanning capability and was used in this experiment (see specifications in Table 1). The lidar was deployed approximately 350 m west of M5 (see Fig. 2). The arc scan used for this experiment was designed so that the center of the arc at range gate 11 was close to the sonic anemometer mounted at a height of 74 m on M5. The range gate size was 30 m, and the elevation angle was 12.7°; thus, the center of range gate 11 was 76 m above the ground. The lidar scanning geometry was set with seven azimuth angles of 55.8°–85.8° with a 5° interval. It took approximately 3 s to complete one measurement and start a new one at the next azimuth angle; thus, there were approximately 25–28 radial velocity measurements per range gate over 10 min.

c. Meteorological tower M5

Measurements from the instruments on M5 (Table 2) were used as the reference data to evaluate the accuracy of wind velocity retrieved from the arc scan. Sonic anemometers were mounted at six heights on long booms (boom length to face width ratio = 5.7) oriented 278° clockwise from the north. The sonic anemometer and the lidar used the same frame of reference in which the vertical axis was aligned with the local gravitational field. Sonic anemometers sampled at the rate of 20 Hz. High-frequency measurements were flagged as missing if they were beyond the valid data range (±30 m s⁻¹). A 10-min time series of the sonic data was excluded if missing data exceeded 30%. Two types of cup anemometers were mounted on short booms (boom length to face width ratio = 2.8) attached to M5 at multiple heights and orientated 278° clockwise from the north (Table 2). Data exceeding the valid range (75 m s⁻¹) were removed to calculate the 10-min mean wind speeds from the cup anemometers.

3. Wind retrieval method

This section briefly describes assumptions and methods used for the wind velocity retrieval from the arc scan and data quality criteria applied. The detailed mathematical description of the wind retrieval algorithm and uncertainty quantification is given in the appendix.

a. Assumptions

The wind field was assumed to be horizontally homogeneous and stationary within 10 min. The wind vector is denoted as \( \mathbf{v} \), and its three components are expressed as the sum of the mean and the turbulent fluctuation:

\[
\begin{align*}
    u &= u_0 + u', \\
    v &= v_0 + v', \quad \text{and} \\
    w &= w_0 + w',
\end{align*}
\]

where positive \( u, v, \) and \( w \) are the instantaneous west–east, south–north, and upward vertical wind components, respectively. The letters with subscript 0 denote the means, and those with a prime superscript denote the turbulent fluctuation terms, which are assumed to have zero means. Similarly, the radial velocity \( (v_R) \) measured by the lidar is defined as the sum of its mean \( (v_{R0}) \) and turbulent fluctuation term \( (v'_R) \):

\[
v_R = v_{R0} + v'_R, \tag{4}
\]

where

\[
\begin{align*}
    v_{R0} &= u_0 \cos \phi \sin \theta + v_0 \cos \phi \cos \theta + w_0 \sin \phi, \\
    v'_R &= u' \cos \phi \sin \theta + v' \cos \phi \cos \theta + w' \sin \phi. \tag{4b}
\end{align*}
\]

In the above-mentioned equations, \( \theta \) is the azimuth angle clockwise from the north and \( \phi \) is the elevation angle. Because \( v_R \) is a linear combination of \( u, v, \) and \( w \), it has a mean defined in Eq. (4a) and variance, denoted as \( (v'^2_R) \), given by (Eberhard et al. 1989):

\[
\langle v'^2_R \rangle = A_0 + A_1 \cos \theta + B_1 \sin \theta + A_2 \cos 2\theta + B_2 \sin 2\theta + \sigma^2, \tag{5}
\]

\[
\begin{align*}
    A_0 &= \frac{\cos^2 \phi}{2} \left( \langle u'^2 \rangle + \langle v'^2 \rangle + 2 \tan^2 \phi \langle w'^2 \rangle \right), \\
    A_1 &= \langle u'v' \rangle \sin 2\phi, \\
    B_1 &= \langle u'v' \rangle \sin 2\phi, \\
    A_2 &= -\frac{\cos^2 \phi}{2} \left( \langle u'^2 \rangle - \langle v'^2 \rangle \right), \quad \text{and} \\
    B_2 &= \langle u'v' \rangle \cos^2 \phi.
\end{align*}
\]
where \( \langle \cdot \rangle \) denotes the ensemble average in time and space, and \( \sigma_k^2 \) is the lidar measurement error variance. According to Eq. (4a), given that \( v_{\theta 0} \) is available at more than three different azimuth angles at the same height, the three mean wind components can be estimated by solving the ordinary least squares equation [i.e., Eq. (A10)]. However, because the radial velocity variance depends on the azimuth angle, the weighted least squares (WLS) [i.e., Eq. (A11)] provide more accurate wind velocity estimates.

Vertical wind speed is not retrievable at the same time as the two horizontal components \( u_0 \) and \( v_0 \) from the arc scan, because the least squares method—including vertical wind speed—is unstable and has a high sensitivity to the turbulent variation of the radial velocity (Banakh et al. 1995). Uncertainty in the radial velocity propagates into the retrieved wind velocity according to the following equation (Demmel 1997):

\[
\| \delta \mathbf{v} \| \leq K(\mathbf{G}) \frac{\| \delta \mathbf{v}_R \|}{\| \mathbf{v}_R \|}, \tag{6}
\]

where \( \| \cdot \| \) denotes the norm of a vector, \( \| \delta \mathbf{v}_R \| / \| \mathbf{v}_R \| \) is the relative variation in radial velocity, and \( \| \delta \mathbf{v} / \| \mathbf{v} \| \) is the resultant variation in the retrieved wind velocity. The term \( K(\mathbf{G}) \), the cond number of \( \mathbf{G} \) defined in Eq. (A6), is the ratio of the maximum and minimum singular values of \( \mathbf{G} \). Thus, Eq. (6) allows \( K(\mathbf{G}) \) to be used as a measure of sensitivity of the retrieved wind velocity to the variation in radial velocity. In the experiment at the NWTC, \( K(\mathbf{G}) = 345.5 \) if vertical wind speed is included in Eq. (A7), which can cause the variation in the radial velocity to increase by two to three orders of magnitude when Eq. (A10) is solved for wind velocity (see Fig. 3a).

