Turbulence Estimation Using Fast-Response Thermistors Attached to a Free-Fall Vertical Microstructure Profiler

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(Manuscript received 19 October 2015, in final form 28 May 2016)

ABSTRACT

Estimation of turbulence intensity with a fast-response thermistor is examined by comparing the energy dissipation rate $\varepsilon_T$ from a Fastip Probe, model 07 (FP07), thermistor with $\varepsilon_S$ from a shear probe, both of which are attached to a free-fall microstructure profiler with the fall rate of $0.6$–$0.7$ m s$^{-1}$. Temperature gradient spectra corrected with previously introduced frequency response functions represented by a single-pole low-pass filter yields $\varepsilon_T$ with a bias that strongly depends on turbulence intensity. Meanwhile, the correction with the form of a double-pole low-pass filter derives less bias than of single-pole low-pass filter. The rate $\varepsilon_T$ is compatible with $\varepsilon_S$ when the double-pole correction with the time constant of $3 \times 10^{-3}$ s is applied, and $68\%$ of $\varepsilon_T$ data are within a factor of $2.8$ of $\varepsilon_S$ in the wide range of $\varepsilon_S = 10^{-30} - 10^{-7}$ W kg$^{-1}$. The rate $\varepsilon_T$ is still compatible with $\varepsilon_S$ even in the anisotropy range, where the buoyancy Reynolds number $I = \nu(N^2)$ is $20$–$100$. Turbulence estimation from the fast-response thermistor is thus confirmed to be valid in this range by applying the appropriate correction to temperature gradient spectra. Measurements with fast-response thermistors, which have not been common because of their poor frequency response, are less sensitive to the vibration of profilers than those with shear probes. Hence, measurements could be available when a fast-response thermistor is attached to a CTD frame or a float, which extends the possibility of obtaining much more turbulence data in deep and wide oceans.

1. Introduction

Vertical turbulent mixing is one of the main physical processes that control the ocean meridional overturning circulation and the vertical transport of heat, nutrients, carbon dioxide, and other materials. However, observation of turbulence is scarce even in the present day due to the difficulty of measuring a few centimeters of microstructure to estimate turbulence intensity. Turbulence measurements have usually been conducted by free-fall or free-rise profilers with velocity shear probes in order not to disturb microstructure fields (Lueck et al. 2002).

Turbulence intensity is able to be estimated from small-scale velocity or scalar fields such as temperature. Small-scale velocity shear and temperature are measured by using airfoil shear probes and fast-response thermistors (e.g., Kocsis et al. 1999; Ruddick et al. 2000; Moum et al. 2013), respectively. The Fastip Probe, model 07 (FP07), thermistor is surrounded by a glass coating, and this causes a time delay of heat transferring from the surface of the glass coating to the sensor core (Gregg 1999). Since the time response of the thermistor is not sufficient at high frequency, where a temperature spectrum is attenuated (Lueck et al. 1977; Gregg and Meagher 1980), spectral corrections are sometimes inevitable (e.g., Kocsis et al. 1999; Peterson and Fer 2014, hereafter PF14). Since in previous studies correction procedures have been diverse depending on various situations of observing platforms and/or turbulence intensity, thermistors have not been widely used compared with shear probes. However, thermistors are less sensitive to the vibration and motions of profilers than shear probes, which measure velocity field. Thus, thermistors could be attached to various ocean-observing platforms, such as conductivity–temperature–depth (CTD) frames, floats,
and moorings, which have some vibrations. Indeed, there are previous studies where thermistors have been attached to moorings (Moum and Nash 2009; Perlin and Moum 2012; Moum et al. 2013).

The signal of an observed temperature spectrum is attenuated at high frequencies. The form of the attenuation for the thermistors has been proposed to be represented by the single-pole (Lueck et al. 1977) or double-pole (Gregg and Meagher 1980) low-pass filter functions by conducting laboratory experiments. Observed temperature gradient spectra could be corrected by multiplying the reciprocal of these low-pass filter functions.

On the basis of field observational data, Kocsis et al. (1999) carried out a comparison of turbulence intensity from FP07 thermistors and shear probes with a slow free-rise (with the speed of 0.08 m s\(^{-1}\)) profiler. After correcting the temperature spectra with the single-pole function and the time constant of 7 ms, they showed that both microstructure methods yield nearly the same turbulence intensity. Meanwhile, PF14 used a glider with a moving rate of \(\sim 0.4\) m s\(^{-1}\). The resulting temperature-derived and shear-derived turbulence intensities agree well by correcting the temperature spectra with the single-pole function with the time constant of 12 ms.

Although several studies such as the above-mentioned showed the availability of turbulence measurements with FP07 thermistors, there are differences in correction functions and time constants. There seems to be even no consensus on which functions (single pole or double poles) and what time constants are appropriate. To make turbulence measurements with thermistors practical, it is required to make clear the correction methods and their limitations.

In this paper, we aim to expand the knowledge of availability of turbulence estimations with fast-response thermistors on free-fall vertical microstructure profilers, where concurrently measured turbulence estimates are available from both velocity shear and temperature sensors. Turbulence intensity from thermistors is compared with turbulence intensity from shear probes, both of which are attached to the same free-fall microstructure profiler, and the performances of thermistor estimation with single- and double-pole frequency response corrections and several time constants in the nominal range are quantified. Section 2 provides definitions and procedures for estimating turbulence intensity. Section 3 presents the results of the comparison between thermistors and shear probes. The performances of thermistors in strong and weak turbulence ranges, where spectra of temperature gradient are largely attenuated and measurements with velocity shear probes could be unreliable, respectively, are discussed in section 4.

