A Comparison of Vertical Atmospheric Wind Profiles Obtained from Monostatic Sodar and Unmanned Aerial Vehicle–Based Acoustic Tomography

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ABSTRACT

The natural sound generated by an unmanned aerial vehicle is used in conjunction with tomography to remotely sense the virtual temperature and wind profiles of the atmosphere in a horizontal plane up to an altitude of 1200 m and over a baseline of 600 m. Sound fields recorded on board the aircraft and by an array of microphones on the ground are compared and converted to sound speed estimates for the ray paths intersecting the intervening medium. Tomographic inversion is then used to transform these sound speed values into two-dimensional profiles of virtual temperature and wind vector, which enables the atmosphere to be visualized and monitored over time. The wind vector and temperature estimates are compared to measurements taken by a collocated midrange Doppler sodar and sensors on board the aircraft. Large-eddy simulations of daytime atmospheric boundary layers and error models of the tomographic inversion and sodar are also used to assess the magnitude and nature of anticipated differences. Both the simulations and field trials data show similar levels of correspondence between the tomographically derived and independently observed measurements.

1. Introduction

There is a need for good wind speed measurements to identify and examine the properties of the atmospheric boundary layer (ABL). Features of interest include the nocturnal low-level jet, convective structures in the daytime ABL, elevated inversions and temperature structures, ABL turbulence, sea-breeze circulations, wind flow over complex terrain, gravity waves, and frontal

passages. Several methods for observing wind energy exist: cup and sonic anemometers (usually mounted on masts), sodar, lidar, radio acoustic sounding system (RASS), radar, satellite-based techniques, and radiosondes. Each has benefits and drawbacks.

Point-source observations from anemometers provide high wind velocity resolution and precision (0.01 and 0.05 ms^{-1} , respectively), but erection and maintenance costs for mast-based instruments become high as altitudes increase; and the presence of masts can obstruct or distort local wind flow (Hasager et al. 2008). Measurements by remote sensing instruments such as Doppler sodar, lidar, radar, and RASS offer advantages over point measurements (Hooper and Eloranta 1986; May 1993; Lang and McKeogh 2011): they yield wind information from different heights simultaneously. Although relatively inexpensive sodars designed to sound the ABL from a few meters to about 200 m (with 8-m vertical resolution) have been developed (Casasanta et al. 2014; Argentini et al. 2013;

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Mastrantonio and Fiocco 1982)—and many commercially available scanning Doppler lidars have ranges on the order of 2–5 km and a resolution of 25–50 m (depending on aerosol loading)—in general these traditional instruments are less accurate and considerably more costly than anemometers.

Sodar, lidar, and radar operate using similar principles: sodars emit pulses of acoustic energy, lidar near-infrared light energy, and radar radio energy. The scatterers are turbulent refractivity fluctuations, particulate matter carried by the atmosphere, and turbulent eddies whose scale size is half the wavelength of the emitted signal. RASS operates on the basis that by correctly matching the wavelengths of the acoustic and radio emissions, Bragg scattering can be induced. All then obtain wind velocity and height information from the Doppler shift and time delay information derived from the signal scattered back to the receiver. Similar assumptions are made regarding the homogeneity of the relevant scatterers and the manner in which they are carried by the wind. For a more complete understanding of the operating principles of each instrument, the reader is referred to Hall et al. (1984), Hooper and Eloranta (1986), Singal (1997), Antoniou et al. (2003), May et al. (1989, 1988, 1990), and Strauch et al. (1984).

Radiosondes and dropsondes offer a degree of deployment mobility, more complete meteorological datasets (wind velocity, thermodynamic temperature, atmospheric pressure, and relative humidity), and rise well into the stratosphere, but they offer only point observations with poor temporal and spatial resolution. Satellite-based techniques provide snapshots over wide areas at regular but infrequent intervals, but the grid resolutions and coverage near coastlines are typically poor. Synthetic aperture radar (SAR) can achieve higher resolution and better coverage, but it is less accurate. Unmanned aerial vehicle (UAV) techniques have also been developed (Becker et al. 1999; Holland et al. 2001; Soddell et al. 2004). Over several decades many researchers have intercompared these techniques and shown their measurements to be highly correlated, for example, Jasperson (1982), Gage et al. (1988), Ecklund et al. (1990), Roettger and Larsen (1990), Tsuda et al. (1995), and Kelley et al. (2007). All have advanced our understanding of the ABL.