Naturally, vertical wind speed can be retrieved if the elevation angle is close to or equal to 90°. However, \( u_0 \) and \( v_0 \) are not retrievable because of their small contributions to the radial velocity [as shown in Eq. (4a)]. For the conditions during the NWTC experiment, \( K(\mathbf{G}) \) reached its lowest value of 150 for \( \phi = 45° \). This is still unacceptably high and thus demonstrates that vertical wind speed cannot be retrieved simultaneously with \( u_0 \) and \( v_0 \).

The inversion procedure in Eq. (A10) can be stabilized by setting \( w_0 \sin \phi = 0 \) in Eq. (4a). When \( \phi \) and/or \( w_0 \) are low, the radial velocity has little contribution from the vertical wind speed, and hence it is appropriate to assume \( w_0 = 0 \) in the wind velocity retrieval. By assuming \( w_0 = 0 \), \( K(\mathbf{G}) \) reduces to 5.7 and the radial velocity variation increases by no more than 2 (Fig. 3a). Neglecting vertical wind speed, however, introduces bias to the estimated \( u_0 \) and \( v_0 \). The expected value of bias (\( \langle \beta \rangle \)) for \( N_0 \) azimuth angles is defined as (Myers 1990)

\[
\langle \beta \rangle = \tan \phi w_0 \mathbf{B}, \tag{7}
\]

where

\[
\mathbf{B} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{1}, \quad \text{and} \quad \mathbf{X} = \begin{bmatrix} \sin \theta_1 \cos \theta_1 \\ \vdots \\ \sin \theta_{N_0} \cos \theta_{N_0} \end{bmatrix}_{N_0 \times 2} \quad \text{and} \quad \mathbf{1} = [1, \ldots, 1]_{N_0 \times 1}. \tag{7a}
\]

The values of \( \langle \beta \rangle \) computed this way are 0.216\( w_0 \) for \( u_0 \) and 0.075\( w_0 \) for \( v_0 \) in this experiment. They are thus

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**Table 2. Specifications of anemometers and wind vanes installed on the M5.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Instrument</th>
<th>Height (m)</th>
<th>Range</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed</td>
<td>Met One SS-201 cup anemometer</td>
<td>3, 10, 38, 87, 122</td>
<td>0–90 m s(^{-1})</td>
<td>0.5 m s(^{-1}) or 2%</td>
</tr>
<tr>
<td>Wind speed</td>
<td>Thies 4.3351.10.0000</td>
<td>30, 51, 80, 105, 130</td>
<td>0–75 m s(^{-1})</td>
<td>0.2 m s(^{-1}) or 1%</td>
</tr>
<tr>
<td>Wind direction</td>
<td>Met One SD-201 vane</td>
<td>3, 10, 38, 87, 122</td>
<td>0°–360°</td>
<td>3.6°</td>
</tr>
<tr>
<td>Wind vector</td>
<td>ATI “K”-type sonic anemometer</td>
<td>15, 41, 61, 74, 100, 119</td>
<td>±30 m s(^{-1})</td>
<td>0.01 m s(^{-1})</td>
</tr>
</tbody>
</table>

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**Fig. 3.** (a) The modeled RMSE of the retrieved wind velocity as a function of the azimuth range with and without vertical wind speed according to Eq. (A12), and (b) the expected value of bias (\( \beta \)) in \( u_0 \) and \( v_0 \) introduced by \( w_0 = 1.0 \) m s\(^{-1}\) and \( \phi = 12.7° \) according to Eq. (7). In the top panel, wind components \( u, v \), and \( w \) were assumed to be independent and had the same variance of 1.0 m\(^2\) s\(^{-2}\). The first line of sight of the azimuthal range was at \( \theta = 0° \).
much lower than the uncertainty caused by including vertical wind speed, provided that \( w_0 \) is low (i.e., \( u_0 \gg w_0, v_0 \gg w_0 \), as shown in Fig. 3).

b. Radial velocity quality control

The accuracy of (and by association uncertainties in) lidar radial velocity measurements is determined by the signal-to-noise ratio (SNR) (Frehlich 1997). Thus, a high SNR is required to ensure data quality. A threshold SNR of 0.01 (−20 dB) was used for the lidar deployed in this experiment. Because statistical properties were available from 25 to 28 radial velocity measurements per 10 min at each range gate, additional quality control procedures were applied, including a combination of SNR, outlier detection, and despiking. Radial velocity measurements at a given range gate within a 10-min period were subject to the following data selection criteria:

1) A radial velocity with \( \text{SNR} < 0.01 \) (−20 dB) was flagged with an error code. A radial velocity with \( \text{SNR} > 10 \) (10 dB) and \( |v_R| < 0.25 \text{ m s}^{-1} \) was assumed to be caused by hard targets and hence flagged as an error. Given that 50% of the data must be considered valid to derive an accurate estimate of interquartile range (IQR) in step 2, a 10-min time series was considered missing and excluded from further processing if the count of flags because of low/high SNR >12.

2) Data that passed step 1 were then used for outlier detection using the box plot method (Hoaglin et al. 1983), wherein a radial velocity is an outlier if

\[
\begin{align*}
    v_R > v_{R,75} + 1.5 \text{IQR} & \quad \text{or} \\
    v_R < v_{R,25} - 1.5 \text{IQR},
\end{align*}
\]

where \( v_{R,25} \) and \( v_{R,75} \) are the 25th and 75th percentiles of the time series, respectively. Figure 4 shows an example in which an outlier passed the SNR test but failed the criteria in Eq. (8).

3) Spikes were detected in data that passed step 1 using the difference between the two consecutive measurements (\( \Delta v_R \)) (Rebmann et al. 2012). A spike is defined from both sides (i.e., the preceding and subsequent measurement) if

\[
|\Delta v_R| > 2 \text{IQR}_\Delta,
\]

and they have different signs (e.g., Fig. 5). Here \( \text{IQR}_\Delta \) is the IQR of \( \Delta v_R \).