### 2. Data and method

#### a. Observational data

A Vertical Microstructure Profiler 2000 (VMP, manufactured by Rockland Scientific International Inc.) was used to measure turbulence fields around the Aleutian Islands in the R/V *Hakuho-Maru* during the KH-09-4 cruise from 12 August to 25 September 2009 (Fig. 1). The VMP was equipped with two velocity shear probes (S1 and S2) and two fast-response FP07 thermistors (T1 and T2, produced by Rockland Scientific). These probes and accelerometers were sampled at 512 Hz. The VMP was also equipped with Sea-Bird Electronics temperature (SBE3) and conductivity (SBE4C) sensors with a pump (SBE5T). It was deployed from the stern deck and freely fell at the speed of 0.6–0.7 m s\(^{-1}\) at a depth of 0–2000 m. We used 12 VMP profiles for comparison between shear probes and thermistors (Table 1).

#### b. Turbulence intensity and universal spectra

When temperature fields are stirred by turbulence, temperature fluctuation develops, and the temperature gradient becomes large and will be modified by molecular thermal diffusion. Accordingly, the decreasing rate of temperature variance due to molecular diffusivity is represented as an indicator of turbulence intensity and is denoted by \(\chi\), which is approximated by, under isotropy,

\[
\chi = 6\kappa (\partial T/\partial z)^2, \tag{1}
\]

where \(\kappa\) is the molecular thermal diffusivity and \(\partial T/\partial z\) is the small-scale vertical temperature gradient. Here \(\partial T/\partial z\) is measured with the fast-response thermistors, which can resolve a length scale of a few centimeters. The angle brackets \(\langle \rangle\) denote spatial averaging, which is set at about 10 m in our analysis.

Another indicator of turbulence intensity is the rate of loss of kinetic energy due to molecular viscosity. It is
Table 1. Observed station, time and date, locations (positive: north or east; negative: south or west), and probe numbers of the VMP. VMP has two shear probes (S1, S2) and two thermistors (T1, T2), and their serial numbers are shown. Depth is the maximum depth to which VMP reached.

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<th>S1</th>
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represented by turbulent dissipation rates \( \varepsilon \), which is given by the following equation under the assumption of isotropy,

\[
\varepsilon = (15/2) \nu \left( \frac{\partial u}{\partial z} \right)^2, \tag{2}
\]

where \( \nu \) is the kinematic viscosity and \( \partial u / \partial z \) is the vertical shear of small-scale horizontal velocity. Here \( \varepsilon \) is usually estimated by measuring velocity shear with an airfoil shear probe and integrating a shear spectrum.

The rate \( \varepsilon \) could be estimated not only from the velocity shear but also from the microtemperature by fitting a universal spectrum to an observed temperature gradient spectrum. The energy dissipation rate is related to the Batchelor length scale \( \eta_B \), where the molecular diffusion of temperature becomes effective (Batchelor 1959), as follows:

\[
\eta_B = 1/k_B = 2\pi (\nu \times \kappa^2/\varepsilon)^{1/4}, \tag{3}
\]

where \( k_B \) is known as the Batchelor wavenumber. In this paper, the unit of the wavenumber is described as a cyclic wavenumber [cycle per meter (cpm)]. From Eq. (3), we obtain

\[
\varepsilon = (2\pi)^4 \times k_B^4 \times \nu \times \kappa^2. \tag{4}
\]

Previous studies such as Oakey (1982) and PF14 used this relation to estimate \( \varepsilon \) from thermistors by fitting the universal spectrum to the observed temperature gradient spectrum, as will be described later in this section.

c. Estimating \( \varepsilon \) and \( \chi \)

The rate \( \varepsilon \) from velocity shear is estimated by integrating a \( (\partial u/\partial z)^2 \) spectrum. Accuracy of the shape of the spectrum is confirmed by fitting a universal shear spectrum to the observed one. The form of the universal spectrum in Kolmogorov inertial to viscous range were advocated theoretically by Corrsin (1964), Saffman (1963), and Panchev and Kesich (1969). Nasmyth (1970) showed a coherent spectrum by using observed field data. In this paper, the following equation in Oakey (1982) is used:

\[
S_{\text{Nasmyth}} = \varepsilon^{3/4} \times \nu^{-1/4} \times G2, \tag{5}
\]

where \( G2 = 8.05(k/2\pi k_p)^{13/3}[1 + 20(k/2\pi k_p)^{3/7}] \) (Wolk et al. 2002), \( k \) is a wavenumber, and \( k_p \) is the Kolmogorov wavenumber \( [k_p = (\varepsilon/\nu)^{1/4}/2\pi] \).

The microscale \( \partial u/\partial z \) was high-pass and low-pass filtered to remove shear components with a frequency less than 0.25 Hz and more than 98 Hz, and segmented into half-overlapping segments of 10-m length before Fourier transformation. Fourier transformations were performed over half-overlapping segments of approximately 1 m, and then spectra were averaged over 10 m. One 10-m-averaged spectrum was thus obtained by using approximately 20 spectra. These high-pass and low-pass (to avoid aliasing) filters with the mean free-fall speed of 0.65 m s\(^{-1}\) could permit the VMP to cover the turbulent eddies with a wavenumber of 0.38–150 cpm and a length scale of 6.6 mm–2.6 m, which were able to be resolved, assuming the length scale of the probe and the profiler are 5 mm and 2.5 m, respectively.

An iterative procedure is used to determine \( \varepsilon \) as follows. First, observed shear spectra are integrated from a minimum wavenumber to an arbitrary wavenumber. Second, \( \varepsilon \) is estimated via Eq. (2) and then \( k_p \) is determined. The \( \varepsilon \) and \( k_p \) lead to a single form of the Nasmyth spectrum via Eq. (5). Third, the ratio between the Nasmyth and observed spectra is computed over the range of the wavenumbers from minimum to 0.04 \( \times \) 2\( \pi \)\( k_p \) (near the peak of the Nasmyth spectrum). When the ratio is less (more) than one-half (twice) the interval of integration is spread to four-thirds (reduced to three-fourths) of the previous range. This sequence is repeated until the ratio is less than a factor of 2, and then \( \varepsilon \) is determined. An example
of a shear spectrum that is visually regarded as good agreement with the Nasmyth spectrum is shown in Fig. 2a. Shear spectra poorly fitted to the Nasmyth form are not used in the following analysis. For S1 (S2), 2067 (2267) of the total 2647 shear spectra were judged to be well fitted to the Nasmyth form by visual inspection.