Acoustic atmospheric tomography (AAT) has also been used to observe the virtual temperature and wind velocity profiles of the atmosphere, and to monitor their evolution in time and space (note: virtual temperature is the temperature dry air would have if it had the same density as a sample of moist air at the same pressure; throughout this paper, temperature refers to virtual temperature, unless otherwise stated). Initially, AAT

was based on a set of masts a few meters above the ground that supported microphones and loud speakers covering an area of 200 m × 240 m (Wilson and Thomson 1994). Subsequently, other arrays have been built (Ziemann et al. 1999; Arnold et al. 1999; Jovanovic et al. 2009), some of which allow 3D measurements (Wilson et al. 2001; Vecherin et al. 2008a). Several tomographic methods have also been developed (Vecherin et al. 2006, 2007, 2008b,a; Barth and Raabe 2011; Kolouri and Azimi-Sadjadi 2012), including methods that make use of noise sources such as birds or meteors (Spiesberger and Fristrup 1990) or commercial aircraft (Ostashev et al. 2000; Wilson et al. 2001). An overview of the field is available in Ostashev et al. (2008).

Until recently AAT was largely restricted to near-horizontal atmospheric soundings close to ground level (<50 m). Based on observations made during the overflight of a UAV, AAT techniques have been developed that allow determination of temperature and wind profiles up to altitudes of about 2 km (Finn and Rogers 2015, 2016c). This potentially overcomes the limitations of short operating ranges suffered by midrange sodar and lidar (typically <300 m for medium-performance systems) and negates the need to horizontally interpolate or extrapolate wind speed and variance from lidar or sodar observations. Multi-UAV and air-underwater techniques are also available (Finn and Rogers 2016a, 2017). Moreover, if 2D microphone arrays are used, 3D tomographic profiles may be reconstructed (Rogers and Finn 2013c); and, as equipment costs a fraction of a lidar or sodar, deployment in inaccessible or hazardous locations is more justifiable. There are, of course disadvantages to UAV-based AAT (Finn and Rogers 2015, 2016a), such as the current need for manned operation (for safety reasons) that precludes extended duration continuous (months/years) autonomous observation.

The performance of AAT has been examined using synthetic atmospheres generated using radial basis functions (Finn and Rogers 2015), large-eddy simulation (LES) data (Finn and Rogers 2016c, 2016a), and limited field trials (Finn and Rogers 2015; Rogers and Finn 2013b,a). Real-world observations have never been compared to vertical profiles obtained from independent instruments such as sodar.

In this paper, we summarize the results of a short study that compares the wind speeds derived from a midrange Doppler sodar with those estimated using AAT. The time delay observations are derived through exploitation of the natural signature of a UAV as it overflies a set of microphones located on the ground (Finn and Franklin 2011). The paper commences with a brief description of sodar and UAV-based AAT, how each obtains estimates of horizontal and vertical wind

components, and the kinds and magnitude of real-world measurement errors that can be expected. Computer simulations are then used to intercompare the two approaches, followed by the results of a short field trial. The aim is to estimate the uncertainties associated with UAV-based AAT wind profile retrieval.

2. Modeling the observation systems

A detailed discussion on monostatic Doppler sodar and UAV-based AAT is beyond the scope of this paper, so for a more complete treatment the reader is referred to [Antonioni et al. \(2003\)](#), [Spizzichino \(1974a\)](#), and [Singal \(1997\)](#) for sodar, and [Finn and Rogers \(2015, 2016c\)](#) [Rogers and Finn \(2016\)](#) for UAV-based AAT. The discussion below concentrates on the measurement errors in these systems and the models used in the simulations described in the following section.

a. Sodar

Sodar emits a series of acoustic pulses at frequency f_i and measures the delay and frequency of the returned signals to estimate the wind profile as a function of height directly above the sodar: coherent integration improves the echo signal-to-noise ratio (SNR). Each transmitted sound pulse is scattered by fluctuations in the refractive index of air and the echo delay $t_p = c/2z$ converted to height z using the average speed of sound along the ray c . The scattering fluctuations typically develop as a result of temperature and humidity gradients that are coupled to atmospheric turbulence.