All outliers and spikes identified were removed from the time series to allow more reliable estimates of variance for uncertainty quantification.

c. Estimating mean wind velocity

After outliers and spikes were detected and removed using the methods described in section 3b, the mean and variance of radial velocity over 10 min were calculated for all range gates. WLS was then used to estimate \( u_0 \) and \( v_0 \) using Eq. (A11) (Myers 1990). In this analysis, the measured 10-min mean radial velocity at each range gate was assumed to be independent, and hence its covariance matrix that defines the weight matrix for WLS has only diagonal entries given by \( \langle v_R^2 \rangle/n_{10} \), where \( n_{10} \) is the number of measurements used for averaging in each 10-min period. WLS has the advantage of incorporating measurement uncertainties into the estimates of \( u_0 \) and \( v_0 \), because \( \langle v_R^2 \rangle \) and \( n_{10} \) represent uncertainties associated with turbulence fluctuation and sample size, respectively. Moreover, WLS should have higher accuracy than the ordinary least squares, given the dependence of radial velocity variance on azimuth angles [as shown in Eq. (5)].

Cook’s distance is a measure of the influence of an observation on the predicted coefficients of WLS (Myers 1990). It can be used to find observations that significantly affect the retrieved wind velocity, and thus it is used to detect outliers possibly arising from inhomogeneous wind fields. Cook’s distance was applied for the arc scan implemented in this experiment once per averaging time with a threshold value of 1.0 according to Eq. (4)/(\( N - k - 1 \)), where \( N = 7 \) and \( k = 2 \) for seven azimuth angles and two coefficients \( (u_0 \) and \( v_0 ) \) (Myers 1990). If the sample number is low, the columns in the WLS model matrix become linearly dependent and uncertainty will increase in the estimated wind velocity. Thus, outliers detected using Cook’s distance were removed, and
the wind velocity was estimated only if the mean radial velocities were available for at least five azimuth angles—
that is, ≥5/7 of the data were available for WLS.

The goodness of fit of WLS was evaluated using the coefficient of determination ($R^2$) derived from the measured and fitted 10-min radial velocity. In orthogonal scans, the mean radial velocity is close to zero, because the wind direction is perpendicular to the line of sight. The variance, however, does not decrease proportionally with the mean, as shown in Eq. (5). It is independent of wind direction if turbulence is isotropic. Thus, $R^2$ has low values when orthogonal scans occur. Statistical simulation showed that, if the wind field is horizontally homogenous, $R^2 > 0.8$ for 99% of cases of orthogonal scans. For nonorthogonal scans, $R^2 > 0.9$ for all cases. Hence, a threshold value of 0.8 was chosen for $R^2$. Cases with $R^2 < 0.8$ indicated bad fits that may be associated with a violation of the homogeneity assumption.

The standard error of the retrieved wind speed, denoted as $\sigma_v$, can be approximated by the following equation and used as a measure of uncertainty (Lyons 1991):

\[
\sigma_v = \frac{1}{V} \sqrt{(u_0\sigma_{u_0})^2 + (v_0\sigma_{v_0})^2 + 2\rho_{u_0v_0}\sigma_{u_0}\sigma_{v_0}u_0v_0},
\]

where

\[
V = \sqrt{u_0^2 + v_0^2}
\]

In Eq. (10), $\sigma_{u_0}$ and $\sigma_{v_0}$ are the estimated standard errors for $u_0$ and $v_0$, respectively; $\rho_{u_0v_0}$ is the estimated correlation between $u_0$ and $v_0$; and $V$ is the estimated wind speed. Values of $\sigma_{u_0}$, $\sigma_{v_0}$, and $\rho_{u_0v_0}$ can be found from the covariance matrix calculated from Eq. (A12), assuming that the radial velocity at each range gate is independent. The standard error $\sigma_v$ includes uncertainties caused by both wind speed fluctuation and the instability of WLS. Note that errors can be introduced into $\sigma_v$ if the term $w_0\sin\phi$ is nonzero.

4. Results and discussions

Using the methods described in the previous section, 10-min mean wind speeds and directions were retrieved from the arc scans conducted at the NWTC and evaluated against concurrent anemometer measurements from M5. Among the 1556 records collected during 11 days by the lidar, 325 were flagged as missing as the result of the data quality procedure and 165 had poor goodness of fit ($R^2 < 0.8$). However, an evaluation was conducted only for freestream sectors [210°–360°] in which flow was not distorted by the meteorological mast or the nearby wind turbines [in which flow distortion was determined using the method defined in IEC (2005b)]. Periods when $|w_0| > 1$ m s$^{-1}$ (based on the sonic data at a height of 74 m on M5) were also excluded, because large vertical wind speeds at the site were associated with nonstationary and nonhomogenous drainage flows from the west.

The accuracy of the lidar measurements was evaluated in terms of two linear regression models,

\[
y_{\text{lidar}} = b_0 + b_1x_{\text{ref}} \quad \text{and} \quad y_{\text{lidar}} = a_1x_{\text{ref}},
\]

where $a_1$, $b_1$, and $b_0$ are the coefficients; $x_{\text{ref}}$ is the reference data from M5; and $y_{\text{lidar}}$ is the retrieved lidar data, in terms of the absolute error ($e_{\text{abs}}$), defined as

\[
e_{\text{abs}} = y_{\text{lidar}} - x_{\text{ref}},
\]

and the relative error ($e_{\text{rlv}}$), defined as

\[
e_{\text{rlv}} = (y_{\text{lidar}} - x_{\text{ref}})/x_{\text{ref}}.
\]

The bias (BIAS) and root-mean-square error (RMSE) were then defined as the mean of $e_{\text{abs}}$ or $e_{\text{rlv}}$ and the square root of the mean of squared $e_{\text{abs}}$ or $e_{\text{rlv}}$, respectively.