The microtemperature $\frac{\partial T}{\partial z}$ is also segmented in the same way as velocity shear before Fourier transformation, although the methods for estimating $\chi$ and $\epsilon$ are different. They are determined by fitting a universal spectrum to the observed temperature gradient spectrum by using a maximum likelihood estimation (MLE) method, introduced by Ruddick et al. (2000). This method has an advantage for non-Gaussian error distribution, and the MLE estimates are unbiased in comparison with other least squares. Besides, the number of free parameters needed for estimation is reduced by using $\chi$, which is computed by integrating the temperature spectrum after removing instrument noise. This noise spectrum is here defined as the lowest level of the temperature gradient spectrum of all spectra in the 12 casts (light blue curve in Fig. 2b). The spectrum is integrated from the minimum wavenumber $k_{\text{min}}$ to the maximum wavenumber $k_{\text{max}}$, where the ratio of the observed to the noise spectra (SNR) is less than 1.55 (vertical black solid line in Fig. 2b) or the frequency is 165 Hz (cutoff frequency of VMP) if it is less than the frequency of SNR $\approx 1.55$. Then the temperature dissipation rate $\chi$ is determined by

$$\chi = 6\kappa \int_{k_{\text{min}}}^{k_{\text{max}}} (S_{\text{obs}} - S_{\text{noise}}) dk,$$

where $S_{\text{obs}}$ and $S_{\text{noise}}$ are the observed and noise temperature gradient spectrum amplitude at each wavenumber, respectively.

The universal temperature gradient spectrum was first introduced by Batchelor (1959), who assumed a constant strain rate in the spatial scales smaller than the Kolmogorov length scale, $1/\kappa$, and then it was revised by Kraichnan (1968), who included the intermittency of the strain rate. In this paper we use the Kraichnan universal temperature gradient spectrum $S_{\text{theoretical}}$ with the following form in Roget et al. (2006):

$$S_{\text{theoretical}} = \frac{\chi q_K^{1/2} y_k}{\kappa k_B^2 y_k^2} \exp(-\sqrt{6} y_k),$$

where $y_k = \sqrt{q_K} \cdot k/k_B$ and $q_K$ is the Kraichnan constant. Here $q_K$ has been estimated as $q_K = 3.4-7.9$ (Antonia and Orlandi 2003; 5.26 ± 0.25: Bogucki et al. 1997, 2012; 7.9 ± 2.5: Sanchez et al. 2011). We used a fixed value of $q_K = 5.26$ that was introduced in Bogucki et al. (1997, 2012) and used in PF14.

In the present temperature-based method, $\epsilon$ is determined by fitting the universal spectrum to observed spectra with the MLE, unlike the shear-based method by which $\epsilon$ is obtained from integration of the observed shear spectrum. The rate $\epsilon$ from thermistors thus strongly depends on the accuracy of the fitness to the theoretical spectrum. The best-fitted theoretical spectrum is determined via $k_B$ by finding the maximum likelihood $C_{11}$ between the observed and theoretical spectra, which is defined as

$$C_{11} = \sum_{i=1}^{N} \ln \left( \frac{d}{S_{\text{theoretical}} + S_{\text{noise}}} \times \chi^2 \left[ \frac{dS_{\text{obs}}}{S_{\text{theoretical}} + S_{\text{noise}}} \right] \right),$$

where $\chi = 6\kappa \int_{k_{\text{min}}}^{k_{\text{max}}} (S_{\text{obs}} - S_{\text{noise}}) dk$, $S_{\text{obs}}$ and $S_{\text{noise}}$ are the observed and noise temperature gradient spectrum amplitude at each wavenumber, respectively.
where $\chi^2$ is the chi-square distribution based on that $S_{obs}$ is distributed as a $\chi^2$ probability density function with the degrees of freedom $d$ ($=2$; Ruddick et al. 2000). By applying various $k_B$ to $S_{obs}$, the maximum C11 and one $k_B$ are determined, and then $e$ is obtained via Eq. (4). An example of observed and fitted theoretical temperature spectra with the noise spectrum is shown in Fig. 2b. The spectra in Fig. 2a and in Fig. 2b were measured simultaneously at station ST017.

In the MLE method for fitting the universal temperature spectrum, we apply three criterion tests introduced by Ruddick et al. (2000) and used in PF14; mean absolute deviation (MAD) between observed and Kraichnan spectra, SNR, and likelihood ratio (LR). The LR test confirms whether an observed spectrum has a sharp roll off in high wavenumbers. We set MAD = 1.2, SNR = 1.5, and LR = 100 as the thresholds according to PF14.

d. Correction function for temperature gradient spectra

The FP07 thermistors are not fast enough to resolve the temperature fluctuation in the high wavenumber range, except for slow fall speeds ($<0.2 \text{m s}^{-1}$). Temperature spectra are attenuated at high frequency and could sometimes yield underestimated dissipation rates. For practical use, the thermistor signal needs to be corrected by applying correction functions to observed temperature gradient spectra. Such a correction function is represented by the reciprocal of the frequency response function of the temperature gradient spectrum. The time response of the temperature probe is assumed to be described by the following equation (Fofonoff et al. 1974)

$$T_{true} = T + \tau \cdot dT/dt,$$  
(9)

where $T$ is the measured temperature that is smoothed due to the slow thermistor response, $T_{true}$ is the true microscale temperature profile, and $\tau$ is the time constant, which represents response time. By expressing the temperature as a Fourier series, a frequency response function is derived as a single-pole low-pass filter using

$$T/T_{true} = 1/[1 + (f/f_c)^2],$$  
(10)

where $f$ is the frequency and $f_c$ is the half-attenuation frequency at which $T/T_{true} = 1/2$. Lueck et al. (1977) compared the frequency response of thermistors in a water tunnel with a spectrally calibrated platinum thin film thermometer. They found the theoretical response functions were similar to a single-pole filter in the frequencies lower than 12 Hz.