At the wavelengths normally used in sodar, this turbulence is assumed to be in the Kolmogorov inertial subrange; that is, it is considered locally homogeneous and statistically isotropic. Monostatic sodars are sensitive only to thermal fluctuations ([Singal 1997](#)), and as the fluctuations are assumed to be homogeneously distributed and move with the prevailing wind, the frequency of the emitted signal is Doppler shifted during the scattering process by an amount, $\delta f = f_i - f_r \approx 2f_i \mathbf{V}_s/c$, where the received frequency f_r is proportional to the velocity of the scatterer/wind $\mathbf{V}_s = (u, v, w)$ in the direction of the sodar beam. For monostatic sodar systems, one beam is directed vertically, so the vertical wind speed w can be calculated directly from the Doppler shift, $\delta f_0 = 2f_i w/c$. The horizontal wind components, u (east) and v (north), are computed using beams tilted off vertical in two horizontally perpendicular directions, θ (elevation) and ψ (azimuth), and they solve the set of equations obtained from each beam, $\delta f_{1,2} = f_r[1 - (u \sin \theta \cos \psi + v \sin \theta \sin \psi + w \cos \theta)/c]$.

Several kinds of uncertainty need to be accounted for: the uncertainty $\sigma_{\mathbf{V}}$ in the wind velocity, which is derived

from the uncertainty in the Doppler shift σ_f of the returned signal; the uncertainty σ_z in the height to which the wind value is ascribed; the uncertainty caused by spatial variation in wind over the scattering volume of the sodar beams; the uncertainty caused by temporal variation in the echo structure over the period of coherent integration used to improve the SNR; and the uncertainty caused by spectral broadening of the returned signal by scattering or signal processing (uncertainty is the real physical spread in a measured variable, while error is the difference between reported and true values). For $\text{SNR} > 5$ dB, the uncertainty may be shown to be $\sigma_{\mathbf{V}} \approx \pm 1.4 \text{ m s}^{-1}$ and $\sigma_z \approx \pm 12 \text{ m}$ for an altitude of 200 m ([Miller and Rochwarger 1970](#); [Spizzichino 1974b](#)); and improvements in signal processing since these early papers allow for increased precision at much weaker SNR, for example, $\sigma_{\mathbf{V}} \approx \pm 0.4 \text{ m s}^{-1}$ at $\text{SNR} < -5$ dB ([Mastrantonio and Fiocco 1982](#); [Singal 1997](#); [Coulter and Kallistratova 2004](#)).

In the simulations described in the next section, these uncertainties are modeled using techniques first presented by [Spizzichino \(1974a\)](#) in which the pulse is assumed to be confined to a conical beam and the measured wind averaged over a height interval is limited by the transmitted pulse duration, the duration of the receiver gate window, and the finite beamwidth of the acoustic antenna.

Although sound rays are refracted, for $\theta > 70^\circ$, the error δz in the estimate of z is considerably smaller than the uncertainty in height (i.e., $\delta z \ll \sigma_z$) and may therefore be ignored. However, as estimates of wind speed are derived from the Doppler shift, even for modest wind speeds ($u = w = 7 \text{ m s}^{-1}$) errors in the horizontal and vertical wind components δu and δw , respectively, derived from near-vertical incidence are on the order of 1 and 0.3 m s^{-1} , respectively, and must be computed ([Spizzichino 1972](#); [Georges and Clifford 1972](#)).

b. AAT

Tomography is a subset of inverse theory from which a data kernel is formed by integrating the model parameters $\mathbf{m}$ along the ray paths that intersect a medium. Measurements, $\mathbf{b}_{\text{obs}}$, of observed time delays are used to infer values of the sound speed (model) parameters. The model and the dataset are related by a set of explicit equations $\mathbf{b}_{\text{obs}} = \mathbf{a}(\mathbf{m})$ that may be written as $\mathbf{b}_{\text{obs}} = \mathbf{A}\mathbf{m}$ if the relationship between the model parameters is linear, where $\mathbf{A}$ is the data kernel. The model used to compute the tomographic inversions in this paper is described in detail in [Finn and Rogers \(2015, 2016c\)](#), and [Rogers and Finn \(2016\)](#) and, for convenience, briefly in the [appendix](#). The error in the time delay estimates derived using this model is calculated using ([Finn and Rogers 2016c](#))