In the following section, $V$ and $D$ are used to denote the 10-min mean wind speed and direction, respectively, and subscripts are used to differentiate data from different instruments. For example, $V_{\text{lidar}}$ is the lidar mean wind speed and $D_{\text{sonic}}$ is the sonic mean wind direction.
Mean wind speed

The accuracy of the lidar was evaluated by comparing 254 pairs of lidar data at range gate 11 and sonic data at a height of 74 m on M5 (Fig. 6). There was relatively large scatter between lidar and sonic with $R^2 = 0.956$ and RMSE = 0.72 m s$^{-1}$. Both the fitted $b_1$ and $a_1$ are 1.0, primarily because lidar > sonic at high wind speeds. Except for a few outliers (which are discussed in more detail below), the RMSE scaled to wind speed.

Instrument errors should have little contribution to the scatter because of the stringent quality control procedure described above. The lidar pitch and roll were well below 0.5°; therefore, the associated errors were much lower than 0.6% of the true wind speed and negligible. The scatter shown in Fig. 6 was likely caused by turbulent wind fluctuation, WLS instability, and a violation of the assumption of homogeneity. These sources are quantified in the following analysis.

1) TURBULENCE AND ERROR

High turbulent wind fluctuation, which is commonly measured by the standard deviation of wind speed, affects lidar measurements in the following two ways. First, a radial velocity measured by lidar is a spatially weighted average in a volume along the line of sight, but it is ascribed to a radial velocity at the center of a range gate (range gate 11 in this analysis). Thus, high uncertainties in radial velocity measurements are expected when turbulent wind fluctuation increases in the sampled volume. The other effect of high turbulent fluctuation is related to the instrument sampling rate. The lidar was configured to take measurements at seven azimuth angles, and each measurement took approximately 3 s. This sampling rate resulted in only one sample every 18 s for one azimuth angle. When rapidly fluctuating winds are present, a low sampling rate can cause high uncertainty in the estimated mean radial velocity and eventually the estimated wind velocity.

Turbulence kinetic energy (TKE), defined as

$$TKE = \frac{1}{2}(\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle),$$

is used in Fig. 7 to demonstrate the effects of turbulent wind fluctuation on lidar errors. As shown, $e_{\text{abs}}^2$ increases with increasing TKE.

2) WLS AND ERROR

The number of measurements used to calculate the mean radial velocity varies as the result of the data selection procedure described in section 3b. The lowest number is 13. Lowering the sample size can increase uncertainty in the estimated mean. However, the increased uncertainty is accounted for by WLS, because the weight given to a mean radial velocity is inversely proportional to its variance. Therefore, the number of samples used to calculate the mean radial velocity should have little effect on the accuracy of the estimated
wind speed. Nevertheless, WSL instability can arise when mean radial velocities are missing or excluded at some azimuth angles. Reducing the number of mean radial velocities in WLS can increase the model matrix cond number K(G) and inversion instability. (See Fig. 6, in which retrieved wind speeds with high K(G) and missing mean radial velocities are marked by red dots and dark diamonds, respectively.) These estimates in general have large errors at high wind speeds, but their errors are relatively small at low wind speeds. Note that some observations with high K(G) still have small errors when TKE is small (<0.25 m² s⁻²). Thus, even though errors in the estimated wind speed are amplified by large K(G), they can still be low as the result of a small variance in the mean radial velocities.

3) INHOMOGENEITY AND ERROR

A violation of the assumption of homogeneity is another source of error. Winds can become horizontally nonuniform because of their interactions with the complex topographical features around the site. As described above, a threshold R² value of 0.8 was used to detect estimates possibly related to high spatial variability, and this removed 74 of 328 records (22.5%) that had matching sonic measurements within the freestream sector. Increasing the threshold value (to 0.9) can improve data quality to some extent. The points with blue squares shown in Fig. 6 are those with 0.8 ≤ R² ≤ 0.9. The relative RMSE for those points is 52%, which is much higher than the 16.1% for the rest of the points with R² > 0.9.

To further investigate the effects of nonuniform flow on errors, measurements from the sonic anemometer were converted to radial velocities (denoted as v₉₀J), and their mean and variance (denoted as ⟨vᵢR,₀⟩²) were calculated at the seven azimuth angles using Eq. (4a) and Eq. (5), respectively. Then v₉₀J was compared to its concurrent lidar-measured radial velocities (denoted as v₉₀J). If the flow were uniform, then the difference between v₉₀J and v₉₀J should be within a range defined by ⟨vᵢR,₀⟩². Statistical tests were conducted for all of the v₉₀J against normal distributions with their means and standard deviations defined by v₉₀J and ⟨vᵢR,₀⟩², respectively; and N₆ was used to denote the number of azimuth angles at which v₉₀J values were significantly different from v₉₀J. Figure 8 shows the relationship between errors in v₉₀J and N₆. Errors were large when N₆ = 2 − 4, because of the existence of nonuniform flows (see example in Fig. 9). Wind speed for these cases varied monotonically with azimuth angle, and measured radial velocities can fit Eq. (4a) very well. However, the fitted coefficients gave incorrect estimates of wind speed, because the consistent spatial variability pulled the sinusoidal curve of Eq. (4a) in a wrong direction. When the difference between v₉₀J and v₉₀J was fitted as a linear function of the azimuth angle, statistically significant large slopes were found for cases when N₆ = 2 − 4 (Fig. 8). When N₆ < 1, flow was uniform and the errors of the retrieved wind speeds were minimal. When N₆ = 7, all of the v₉₀J were different from v₉₀J; however, the error of the estimated wind speed was small. This outcome is possible when a spatial gradient of wind speed exists only between the sonic anemometer location and the arc scanned by the lidar but not along the arc. The retrieved wind speed represents the uniform wind field along the arc. As long as the spatial variability is small across the small distance between the arc to the sonic anemometer, the error of the retrieved wind speed should be small.