On the other hand, Gregg and Meagher (1980) found that the response function is described by a double-pole low-pass filter [Eq. (11)] for frequencies less than 25 Hz, although the single-pole filter was an equally good representation for frequencies less than 10 Hz,

$$T/T_{true} = 1/[1 + (f/f_c)^2],$$  
(11)

where $f_c$ represents the quarter-attenuation frequency ($T/T_{true} = 1/4$). The time constant is defined as the inverse of $f_c$: $\tau = 1/(2\pi \cdot f_c)$ in both single-pole and double-pole functions.

The time constant is ascertained to depend on the moving (fall) rate $W$ ($\text{m s}^{-1}$) as $\tau = \tau_0 W^7$. Gregg and Meagher (1980) showed $\tau_0 = 5 \text{ ms}$ and $\gamma = -0.32$ for the double-pole function, while Hill (1987) reported $\gamma = -0.5$. We adopt $\gamma = -0.32$ following Kocsis et al. (1999) and PF14, which compared the turbulence intensity from thermistors with shear probes.

3. Results

Turbulent energy dissipation rates $\epsilon_T$, estimated from thermistors attached to the free-fall VMP, are compared with concurrently measured $\epsilon_S$ from shear probes, as a reliable standard, that were also attached to the same VMP. Here subscripts T and S denote thermistor and shear probe, respectively. Before comparing $\epsilon_T$ with $\epsilon_S$, we should note there is an acceptable error coming from natural variability in estimating turbulence intensity. The rate $\epsilon_S$ has some variability between the S1 and S2 sensors (Fig. 3), even though they measure velocity shear simultaneously. Dots of the ratio $\log_{10}(\epsilon_{S1}/\epsilon_{S2})$ in Fig. 3 are not located closely along the horizontal solid straight line, which indicates $\epsilon_{S1} = \epsilon_{S2}$, but scattered around this line. According to Oakey (1982), who compared two simultaneously measured shear probes, $\epsilon$ has natural variability with a factor of 2. Thus, the ratio of $\epsilon_{S1}$ and $\epsilon_{S2}$ from two independent probes could be scattered within a factor of $2.8 = \sqrt{2}^2 + \sqrt{2}^2$ based on the law of error propagation. The scatterplots in Fig. 3 are distributed within the dashed lines, which denote $\epsilon_{S1} = 2.8^{\pm1} \epsilon_{S2}$. Shear-based $\epsilon$ data are used for comparison with temperature-based $\epsilon$ only when the ratios of S1 and S2 are within the acceptable range of 2.8. In the case of thermistors, there is some difference between two concurrently measured thermistors, but the difference is within a factor of 2.8, including the nominal variability of time constants, which is shown in the appendix.

a. The case without correction

Before frequency response corrections are applied, we compare $\epsilon_T$ from the T2 thermistor without correction
with \( \varepsilon_S \) from the S2 shear probe to see how different they are. We chose the T2 and the S2 probe for comparison because they have more passed spectra than other probes.

The rate \( \varepsilon_T \) from the thermistor without correction shows a bias that strongly depends on \( \varepsilon_S \) (Fig. 4). The equality \( \varepsilon_T = \varepsilon_S \) is achieved only in the range of \( \varepsilon_S \approx 10^{-10} \times 10^{-9} \text{Wkg}^{-1} \) (henceforth \( \text{Wkg}^{-1} \) of \( \varepsilon \) is omitted). The rate \( \varepsilon_T \) is significantly underestimated for \( \varepsilon_S > 3 \times 10^{-9} \), where \( \varepsilon_T < \varepsilon_S / 2.8 \), and most of the dots are distributed below the lower dashed line denoting \( \varepsilon_T / \varepsilon_S < 1/2.8 \) (Fig. 4b). This underestimation is amplified with increasing \( \varepsilon_S \), and the terrible underestimation is seen in the range of \( \varepsilon_S > 3 \times 10^{-8} \), where \( \varepsilon_T < \varepsilon_S / 10 \). This is consistent with the fact that temperature gradient spectra are attenuated in the high-frequency range because of the insufficient time response of thermistors.

In the weak turbulence range, \( \varepsilon_S < 10^{-10} \), \( \varepsilon_T \) is also underestimated compared with \( \varepsilon_S \) by a factor of 2.8. One reason for this underestimation could come from the measurement limit of the shear probe. The lower limit of shear estimation in the free-fall VMP2000 measurement is \( \varepsilon_S \sim 1–3 \times 10^{-10} \) according to the manufacturer’s specifications. Shear probes are generally more sensitive to the vibration of instruments than thermistors, which measure the scalar field. Thermistors hence could be used to detect weaker turbulence than shear probes. The case of the weak turbulence will be further discussed in section 4.

As mentioned above, the reasons for the underestimation of \( \varepsilon_T \) are different between the strong (\( \varepsilon_S > 3 \times 10^{-9} \)) and weak (\( \varepsilon_S \sim 10^{-10} \)) turbulence ranges. We next examine the corrections in the strong turbulence range by applying two frequency response functions and several time constants to temperature gradient spectra, and quantify the errors between \( \varepsilon_T \) and \( \varepsilon_S \).

b. Single-pole correction

First, the single-pole correction is applied to the observed spectra. The time constant \( \tau_0 \) is \( 7 \pm 3 \text{ms} \), which was reported by Rockland Scientific International Inc. as a nominal value. A \( \tau_0 = 7 \text{ms} \) indicates that the
temperature gradient spectrum is attenuated to half the amplitude at 23 Hz \([1/(2\pi \times 0.007)]\).

An example of the single-pole correction to the temperature gradient spectrum is shown in Fig. 2b. Higher frequencies are amplified, and the turbulent energy dissipation rate changes from \(e_T = 6.4 \times 10^{-9}\) to \(e_T = 5.2 \times 10^{-8}\), which is closer to the concurrently measured shear probe estimation, \(e_S = 1.0 \times 10^{-7}\) (Fig. 2a).