$$\sigma_{ip} = \sqrt{\sigma_{ip0}^2 + \frac{(\sigma_{f_r^C}^2 + \sigma_{f_r^M}^2)}{\left(\frac{\partial f_r}{\partial t}\right)^2} + \left[\frac{(f_r^C - f_r^M)\sigma_{\delta}}{\left(\frac{\partial f_r}{\partial t}\right)^2} \right]^2}, \quad (1)$$

where σ_{ip0} is the error associated with nominal propagation; $\sigma_{f_r^C}$ and $\sigma_{f_r^M}$ are the errors associated with estimating the computed and measured frequencies f_r^C and f_r^M , respectively; and σ_{δ} is the error associated with obtaining the derivative of $\partial f_r(t + t_{p0})/\partial t$. In other words, errors in time delay estimation for UAV-based AAT may be modeled using three terms: errors in the nominal delay [term 1 in Eq. (1)], errors in measuring the frequencies f_u and f_r (term 2, noting that f_r^C is computed from f_u ; see the [appendix](#)), and errors caused by unmodeled or unknown states of the problem (term 3). A description of the components of error and their magnitude may be found in [Finn and Rogers \(2016c\)](#).

3. Simulated comparisons of UAV-based AAT and sodar

Simulations representative of a canonical daytime convective ABL were generated by [Sullivan and Patton \(2011\)](#) using LES. The simulations provide 3D wind fields and temperature profiles at each point in a lattice for a volume of atmosphere $5120\text{ m} \times 5120\text{ m}$ (horizontal) $\times 2048\text{ m}$ (vertical) over a uniform grid mesh of 1024^3 points. The data carry forward in time for 25 large-eddy turnover times, or about 38 min. The equations and input parameters used to simulate this weakly sheared daytime convective PBL are contained in [Sullivan and Patton \(2011\)](#).

To examine the anticipated correspondence between AAT and sodar, 500 simulations were run using the LES data and conditions generally representative of the trials data described in the following section. The simulated AAT scenarios modeled a linear array of ground microphones located over a baseline of 600 m. The coordinate system is such that the positive x axis is in the direction of UAV travel through the sensor array (left to right), the positive y axis is out of the page, and the z axis is vertical. The origin of the system coincides with the commencement of the LES lattice/data. Sensor separation was 25 m.

Time delay observations t_{pj} were computed by integrating along the straight-line path between the UAV and the microphone using Eq. (A5), and the effects of refraction determined per the [appendix](#) ([Urlick 1983](#)). Additive Gaussian white noise (AGWN) was computed in line with parametric estimates noted or derived from previous field trials ([Rogers and Finn 2016](#); [Finn and Rogers 2016b](#)) and the errors were added to the

refracted time delay estimates for each ray. These represented the following:

- A signal sampling regime for a 107-dB dynamic range 44.1-kHz analog-to-digital converter on both UAV and ground microphones combined with a 2^{15} point fast Fourier transform (FFT) with 4-times oversampling and 50% overlap between sample blocks; that is, ray paths from the UAV at about 2 Hz.
- A signal processing regime that represents observed SNR applied to data sampled synchronously at 44.1 kHz at the ground microphones; that is, signal processing errors of 0.1 ms.
- Positional errors in the location of the microphones and the UAV at each epoch as if obtained using real-time kinematic carrier phase differential GPS ([Parkinson and Spilker 1996](#)); that is, UAV and microphone position errors of 0.05 m.

Inversion procedures described in [Finn and Rogers \(2016c\)](#) were used to perform the tomography. Atmospheric profiles were assumed frozen over the observation period, which for a UAV traversing a horizontal path at 28 m s^{-1} is approximately 20 s. The simulated UAV flew at an altitude of 200 m. [Figure 1](#) (left) shows a vertical cross section of temperature and in-plane wind velocity through an LES dataset for a UAV overflight at 200 m altitude. Temperature is coded according to the color scale on the right of the image, and wind direction is shown using streamlines, with arrows pointing in the direction of flow. The maximum wind speed is 5.1 m s^{-1} . [Figure 1](#) (right) shows the AAT reconstruction.