4) APPROACHES TO QUANTIFY UNCERTAINTIES IN V₉₀J

As discussed above, uncertainty in V₉₀J is proportional to turbulent wind fluctuation, and thus TKE can be used to identify V₉₀J values that have high associated uncertainty and even to quantify part of the uncertainty in V₉₀J. TKE can be estimated from direct lidar measurements when radial velocity variance is available from at least six azimuth angles (Sathe and Mann 2012). However, this estimation is not possible for the arc scan covering a small azimuth range, because the model matrix has a high cond number. An alternative method is to approximate TKE from Eq. (5) under the assumption of isotropic turbulence. Then all the covariance terms in
Eq. (5) become zero, and the variance terms have the same value. As a result, the radial velocity variance is equal to two-thirds of TKE—that is, 

$$\langle u_R^2 \rangle = \frac{2}{3} \text{TKE.}$$  \hspace{1cm} (16)$$

Note that in the analysis the lidar measurement error term $\sigma_s^2$ was neglected, as was attenuation of TKE by the volumetric averaging of the lidar measurement. Nevertheless, as shown in Fig. 10, Eq. (16) is consistent with the fitted relationship between the $\langle u_R^2 \rangle$ measured by the lidar and the TKE measured by the sonic anemometer, which has a slope of 0.630 $\pm$ 0.012, which is very close to two-thirds.

Screening of $V_{\text{lidar}}$ using an arbitrary threshold of the 30th highest $\langle u_R^2 \rangle$ value (as a proxy for high TKE) can improve the quality of the linear fit between $V_{\text{lidar}}$ and $V_{\text{sonic}}$ (Fig. 11), although it should be noted that removing data with high $\langle u_R^2 \rangle$ actually removed some data with high wind speeds, because TKE is dependent on and proportional to wind speed.

The standard error $\sigma_V$ defined in Eq. (10) can also be used to quantify the uncertainty in $V_{\text{lidar}}$ estimates, because it is a function of both turbulent wind fluctuation and WLS instability. As shown in Fig. 12, $V_{\text{sonic}}$ is within the 95% confidence interval of $V_{\text{lidar}}$ as described using $\sigma_V$ for most cases, suggesting that $\sigma_V$ could be used for uncertainty quantification. The large discrepancies in the wind direction sector 210°–270° were associated with nonuniform flows (as shown in Fig. 9). As in the analysis in which the highest 30 values of $\langle u_R^2 \rangle$ were excluded, when the 30 measurements with the highest $\sigma_V$ were excluded, again improvement was made for both the fitted linear model and the error statistics (Fig. 11). The analysis summarized in Fig. 11 suggests that $\sigma_V$ might be a better parameter than $\langle u_R^2 \rangle$ for quality control.

b. Mean wind direction

The 10-min values of $D_{\text{sonic}}$ (at 74 m) and $D_{\text{lidar}}$ (at 76 m) exhibited some correlation but rather large scatter (Fig. 13), particularly when $D_{\text{sonic}} < 300°$. This is consistent with the large $V_{\text{lidar}}$ errors shown in this sector (Fig. 11), and the scatter was caused by high wind fluctuation and nonuniform flows. The scatter was low for $D_{\text{sonic}} > 300°$, but $D_{\text{lidar}}$ was biased relative to $D_{\text{sonic}}$ by 5.2° (Fig. 13). If $D_{\text{sonic}}$ is correct, then this suggests that the lidar was not correctly aligned with the north direction. Note that the offset from true north does not affect the accuracy of the retrieved mean wind speed.

c. Vertical profile of the horizontal wind speed

In the previous discussion, uncertainties in the wind speed and direction retrieved at a specific height (range gate 11) were evaluated using the sonic anemometer measurements as the reference. Here we evaluate the accuracy of the retrieved wind speed at multiple heights.
or range gates in terms of vertical profiles of the 10-min mean horizontal wind speed as measured by the four cup anemometers (up to a height of 80 m) mounted on M5. Only those cases that are presented in Fig. 6 and all of the data available from range gate 3 to 11 were evaluated. Range gate 2 and 12 were excluded because of their low data availability. The retrieved wind speed had different spatial coverage at different heights because of the scanning geometry. The arc length increased from 55 m at range gate 3 to 180 m at range gate 11. The horizontal and vertical distances between range gate 3 and 11 were 263 and 60 m, respectively. Wind profiles were analyzed for the wind speed range 4–12 m s\(^{-1}\) in two wind direction sectors (210°–330° and 330°–360°) using wind speeds from the cup anemometer at 80 m and wind direction from the sonic anemometer at 74 m. The wind speed range was chosen because it is relevant to wind power production. The data were divided into two wind direction bins to investigate whether the positive bias observed at range gate 11 in the wind direction sector 210°–330° existed at the other range gates.

For the wind direction sector 210°–330°, the lidar consistently overestimated wind speeds at all heights (Fig. 14). The overestimation was likely caused by the spatial inhomogeneity described in section 4a and illustrated in Fig. 9. For the direction sector 330°–360°, the mean wind speed profile from the lidar and the cup anemometers were almost the same. Moreover, in this direction sector individual wind speed profiles from the cup anemometers and the lidar matched almost exactly. Power-law profiles were fitted to individual lidar wind speed profiles in this direction sector and used to predict wind speed at the heights of the cup anemometers. The predicted wind speed, when compared to the cup wind speed, had BIAS = -0.075 m s\(^{-1}\) and RMSE = 0.222 m s\(^{-1}\) at 55 m and BIAS = -0.033 m s\(^{-1}\) and RMSE = 0.270 m s\(^{-1}\) at 80 m.

According to Eqs. (4a) and (5), radial velocity measurements from orthogonal scans should have large uncertainty. Although it is not possible to make any generalizable conclusions about the accuracy for the orthogonal scans from only 254 measurements, note that the lidar measurements in the directional sector
wind speed in Eq. (A7). As shown in Fig. 3, for $\theta_s = 30^\circ$ and $\Delta \theta = 5^\circ$, assuming $w_0 = 0$ reduces uncertainty in the estimated $V_{\text{lidar}}$ by approximately 80 times relative to the alternative while it introduces only 1 m s$^{-1}$ bias if the actual $w_0 = 1$ m s$^{-1}$, and less if $w_0 < 1$ m s$^{-1}$. To reduce the bias in the estimated $V_{\text{lidar}}$, it is advisable to use the mean radial velocity for wind velocity retrieval, because $w_0$ is more likely to $\to 0$ than any instantaneous value. Otherwise, bias in the estimated $V_{\text{lidar}}$ can be inflated because of variation in the vertical wind speed and the nonlinear relationship between the scalar wind speed and the wind vector. If $w_0$ is high, such as it is sometimes at the NWTC, then it might be beneficial to add a vertical scan to the arc scan in a similar manner to the Doppler beam swinging (DBS) technique (Werner 2005). The measurement of $w_0$ from the vertical scan can then be used to correct the bias in the estimated $V_{\text{lidar}}$.