The time constant might not be unique among individual FP07 thermistors due to the difference in glass coating on the sensors, and the uncertainty of the time constant is \(\pm 3\) ms. Accordingly, we examine \(t_0 = 4, 7, 10\) ms with a fall rate dependence of \(\tau = 7_0 W^{-0.32}\), and compare \(e_T\) with concurrently measured \(e_S\) (Fig. 5a), in order to know how the difference of the time constant affects the estimation of \(e_T\).

The \(e_T\) estimated with the single-pole correction is improved in the large \(e_S\) range, compared with the no-correction case (Fig. 5a), but a bias and a strong dependence on \(e_S\) still remain. The case of \(t_0 = 4\) ms is the best, which is consistent with \(e_S\) for \(3 \times 10^{-10} < e_S < 10^{-9}\), where the average \((m)\) plus-minus standard deviation \((\sigma)\) of \(\log_{10}(e_T/e_S)\) is \(0.106 \pm 0.360\) and 95% of the data are within a factor between 0.251 and 6.48 \((=10^{(m+1.96\sigma)})\); Table 2). However, for \(10^{-7} < e_S < 10^{-6}\), \(\log_{10}(e_T/e_S)\) remarkably decreases with \(e_S\) (the green line in Fig. 5a).

In the case of \(t_0 = 7\) and 10 ms, the \(e_T\) estimate is improved for \(10^{-7} < e_S < 10^{-6}\) than in the case of \(t_0 = 4\) ms (Fig. 5a). The average plus-minus standard deviations of \(\log_{10}(e_T/e_S)\) are \(-0.365 \pm 0.192\) and \(-0.176 \pm 0.214\) for \(t_0 = 7\) and 10 ms (Table 2), respectively, whereas \(e_T\) is overcorrected to yield the overestimation for \(3 \times 10^{-10} < e_S < 10^{-9}\), where the average plus/minus standard deviation of \(\log_{10}(e_T/e_S)\) is \(0.378 \pm 0.406\), and 95% of the data are within a factor between 0.382 and 14.9 for \(t_0 = 7\) ms. Underestimation in the large \(e_S\) and overestimation in the small \(e_S\) are also seen in the previous comparisons of Kocsis et al. (1999) and PF14. In our data, a large bias with the overestimation for \(3 \times 10^{-10} < e_S < 10^{-9}\) and the decreasing trend of \(\log_{10}(e_T/e_S)\) with \(e_S\) for \(e_S > 10^{-9}\) remain for every \(t_0\). These biases depending on \(e_S\) even after the single-pole correction need to be removed. We next seek the possibility of the correction with the double-pole function.

c. Double-pole correction

Gregg and Meagher (1980) proposed the double-pole low-pass filter \([Eq. (11)]\) type of frequency response function for thermistors. Here we apply the double-pole response functions with \(t_0 = 2, 3,\) and 4 ms, and \(\tau = 7_0 W^{-0.32}\). For the double-pole low-pass filter function, a \(t_0 = 3\) ms means that temperature gradient spectrum is attenuated to the quarter amplitude at 53 Hz \([1/(2\pi \times 0.003)]\) and to the half at 34 Hz.

In Fig. 5b it is shown that \(e_S\) estimated with the double-pole correction is found to be more suitable compared with the cases of the single-pole correction in that the decreasing trend of \(\log_{10}(e_T/e_S)\) with \(e_S\) for \(e_S > 10^{-9}\) is much reduced. The overcorrection for \(3 \times 10^{-10} < e_S < 10^{-9}\) in the case of the single-pole function is also improved in the double-pole correction.

The average of \(\log_{10}(e_T/e_S)\) is closest to zero for \(e_S > 3 \times 10^{-10}\) when the double-pole correction with \(t_0 = 2\) ms is applied, yielding \(m \pm \sigma = -0.079 \pm 0.440\) (Table 2b). For \(3 \times 10^{-10} < e_S < 10^{-8}\), the average of \(\log_{10}(e_T/e_S)\) is \(-0.08 \sim 0.03\), and \(e_T\) seems to be well fitted to \(e_S\), whereas for \(10^{-7} < e_S < 10^{-6}\), the average is \(-0.665\), and \(e_T\) is significantly less than \(e_S\) \((10^{-0.665} = 0.216 < 1/2.8))\). This underestimation is not negligible for \(10^{-7} < e_S < 10^{-6}\).
For $t_0 = 3$ ms, averaged $e_T$ estimates are within a factor of 2 of $e_S$ for every $e_S$ range of $3 \times 10^{-10} < e_S < 10^{-6}$. The double-pole correction with $t_0 = 3$ ms is thus more suitable for the wider range than with 2 ms in that the estimation has less bias throughout a wide range of $e_S$. A $t_0 = 4$ ms causes overcorrection because the averages show plus values for every $e_S$ range.

Scatterplots between $e_S$ versus $e_T$ with the double-pole correction with $t_0 = 3$ ms are shown in Fig. 6a, and the dependence of $\log_{10}(e_T/e_S)$ on $e_S$ is shown in Fig. 6b. A large part of the corrected $e_T$ is within a factor of 2.8 of acceptable range for a wide range of $e_S \sim 10^{-10} - 3 \times 10^{-7}$. Confidence intervals of $\log_{10}(e_T/e_S)$ in Fig. 6b are computed in the $e_S$ segments $10^{-n} < e_S < 10^{-n+1}$ for $n = 11, 10, \ldots, 7$; 68% and 95% of the data are distributed within the factors of 2.8 and 10, respectively.

### 4. Discussion

In the present study, the $e_T$ estimation from FP07 thermistors with the double-pole correction and $t_0 = 3$ ms is shown to be acceptable for a wide range of

### Table 2. List of the mean $m$ and the standard deviation $\sigma$ of $\log_{10}(e_T/e_S$), where $e_T$ and $e_S$ are estimated from the FP07 thermistor and from a concurrently measured velocity shear probe, respectively, and their dependence on the correction functions between (a) the single-pole function [Eq. (10)] and (b) double-pole function [Eq. (11)], and on $t_0 = 0, 4, 7, \text{ and } 10$ ms for (a) and on $t_0 = 0, 2, 3, \text{ and } 4$ ms for (b).