[Figure 2](#) shows two vertical profiles taken from [Fig. 1](#). Red circles represent the target (i.e., original LES) data for the x and z wind components and temperature, and black triangles represent the AAT reconstructions. [Figure 3](#) (left) shows a wind speed error map typical of the Monte Carlo dataset, with the overall standard deviation (std dev) between the target LES atmosphere (resolution: $5\text{ m} \times 2\text{ m}$) and the AAT estimate (resolution: $25\text{ m} \times 10\text{ m}$); that is, the differences between the left and right panels in [Fig. 1](#). Over the distribution of 500 simulations, there is negligible bias between the target and reconstructed profiles. However, the standard deviations are 0.3 and 0.4 m s^{-1} , and 0.2°C for the x , z , and temperature profiles, respectively ([Fig. 3](#), right).

Ideally, dense lattices of radial basis functions (RBFs) precisely replicate small-scale atmospheric structures. Unfortunately, the ill-posed nature of the AAT inversion, brought about mainly by the UAV–microphone geometry ([Finn and Rogers 2016c](#)), forces a reduction in RBF density. This generates two components of difference: *direct* and *indirect* errors, depicted in [Fig. 3](#) (right) as green triangles and purple asterisks, respectively

FIG. 1. (left) An LES dataset at $5\text{ m} \times 2\text{ m}$ resolution (horizontal $\times$ vertical), and (right) a tomographic reconstruction of the same dataset at $25\text{ m} \times 10\text{ m}$ resolution (horizontal $\times$ vertical).

[note: these (simulated) mean errors are small compared with observed verifications against radiosonde data obtained in previous comparison campaigns; May et al. 1996].

Indirect errors are the degree of mismatch between AAT estimates and the target datasets, regardless of the resolution of the RBF reconstruction. Direct errors are the degree of mismatch between AAT estimates and the target data (replicated using RBFs) at the same level of resolution as the AAT reconstruction. The latter errors have two components of interest: the extent to which AAT estimates match the (same resolution) target data and the extent to which this (same resolution) target data matches the high-resolution LES data.

Indirect errors are thus best visualized as the extent to which the purple asterisks approach $\text{RMSE} = 0\text{ m s}^{-1}$, whereas direct errors are best visualized as the extent to which the purple asterisks correspond with the green triangles combined with the extent to which the green triangles approach $\text{RMSE} = 0\text{ m s}^{-1}$. Care must be taken to avoid low-resolution RBF lattices, however, as this can give rise to spatial averaging and a misleading indication of accuracy. Figure 4 (left) shows the LES dataset in Fig. 1 represented by RBFs at a same level of resolution as the AAT inversion ($25\text{ m} \times 10\text{ m}$). Figure 4 (right) shows the difference map for wind speed between the AAT estimate and the RBF fit to the LES data.

FIG. 2. Typical vertical profiles taken from the dataset in Fig. 1 (left $x = 350\text{ m}$, right $x = 500\text{ m}$). Tomographic estimate values (black triangles) and target data (red circles). Error bars for the target dataset represent the standard deviations of the observations at each altitude computed over the horizontal baseline.

FIG. 3. (left) Typical errors in AAT wind speed estimates relative to the target LES dataset as a function of x and z and (right) rms errors in wind speed and temperature as a function of height. Direct (green triangles) and indirect errors (purple asterisks) are shown in the right-hand image.

The sodar simulation modeled a midrange system with a three-beam phased-array antenna set at 12.7° from vertical, located midway along the AAT ground sensor array. The signal processing parameters were representative of pulses observing ranges from $z = 50$ to 200 m in 10 -m increments. The vertical profile directly above the sodar was computed based on a scatter volume bounded by the model of errors described in Spizzichino (1974a) with fluctuations—and hence the magnitudes of returned echoes—assumed to have equal amplitude; that is, the estimate of wind $\mathbf{V}_s = [u, v, w]$ is derived from the vector mean of the returned Doppler-shifted signals from the scatter velocities.

The error bars for each sodar observation are computed by adding the variances σ_z^2 and σ_V^2 to the variance of the vector sum of the echoes in the three-beam scatter

volume bounded by the height uncertainty σ_z . Error bars for AAT are derived by computing the standard deviation of the wind values (as determined by AAT) within the scatter volume of the simulated sodar beam pattern. The linear array simulated in this example does not enable AAT to accurately estimate wind velocities out of plane (y axis), as the inversion is poorly conditioned in this axis. As a result, we do not comment on any statistics for v_y , as this is effectively not computed. [See the appendix and references therein for the approach describing the computation of the components of wind by AAT. These references show that, as the flight path–ground sensor geometry in the simulations is coplanar and any “observations” of v_y (applied as constraints in the least squares adjustment) are exact, inversion provides no additional information in the

FIG. 4. (left) The LES dataset in Fig. 1 represented by RBFs at a resolution of $25 \text{ m} \times 10 \text{ m}$, and (right) the difference map for wind speed between the AAT estimate and RBF fit to the LES data.