The value of $\theta_s$ determines the effectiveness of the least squares method used for wind velocity retrieval. Uncertainty in the retrieved wind velocity is caused by radial velocity variance and is amplified by matrix inversion instability. Radial velocity variance is determined by the covariance matrix for $u$, $v$, and $w$, as shown in Eq. (A8), and the variation of radial velocity propagates into the retrieved wind speed and can be inflated by a factor defined by $K(G)$, as shown in Eq. (6). In general, $K(G)$ and uncertainty in the estimated $V_{\text{lidar}}$ decreases with increasing $\theta_s$, assuming isotropic homogenous turbulence (Fig. 15) (Banakh et al. 1995); hence, it is always recommended to use a large $\theta_s$ while being cognizant that for a given averaging period the value of $\Delta \theta$ determines the spatial resolution and temporal resolution of the arc scan and $\theta_s$ and $\Delta \theta$ dictate the repetition rate at which each range gate is sampled. With a large $\Delta \theta$, the spatial resolution decreases, because $N_\theta$ decreases, but $n_{10}$ increases at each azimuth angle. As a result, the uncertainty of the mean radial velocity is reduced. Thus, under the assumption of isotropic turbulence, uncertainty in the retrieved wind velocity decreases with increasing $\Delta \theta$ or decreasing $N_\theta$ (Fig. 15). For the same $\Delta \theta$, increasing $\theta_s$ can reduce uncertainty except for low $\Delta \theta$. When $\Delta \theta$ is low (e.g., $\Delta \theta = 30^\circ$ as shown in Fig. 15), increasing $\theta_s$ causes $n_{10}$ to decrease and the mean radial velocity uncertainty to increase. Consequently, uncertainty in the retrieved wind speed increases with increasing $\theta_s$. Whenever possible, it is desirable for $N_\theta = 5–7$. According to Fig. 15, uncertainty should be minimized when $N_\theta = 3$, but this is not recommended. Arc scans with low $N_\theta$ are sensitive to the spatial variability of wind speed, because the weight of WLS is too high at each azimuth angle. For $N_\theta = 3$, a slight deviation of wind speed at one azimuth angle from the underlying true wind speed can
significantly change the fitted curve defined by Eq. (4a) and consequently the fitted wind speed. Note that the effect of $u_s$ and $D_u$ on the retrieved wind velocity depends on the turbulence length scale (uncertainty length scale) (Banakh et al. 1995).

The optimal choice of $\varphi$ depends on the required vertical profile resolution, magnitude of $w_0$, and the horizontal extent over which homogeneity is valid. Lowering $\varphi$ can increase the vertical resolution, but it requires a homogenous wind field over a large area. The bias because of the assumption of $w_0 = 0$ is proportional to $\tan \varphi$, as shown in Eq. (7); therefore, the bias increases with $\varphi$ for a given combination of azimuth angles. Finally, note that the return signal weakens and therefore data availability decreases as distance increases. To ensure high data availability, it is good practice to focus the laser at the desired range and limit the number of range gates on either side of the focus.

6. Conclusions

A range of scanning geometries can be used with lidar, but the properties that can be retrieved and the accuracy of the wind speed and direction estimates are complex functions of the inverse method, scanning configurations, and site characteristics, because they determine the flow complexity. Here we evaluate arc scans and conduct an analysis designed to evaluate sources of uncertainty and error in wind speed retrievals using data collected at a site with low complexity in the proximal environment (i.e., with 2 km) but with relatively complex terrain at a distance of 5 km. To stabilize the wind velocity retrieval algorithm as applied to arc scan returns, the vertical wind speed must be neglected. Because the mean vertical wind speed is likely to be zero in

![FIG. 14. Mean vertical profiles of the 10-min mean wind speed retrieved from the lidar and measured by the cup anemometers on M5 for the wind direction sectors (a) 210°–330° and (b) 330°–360°. Only wind speeds of 4–12 m s$^{-1}$ as measured by the cup anemometer at a height of 80 m were included. Wind direction was defined by measurements from the sonic measurement at a height of 74 m. Standard errors of the mean values for the lidar profiles and the cup profiles are plotted as error bars and shaded areas, respectively.](image)

![FIG. 15. Uncertainty in the retrieved wind speed from arc scans as a function of the arc scan azimuth range ($\theta_s$) and the azimuth interval ($\Delta \theta$). The number of beams $N_\theta = 1 + \theta_s/\Delta \theta$. Uncertainty is measured by $\sigma_{w_\theta} = \sigma_{w_\theta}^2 + \sigma_{w_\theta}^\theta$, where $\sigma_{w_\theta}^2 = \sigma_{w_\theta}^\theta + \sigma_{w_\theta}^\theta$ as defined in Eq. (10) and $\sigma_{w_\theta}^\theta = \sigma_{w_\theta}^\theta + \langle \nu^2 \rangle$. The method used in this analysis is from Banakh et al. (1995) and is described in Eq. (A3). Turbulence was assumed to be isotropic, and the turbulence spatial correlation was assumed to decay exponentially. The error in the radial velocity was set to zero by assuming a high return intensity. Wind speed = 8 m s$^{-1}$, Wind direction = 270°. The horizontal wind speed standard deviation = 1.414 m s$^{-1}$ as $\langle \nu^2 \rangle = \langle \nu^2 \rangle = 1.0$ m$^2$ s$^{-2}$. The turbulence length scale = 400 m.](image)
most environments, the implication is that the mean radial velocity should be used for the mean wind velocity estimate. Employing an assumption of $w_0 = 0$ for every 10-min time series at each range gate, outliers and spikes were removed from our experimental time series and the mean and variance of radial velocity were calculated to estimate wind speed and direction using WLS. The resulting wind estimates were compared to those from a collocated sonic anemometer and were found to have a fitted slope for a regression forced through zero of 1.033 and RMSE $= 0.72 \text{ m s}^{-1}$ over an observed wind speed range of up to 18 m s$^{-1}$. Errors in the wind speed retrieved from the arc scan were a function of turbulent wind fluctuation, WLS instability, and the spatial variability of wind speed. The magnitude of the uncertainties resulting from the turbulent wind fluctuations in the sample volume were approximated using the radial velocity variance (as an approximation of turbulence kinetic energy), whereas the uncertainty deriving from both the turbulent wind fluctuation and WLS instability were quantified using the standard error of the retrieved wind speed.