#### (a) Single pole

<table>
<thead>
<tr>
<th>$3 \times 10^{-10} &lt; e_S &lt; 10^{-9}$</th>
<th>$10^{-9} &lt; e_S &lt; 10^{-8}$</th>
<th>$10^{-8} &lt; e_S &lt; 10^{-7}$</th>
<th>$10^{-7} &lt; e_S &lt; 10^{-6}$</th>
<th>$3 \times 10^{-10} &lt; e_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$0.152$</td>
<td>$0.303$</td>
<td>$-0.929$</td>
<td>$-0.267$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$0.001$</td>
<td>$0.038$</td>
<td>$0.044$</td>
<td>$0.457$</td>
</tr>
</tbody>
</table>

#### (b) Double pole

<table>
<thead>
<tr>
<th>$3 \times 10^{-10} &lt; e_S &lt; 10^{-9}$</th>
<th>$10^{-9} &lt; e_S &lt; 10^{-8}$</th>
<th>$10^{-8} &lt; e_S &lt; 10^{-7}$</th>
<th>$10^{-7} &lt; e_S &lt; 10^{-6}$</th>
<th>$3 \times 10^{-10} &lt; e_S$</th>
</tr>
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<tr>
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<td>$0.038$</td>
<td>$0.044$</td>
<td>$0.457$</td>
</tr>
</tbody>
</table>

Fig. 6. As in Fig. 4, but for the spectra after applying the double-pole correction with the time constant of $t_0 = 3$ ms. (a) Color denotes $K_T = \sqrt{[2(\partial T/\partial z)^2]}$. (b) Red (blue) error bars denote the confidence intervals, where 68% (95%) of the data are within the error bars in each $e_S$ range.
$\epsilon_S \sim 10^{-10} \cdot 3 \times 10^{-7} \text{ W kg}^{-1}$ in the case of free-fall vertical microstructure profilers with $W \sim 0.6-0.7 \text{ m s}^{-1}$, whereas previous studies reported various correction functions and time constants as documented in sections 2 and 3. We here discuss the reasons why the reported corrections have been diverse and the limitation of the present results.

a. Measurement limit in the strong turbulence and its dependence on sensor moving speed

The present study shows $\epsilon_T$ is best matched to $\epsilon_S$ in the case where the double-pole correction function with the time constant of 3 ms is applied to observed temperature gradient spectra. However, $\log_{10}(\epsilon_T/\epsilon_S)$ in Fig. 6b shows a slight decreasing trend with $\epsilon_S$ for increasing $\epsilon_S > 10^{-8} \text{ W kg}^{-1}$. This slight decreasing trend of $\epsilon_T/\epsilon_S$ could be due to the insufficient correction in high-frequency ranges.

In stronger turbulence fields, spectra shift to higher wavenumber (and thus higher frequency) where spectra might not be fully corrected. This could cause the underestimation of $\chi$ and $\epsilon$. Since the Batchelor wavenumber is represented as $k_B = k_p \sqrt{6g_k}$ via d(Eq.8)/dk = 0, where $k_p$ is the wavenumber at the spectral peak, $\epsilon_T$ is represented by

$$\epsilon_T = (2\pi)^4 \times (6g_k)^2 \times f_p^4 / W^4 \times \nu \times \kappa^2,$$

(12)

where $f_p = k_p \cdot W$ is the spectral peak frequency, assuming the falling speed $W$ is constant; $\epsilon_T$ is thus a function of $(f_p / W)^4$.

The horizontal axis of Fig. 5 can be converted from $\epsilon_S$ to $f_p$ via Eq. (12) as shown in Fig. 7, where $\log_{10}(\epsilon_T/\epsilon_S)$ severely depends on $f_p$ in the case without correction. Even though the half-attenuation frequency of 23 Hz (equivalent to the time constant of 7 ms in the single-pole correction) is expected, $\epsilon_T/\epsilon_S$ is less than 1/2.8 for $f_p > 10$ Hz as shown in the blue curves in Fig. 7. Without any corrections and for $W = 0.64 \text{ m s}^{-1}$, the maximum acceptable $\epsilon_T$ is $3 \times 10^{-9} \text{ W kg}^{-1}$ at which the blue curve crosses $\epsilon_T/\epsilon_S = 1/2.8$ in Fig. 5. This maximum acceptable $\epsilon_T$ could change with variable $W$ because $\epsilon$ and $k_p$ are the function of $W$ via Eq. (12), as long as the upper limit of $f_p$, where the spectrum is not strongly attenuated, is fixed; the maximum limit is $5 \times 10^{-6} \left[ = 3 \times 10^{-9} \times (0.1/0.64)^{-4} \right] \text{ W kg}^{-1}$ for $W = 0.1 \text{ m s}^{-1}$, and for $W = 1.0 \text{ m s}^{-1}$ the limit is $5 \times 10^{-10} \left[ = 3 \times 10^{-9} \times (1.0/0.64)^{-4} \right] \text{ W kg}^{-1}$. These indicate that the maximum limit heavily depends on $W$; for the slow-moving platforms, correction is not necessary even for the strong turbulence field, whereas for the faster speed, correction is inevitable even for the weak turbulence field. In the case without correction, the maximum limit can be estimated as above.

In the case with correction, the estimate of the maximum limit is not straightforward, because $f_p$ also changes by multiplying the correction functions [Eq. (10) for the single-pole and Eq. (11) for the double-pole] whose time constant is a function of $W$ through $\tau = \tau_0 W^{-0.32}$. The upper limit of $f_p$ becomes higher by amplifying spectra at greater frequency as well as through a larger time constant with decreasing $W$. If the dependence of the time constant $\tau$ on $W$ were ignored to simplify the discussion, this could yield the maximum limit of $\epsilon_T$ for variable $W$ via Eq. (12). For the present free-fall VMP mean $W (=0.64 \text{ m s}^{-1})$, the maximum limit of $\epsilon_T$ is $7 \times 10^{-7} \text{ W kg}^{-1}$ (the magenta curve in Fig. 5b). Since $\epsilon_T$ is proportional to $(f_p / W)^4$, the maximum limit of $\epsilon_T$ would shrink to $1 \times 10^{-7}$ $\left[ = 7 \times (1/0.64)^4 \right] \text{ W kg}^{-1}$ for $W = 1 \text{ m s}^{-1}$. To take the dependence of $\tau$ on $W$, we need additional studies in the future.