FIG. 5. Typical comparisons between true (thin blue line), simulated sodar observations (red triangles), and AAT estimates (black circles) based on LES data for a daytime convective ABL. The error bars are applied to the sodar, and AAT observations are based on the models in the previous section. The altitude of the constant velocity horizontal UAV flight was 200 m.

y axis beyond that taken by simulated sensors. Thus, we constrain the simulated wind profiles to 2D and do not compute or examine the out-of-plane component. However, in the field trials data (see next section), we are not able to eliminate the effects of the out-of-plane component of wind. Furthermore, the UAV–ground sensor geometry is not *exactly* coplanar. As a result, ground and UAV-based direct observations are applied in accordance with the referenced techniques and v_y is computed.]

A total of 500 Monte Carlo simulations were run, with the typical intercomparisons shown in Fig. 5. Several matters are noted from these simulations, described as follows:

- There appears to be no observational bias present at a distribution level; that is, the mean wind speed differences for (AAT–sodar), (AAT–true), and (sodar–true) are all $< \pm 0.05 \text{ m s}^{-1}$.
- At a distribution level, the root-mean-square differences (RMSD) for wind speed are 0.4 and 0.2 m s^{-1} for v_x and v_z , respectively.
- At the individual profile level, the mean differences over the co-observed heights of the AAT and sodar are typically up to 0.5 m s^{-1} for v_x and 0.3 m s^{-1} for v_z , with similar levels of RMSD.
- Both AAT and sodar wind velocity estimates show close correspondence to the true wind profiles at all altitudes. However, the AAT estimates do not capture the high spatial variation with respect to height of the true wind profile as faithfully as the simulated sodar observations.
- The close correspondence between AAT and the true wind profile extends to altitudes below and above

which a midrange sodar is likely to take good observations, that is, altitudes $< 50 \text{ m}$ and $> 300 \text{ m}$ (Rogers and Finn 2013c).

4. Field trial comparisons of AAT and sodar

Over a period of 2 days (10–11 June 2016), field trials were conducted at Saint Leonards, Victoria, Australia, between approximately 1000 and 1500 local time (midnight and 1300 UTC, respectively). Both were overcast winter days with light or light–moderate winds ($< 10 \text{ m s}^{-1}$). An Aerosonde Mark 4.7 UAV (Holland et al. 2001) was repeatedly flown directly over a linear array of 28 microphones set over a baseline of 600 m, similar to the configuration simulated in the previous section. The intersensor separation distances for microphones 1–9, 9–14, 14–15, and 15–28 were 25, 5, 50 m (to span an intersecting runway), and 25 m, respectively. All were positioned approximately 1 m above a flat grassy surface (about 3–4 cm long). There was an increase in elevation over the length of the array of about 3.7 m. Microphones 3 and 28 failed to record data.

Each microphone was located using a real-time kinematic (RTK) carrier phase (CP) differential global positioning system (DGPS), which has an accuracy of $\pm 0.03 \text{ m}$. The coordinate system is such that the positive x axis is through the sensor array (bearing: 110°), the y axis is out of the page (roughly north), and the z axis is vertical. The origin of the system coincides with the first microphone of the array.

The UAV was also fitted with an RTK CP DGPS, enabling position recording at 20 Hz with similar

location accuracy to the ground microphones. The GPS antenna is located on the right wing, 850 mm from the aircraft centerline, and is approximately 1 m from the center point of noise emission. All position recordings were corrected to reflect the location of the centerline of the UAV near its center of gravity. The UAV recorded its own velocity over the ground at 20 Hz, horizontal wind velocity, and engine rotation rate at 1 Hz, and air temperature, mixing ratio, and specific and relative humidity at 0.2 Hz.