A theoretical analysis with the assumption of isotropic turbulence was conducted to determine the optimized scanning geometry for the arc scan. Based on that analysis, it is recommended to use a large azimuth range with five to seven evenly distributed azimuth angles. If the arc scan only samples in a small azimuth range ($<30^\circ$) and/or at 3–4 azimuth angles, then no meaningful estimate of wind speed will be generated.

A long-term arc scan measurement campaign over a flat and uniform site with homogeneous and stationary flow is essential to studying the dependence of the arc scan uncertainty on the relative angle between the arc and the wind direction. Further, such a campaign would be useful to characterize the error properties of the wind velocity retrieved from the arc scan and their relationships to turbulence properties and scanning geometry so that an optimal geometry can be chosen for wind energy applications.

Acknowledgments. This work was funded by the U.S. Department of Energy (Award DE-EE0005379) and the U.S. National Science Foundation (Award 1067007). This work was also supported by the U.S. Department of Energy under Contract DE-AC36-08GO28308 with the National Renewable Energy Laboratory. Funding for the work was provided by the DOE Office of Energy Efficiency and Renewable Energy’s Wind and Water Power Technologies Office. The National Wind Technology Center at NREL supported this research under CRADA CRD-09-343 with Indiana University. We also thank Nick Capaldo and Dr. Peter Clive at SgurrEnergy for their assistance with the Galion lidar.

APPENDIX

Arc Scan Wind Velocity Retrieval Algorithm

a. Relationship between radial velocity and wind velocity

The arc scan is set to have $N$ azimuth angles ($\theta_n$, $n = 1, 2, \ldots, N$) with a fixed elevation angle ($\phi$) and repeat $Q$ times over 10 min. The radial velocity $v_{Rn}$ is the radial velocity measurement from the $q$th scan at $n$th azimuth angle, and it is the projection of wind vector $v_n = (u_n, v_n, w_n)^T$ on the line of sight, written as

$$v_{Rn} = d_n^T v_n + e_n^q,$$  \hspace{1cm} (A1a)

where

$$d_n^T = [\cos \phi \sin \theta_n, \cos \phi \cos \theta_n, \sin \phi]^T,$$  \hspace{1cm} (A1b)

and $e_n^q$ is the lidar measurement error. The superscript T denotes the matrix transpose.

For a homogenous and stationary wind field, two parts comprise the wind vector:

$$v_n = v_0 + v_n^q.$$ \hspace{1cm} (A2)

The term $v_0 = (u_0, v_0, w_0)^T$ is the mean wind vector representing a large-scale steady motion and is constant over 10 min. The term $v_n^q = (u_n^q, v_n^q, w_n^q)^T$ is the fluctuating wind vector representing stochastic turbulent motions. It is a random vector with zero mean and covariance matrix $\Sigma_n^q$.

For the radial velocities measured at all of the $N$ azimuth angles of the $q$th scan,

$$v_n^q = D v^q + e^q,$$ \hspace{1cm} (A3)

where

$$v_n^q = [v_{R1}^q, v_{R2}^q, \ldots, v_{RN}^q]^T,$$ \hspace{1cm} (A3a)

$$D = \begin{bmatrix} d_1^T & \cdots & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & d_N^T \end{bmatrix}_{N \times 3N}$$ and $v^q = \begin{bmatrix} v_1^q \\ \vdots \\ v_N^q \end{bmatrix}_{3N \times 1}$, \hspace{1cm} (A3b)

$$e^q = [e_1^q, e_2^q, \ldots, e_N^q]^T.$$ \hspace{1cm} (A3c)

In the equation above, $\theta$ is the zero vector/matrix.
\[
\mathbf{v}_r = \mathbf{D}_F \mathbf{v} + \epsilon \\
\mathbf{D}_F = \begin{bmatrix}
D & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & D
\end{bmatrix}_{NQ \times 3NQ}
\text{and } \mathbf{v} = \begin{bmatrix}
\mathbf{v}^1 \\
\vdots \\
\mathbf{v}^Q
\end{bmatrix}_{3NQ \times 1}
\]  
(4a)

\[
\mathbf{v} = \bar{\mathbf{v}} + \mathbf{v}' = \begin{bmatrix}
\mathbf{v}_0^1 \\
\vdots \\
\mathbf{v}_0^Q
\end{bmatrix}_{3NQ \times 1} + \begin{bmatrix}
\mathbf{v}_1^1 \\
\vdots \\
\mathbf{v}_1^Q
\end{bmatrix}_{3NQ \times 1}.
\]  
(5)

Therefore, the vector \( \mathbf{v} \) corresponding to all the measured radial velocities is a random vector with mean \( \mathbf{v} = (\mathbf{v}_0^1, \mathbf{v}_0^2, \ldots, \mathbf{v}_0^Q)_{1 \times 3NQ} \) and covariance matrix \( \Sigma_v \), which is determined by the properties of atmospheric turbulence. Replacing \( \mathbf{v} \) in Eq. (4a) with Eq. (5) and applying an averaging operator \( (\mathbf{A}) \) to both sides of the equation gives

\[
\mathbf{v}_0 = \mathbf{G} \mathbf{v}_0 + \mathbf{D}_A \mathbf{v}' + \mathbf{A} \epsilon
\]  
(6a)

\[
\mathbf{G}^T = \begin{bmatrix}
\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_N
\end{bmatrix}_{3 \times N}
\]  
(6a)

\[
\mathbf{D}_A = \frac{1}{Q} [\mathbf{D}, \mathbf{D}, \ldots, \mathbf{D}]_{N \times 3NQ}
\]  
(6b)

\[
\mathbf{A} = \frac{1}{Q} [\mathbf{I}_{N \times N}, \mathbf{I}_{N \times N}, \ldots, \mathbf{I}_{N \times N}]_{N \times 3NQ}.
\]  
(6c)