The variable correction functions and time constants in the previous studies (Kocsis et al. 1999; PF14) could
be explained by the diversity of sensor $W$ and the turbulence intensity of target water. Qualitatively, for larger $W$, the maximum limit of $\epsilon_T$ decreases with a function of $W^{-1}$ for the same correction function and time constant. To measure strongly turbulent water, the double-pole function and/or larger time constants are necessary to recover the attenuated spectra. Previous studies using the single-pole correction (Kocsis et al. 1999; PF14) were conducted with relatively smaller moving speed (0.08 m s$^{-1}$ in Kocsis et al. 1999; 0.4 m s$^{-1}$ in PF14) than in the present study (0.6–0.7 m s$^{-1}$).

b. Measurement limit in the weak turbulence

In a large part of the ocean interior, especially from the intermediate to deep Pacific over smooth bottom topography, the turbulent energy dissipation rate is weak and $\epsilon < 10^{-10}$ W kg$^{-1}$ (e.g., Gregg and Sanford 1988). In such a weak turbulence, estimations from shear probes are not reliable owing to the influence from instrumental noise. The present study shows that $\epsilon_T$ from a thermistor is generally much (one or two orders of magnitude) less than $\epsilon_S$ for $\epsilon < 3 \times 10^{-10}$ W kg$^{-1}$. This small $\epsilon_T$ could represent real turbulence situations because the thermistor is less sensitive to the vibration and motions of profilers and the noise level could be lower than that of shear probes.

However, we need to be careful about the thermistor measurements because the performance of thermistor measurements under weak turbulence has not been well examined. In particular, isotropic assumption, under which Batchelor or Kraichnan theories are established, might not be fully satisfied in such weak turbulence fields. In this subsection, we discuss the availability of thermistor measurements in weak and anisotropic turbulence.

For shear probe measurements under anisotropic turbulence, Yamazaki and Osborn (1990) showed that $\epsilon_S$ from vertical shear of turbulent velocity under the assumption of isotropy is greater by at most 35% than the true $\epsilon$ even for the anisotropic condition of $20 < I < 100$, where the buoyancy Reynolds number, $I = \epsilon/(\nu \cdot N^2)$, is an indicator of isotropy. Here, $N$ is the buoyancy frequency. When $I < 100$, then it is regarded as anisotropy (Gargett, et al. 1984; Gargett 1985).

In the anisotropic range ($20 < I < 100$) where observed $\epsilon_S$ is reliable ($\epsilon_S > 3 \times 10^{-10}$ W kg$^{-1}$), $\log_{10}(\epsilon_T/\epsilon_S)$ is within the reasonable range ($1.28 < \epsilon_T/\epsilon_S < 2.8$) as shown in Fig. 8, where the thick red line denoting the 11-point running mean of $\log_{10}(\epsilon_T/\epsilon_S)$ is within a factor of 2.8, which is denoted by the black dashed lines. This indicates that the $\epsilon_T$ estimate is compatible to the shear probe estimate even in the anisotropic range of $20 < I < 100$. A little larger value of $\epsilon_T$ than $\epsilon_S$ in $20 < I < 100$ is probably because the correction is applied. In the case without correction (the thin red line in Fig. 8), $\epsilon_T/\epsilon_S$ is a little less than 1, but $\epsilon_T/\epsilon_S$ is still within the reasonable range (within the...
black dashed lines in Fig. 8) and whether the correction is applied does not largely change $\varepsilon_T$. This is because the weak turbulence estimation mainly uses the low-frequency components that are not strongly influenced by attenuation.

In the range of $I < 20$, anisotropy is further developed, and the shear probe data with $\varepsilon_S < 3 \times 10^{-10} \text{W} \cdot \text{kg}^{-1}$ are less than the lower limit of the manufacturer’s specification and could be unreliable. In particular, for the extremely small turbulent temperature diffusivity $K_T < 10^{-5} \text{m}^2 \cdot \text{s}^{-1}$, which is estimated on the basis of $K_T = \chi/[2(\partial T/\partial z)^2]$ (Osborn and Cox 1972), most of the values of $\varepsilon_T$ are much less than $\varepsilon_S$ and $\varepsilon_T/\varepsilon_S < 1/10$ (Fig. 6a). That is, turbulence estimates from velocity shear and temperature are largely different. We need further discussion in this weak turbulence range of $I < 20$ or $\varepsilon_S < 3 \times 10^{-10} \text{W} \cdot \text{kg}^{-1}$.

According to the direct numerical simulations (DNS) by Shih et al. (2005), diapycnal diffusivity plus molecular diffusivity $K_\rho + \kappa$ is $0.2 \varepsilon/N^2$, which is the same as in Osborn (1980), and thus $(K_\rho + \kappa)/\kappa = 0.2 \times \text{Pr} \times I$ for $7 < I < 100$, where the Prandtl number Pr is defined as $\nu/\kappa$. When $I < 7$, $K_T$ becomes equivalent to $\kappa(\sim 10^{-7} \text{m}^2 \cdot \text{s}^{-1})$. From another DNS under anisotropy for small $I$ (Godeferd and Staquet 2003), turbulent thermal dissipation rate $\chi$ and $K_T$ are reduced to $5/9$ of the ones under isotropic approximation.

The relationship between $I$ and $K_T$ is examined with the present data to see whether the shear- or temperature-based $\varepsilon$ is consistent with the above-mentioned DNS results, as shown in Fig. 9. Here $I_T = \varepsilon_T/(\nu \cdot N^2)$ with the temperature-based $\varepsilon_T$ (red dots in Fig. 9), $I_S = \varepsilon_S/(\nu \cdot N^2)$ with the shear-based $\varepsilon_S$ (blue crosses in Fig. 9), and $K_T = 5/9 \times \chi/[2(\partial T/\partial z)^2]$ for the data that satisfy $\varepsilon_{S,T} < 3 \times 10^{-10} \text{W} \cdot \text{kg}^{-1}$, $I_{S,T} < 20$, and $-15^\circ < \text{Tu} < 45^\circ$ at which the Turner angle $\text{Tu}$ is in the range $K_T = K_\rho$ without double diffusion.

The relationships between $I$ and $K_T$ (Fig. 9) show that the temperature-based estimate is more consistent with the above-mentioned DNS results. The equation $(K_T + \kappa)/\kappa$ (red dots in Fig. 9) takes a nearly constant of 2 for the temperature-based $I_T < 5$ indicating $K_T = \kappa$, and increases with the slope similar to the one from the DNS for $5 < I_T < 20$. On the other hand, for the shear-based $I_S$ (blue crosses in Fig. 9), $K_T$ is much less than the one from the DNS for $7 < I_S < 20$, where the DNS suggests that $K_T$ would take much larger values and increase with $I$.

![Fig. 10. Examples of the spectra of (a) velocity shear and (b) temperature gradient. Data satisfying $K_T < 10^{-5} \text{m}^2 \cdot \text{s}^{-1}$ and $\varepsilon_T/\varepsilon_S < 1/10$ are shown. Black curves are observed spectra, and red curves denote universal spectra fitted to the observed ones. Thick light blue curves are for the data with the lowest temperature gradient spectrum in all the data. The thick magenta curve is the shear spectrum with the lowest energy dissipation rate in all the shear data.](Unauthenticated)
Let us return to the observed spectra with the large difference between $e_T$ and $e_S$ ($e_T/e_S < 1/10$) in the weak turbulence with $K_T < 10^{-5} \text{ m}^2 \text{s}^{-1}$, because appropriate measurements have been judged by the fitness of observed spectra to the universal spectrum as Nasmyth for shear and Kraichnan for temperature (Fig. 10). Both the observed shear (Fig. 10a) and temperature gradient (Fig. 10b) spectra (thin black curves) look nicely fitted to the Nasmyth and Kraichnan universal spectra (thin red curves), respectively.

On the other hand, the level of the observed shear spectra is the same order of magnitude with the lowest shear spectrum in all the data (thick magenta curve in Fig. 10a), while all the observed temperature gradient spectra are one order of magnitude larger than the lowest temperature gradient spectrum (thick light blue curve), which was used as the noise spectrum in the present study. The shear spectrum corresponding to this lowest temperature gradient spectrum denoted by the light blue curve in Fig. 10a indicates that the observed shear spectra are not discernable from noise if we assume that the light blue spectra are in the noise level. Spectrum shape is hence not the appropriate way to judge the reliability of measurements in such weak turbulence. The present study implies that noise should be also considered for the measurement in the weak turbulence regime.

These results suggest that the shear spectra are influenced by instrument noise although they look fitted to the Nasmyth spectra and that the temperature-based estimate is more reliable in the weak turbulence regime at least from the observed spectrum analysis in the present study. Since anisotropy also could influence the universal spectra derived under the assumption of isotropy, further analysis and theoretical studies are required to reveal whether the temperature- or shear-derived method is reliable in the weak turbulence regime.

**Acknowledgments.** The authors thank Dr. Rolf Lueck for his indication on the importance of correction in the high turbulence range and information on the data processing using thermistors and on VMP2000 and PF14. Thanks are extended to Drs. Sachihiko Itoh and Takahiro Tanaka for performing VMP observations. The authors also express sincere appreciation to the captain, officers, and crew of the R/V *Hakuho-Maru* during the KH-09-4 cruise. Thanks are extended to the anonymous reviewers, whose invaluable comments improved this paper. This study is partially supported by KAKENHI Grants 23710002, 25257206, and 15H05818.
APPENDIX

Difference between Two Individual Thermistors

The dissipation rates from two thermistors are compared. The difference in the dissipation rates could come from the difference in glass coating, which causes the time lag of heat transfer from the surface of the glass coating to the core of the sensor. Most of the $e$ and $\chi$ from the T2 and T1 thermistors without correction (represented by the black dots in Fig. A1) agree within the acceptable error (i.e., within a factor of 2.8 as shown in the black dashed lines in Fig. A1), although $e$ and $\chi$ from T2 are a little larger than from T1. The difference could be interpreted as the difference in the time constants of the two thermistors.

$$\left[ \frac{1}{1 + (2\pi \cdot f \cdot \tau_1 W^{-0.32})^2} \right]^2 \left[ \frac{1}{1 + (2\pi \cdot f \cdot \tau_2 W^{-0.32})^2} \right]^2 = \text{spectrum ratio}, \quad (A1)$$

The thick light blue curve in Fig. A2 takes the value of 0.8 at about 30 Hz, indicating the response of the T1 sensor is less than that of T2 by 0.8 at 30 Hz. Substituting $W = 0.65 \text{ m s}^{-1}$, $\tau_2 = 3 \text{ ms}$, $f = 30 \text{ Hz}$, and spectrum ratio = 0.8 in the equation leads to $\tau_1 = 3.55 \text{ ms}$. When the single-pole frequency response function with the time constant of 7 ms is assumed, the alternative equation of Eq. (A1) without the square of the left-hand side of Eq. (A1) yields $\tau_1 = 8.16 \text{ ms}$, which is within the nominal value ($\tau = 7 \pm 3 \text{ ms}$) as described by Rockland Scientific International Inc. A difference in time constants among individual thermistors does exist; however, the difference is not large for the thermistors this study used. We need further study to evaluate how different the time constants are by using more thermistors.

At ST017 and ST034 (red dots and curves in Figs. A1 and A2, respectively), estimates of $e$ and $\chi$ from the T2 deviate from the ones from T1 by a factor of about 3. This deviation is caused by the signal attenuation in the frequency range greater than 1 Hz (red curves in Fig. A2), which is possibly due to biological fouling over the thermistor—of course we need further studies.

REFERENCES