The averaging operator, which consists of \( Q \) identity matrices \( \mathbf{I} \), is used to calculate the mean radial velocities \( (\mathbf{v}_0) \) at all of the \( N \) azimuth angles. Because \( \mathbf{v}' \) is a random vector with zero mean and covariance \( \Sigma_v \), the mean radial velocity vector in Eq. (6a) is a random vector with mean

\[
\langle \mathbf{v}_0 \rangle = \mathbf{G} \mathbf{v}_0
\]  
(7)

and covariance matrix

\[
\Sigma_{\mathbf{v}_0} = \mathbf{D}_A \Sigma_v \mathbf{D}_A^T + \mathbf{A} \Sigma_v \mathbf{A}^T,
\]  
(8)

where \( \Sigma_v \) is the covariance matrix for the lidar measurement errors. If the lidar measurement errors are independent and have equal variance \( \sigma_v^2 \), then the covariance matrix in Eq. (8) can be written as

\[
\Sigma_{\mathbf{v}_0} = \mathbf{D}_A \Sigma_v \mathbf{D}_A^T + \sigma_v^2 \mathbf{I}.  \quad (A9)
\]

b. Mean wind velocity estimation

Assuming \( \mathbf{v} \) follows a multivariable normal distribution, the mean wind velocity over 10 min can be estimated from measured radial velocities using the ordinary least squares regression, as follows

\[
\hat{\mathbf{v}}_0 = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{v}_0,
\]  
(10)

where \( \hat{\mathbf{v}}_0 \) denotes the estimated value of \( \mathbf{v}_0 \). Equation (10) gives an unbiased estimator of \( \mathbf{v}_0 \) if the radial velocity is independent and has equal variance; however, this is not true, as shown in Eq. (A8). As a result, in terms of the variance of the estimated wind velocity, the most efficient estimator should be the weighted least squares regression, and \( \hat{\mathbf{v}}_0 \) can be calculated from the following equation (Myers 1990):

\[
\hat{\mathbf{v}}_0 = (\mathbf{G}_w^T \mathbf{G}_w)^{-1} \mathbf{G}_w^T \mathbf{v}_0,
\]  
(11)

where

\[
\mathbf{G}_w = \mathbf{L} \mathbf{G}
\]  
(11a)

\[
\Sigma_{\hat{\mathbf{v}}_0} = \mathbf{L}^T \mathbf{L}.
\]  
(11b)

In Eq. (11), the weight matrix \( \mathbf{L} \) is related to the covariance matrix \( \Sigma_{\mathbf{v}_0} \) through the Cholesky decomposition. Assuming the lidar measurement errors are independent and have the same variance, the covariance matrix \( \Sigma_{\mathbf{v}_0} \) for \( \hat{\mathbf{v}}_0 \) in Eq. (11) is given by

\[
\Sigma_{\mathbf{v}_0} = [\mathbf{G}_w^T (\mathbf{D}_A \Sigma_v \mathbf{D}_A^T + \sigma_v^2 \mathbf{I}) \mathbf{G}]^{-1}.
\]  
(12)

If the turbulence is isotropic, the wind vectors are independent, and the measurement errors are negligible, then \( \Sigma_{\mathbf{v}_0} \) in Eq. (12) can be simplified into the following equation:

\[
\Sigma_{\mathbf{v}_0} = \frac{\sigma_v^2}{Q} [\mathbf{G}^T (\mathbf{D} \mathbf{D}^T)^{-1} \mathbf{G}]^{-1},
\]  
(13)

where \( \sigma_v^2 \) is the variance for \( u, v, \) and \( w \).

For an unbiased estimator, uncertainty in the estimated coefficients is measured by the mean square error (MSE), which is equal to the trace of the covariance matrix \( \Sigma_{\mathbf{v}_0} \) for wind velocity retrieval. According to Eqs. (12) and (13), increasing the number of repetitions of \( Q \) reduces MSE. The relationship between \( \Sigma_{\mathbf{v}_0} \) and the scanning geometry \( \mathbf{D}_A \) and \( \mathbf{G} \) is nonlinear and unpredictable because
of the unknown property of turbulence or the value of $\Sigma_v$.

c. Wind velocity retrieval uncertainty estimation

If we assume that the turbulence is isotropic, then the correlation between two wind components separated by a distance vector $\mathbf{r}$ can be derived from the following equation (Panofsky and Dutton 1984):

$$
\rho_{ij}(\mathbf{r}) = \rho_{11}(\mathbf{r}) \delta_{ij} + \frac{1}{2} \left( \frac{r_i r_j}{r} \right) \frac{d}{dr}[\rho_{11}(r)],
$$

(A14)

where $i = 1, 2, 3; j = 1, 2, 3$; and $u_1$, $u_2$ and $u_3$ are the streamwise, transverse, and vertical wind velocities, respectively. The distance vector $\mathbf{r} = (r_1, r_2, r_3)^T$ is defined by the distance in the streamwise ($r_1$), the transverse ($r_2$), and the vertical ($r_3$) directions, and it has length $r$. The Kronecker delta $\delta_{ij} = 1$ when $i = j$ and 0 otherwise. The streamwise spatial correlation function $\rho_{11}(r)$ can be approximated by the exponential decay function, as follows:

$$
\rho_{11}(r) = \exp\left(-\frac{r}{L_v}\right),
$$

(A15)

where $L_v$ is the turbulence integral length scale. For the arc scan, measurements at two range gates are taken at different times and locations and two adjacent measurements at the same range gate are also taken at the different times. The spatial distance between two measurements can be calculated by using the Taylor’s hypothesis of frozen turbulence (Stull 1988). Then each entry in $\Sigma_v$ can be derived from Eqs. (A14) and (A15), assuming isotropic turbulence, and $\Sigma_{\nu}$, and its trace (MSE) can be calculated for a given arc scan geometry.

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