The UAV is propelled by a twin-cylinder four-stroke engine with two exhaust mufflers: one for each cylinder. The exhausts are the major source of noise, although significant energy is also emitted by the engine, the rear-mounted two-blade propeller, the distributed aircraft vibration, and aerodynamic noise. Each exhaust emits an exhaust pulse alternately for every second rotation of the engine. The mufflers are 106 mm apart. The UAV's acoustic emissions were sampled at 51.2 kHz synchronously with the 1 pulse per second (PPS) reference of the GPS on board the UAV.

The ground array comprised 28 ECM800 10 mV Pa^{-1} condenser microphones sampled at 44.1 kHz using four 8-channel 24-bit data acquisition (DAQ) recorders with 107-dB spurious free dynamic range. The DAQ sampling frequencies drift with temperature and cannot be relied upon to provide accurate time-stamping. Hence, one of its channels recorded a GPS-derived 1-PPS signal to provide absolute timing reference; the remaining seven channels recorded microphones. The accuracy of the combined signal time-stamping is approximately $22.5 \mu\text{s}$.

Two WindMaster ultrasonic 3D anemometers, four HOBO Pro V2 temperature and humidity sensors, and a Digitech weather station were deployed. All instruments were placed along the length of the microphone array. Wind velocity was recorded at 10 Hz (Windmaster); temperature and humidity at 1 Hz (HOBO Pro); and pressure, temperature, wind speed/direction, relative and specific humidity, and dewpoint at 1/30 Hz (Digitech Pro). Instrumental bias for temperature and wind speed/direction was removed by averaging all datasets over the period of the trial, and—using the Windmaster anemometers (which were calibrated) as a single reference—computing and applying the relevant offsets for each instrument. Thermodynamic temperature was converted to virtual temperature based on humidity measurements taken on board the UAV and by the Digitech weather station.

A monostatic Fulcrum3D sodar was located approximately midway along the length of the sensor array, displaced perpendicularly from it by about 35 m. This sodar comprises three phased arrays, each with 37

piezoelectric transducers with 100% acoustic fill factor. The beams are physically set at 9° and 12.7° from vertical, with a beam tilt independent of frequency (4.5 kHz). Three-dimensional wind velocities are observed at 10-m intervals between 50 and 250 m over an integration period of 10 min, although often the echo returns are insufficient at altitudes above about 200 m to generate measurements. The result of the temporal integration is that the sodar measurements represent the vector mean of the individual range observations. The (1σ) nominal uncertainty is 0.5 m s^{-1} in each axis, but this figure is dependent upon the (integrated) signal returns and calculated individually for each data point. To ensure there is adequate signal processing gain, echoes are integrated over a period of 10 min, which results in wind velocity observations that are the vector sum of any time-stamped measurement.

Although the UAV may be detected and tracked at ranges approaching 3 km (Lo and Ferguson 2004; Finn and Franklin 2012), it was flown only at altitudes between 100 and 1200 m at roughly constant velocity (28 m s^{-1}) along the sensor array with minimal out-of-plane (y axis) deviation. There were about the same number of flights in the positive and negative x directions, with most trajectories horizontal. Some flights climbed or descended about 100 m over the length of the array to obtain independent measurements of the intervening atmosphere. These were also processed to provide additional data and with the intention of determining whether the (slightly) varied geometry positively or negatively affected the quality of the AAT inversion [these flights do not appear to have had any discernible impact on the outcome of the solutions; furthermore, examination of the sound fields of the sodar and UAV indicate that mutual interference is not present, except when the UAV overflies the array at very low ($<150 \text{ m}$) altitude, when the low altitude limits comparison anyway (see Fig. 7)].

Two-dimensional profiles from 143 UAV overflights were generated. The temperature and wind fields were modeled as a uniform lattice of 25×21 (horizontal $\times$ vertical) RBFs, the limits for which were the UAV's flight path and microphone array—that is, about $25 \text{ m} \times 15 \text{ m}$ resolution. There were 80 overflights observed on day 1 and 63 on day 2. The vertical incidence profile corresponding to the location of the sodar was then compared to that observed by this instrument.

Figure 6 (left) shows a typical virtual temperature and wind profile generated using AAT. It is color coded in accordance with the right-hand bar in the image. The white lines superimposed onto the image show streamlines depicting wind flow in the x and z directions. The inverted V's near the center of the microphone array

FIG. 6. (left) A typical tomographic estimate of 2D temperature and wind velocity profiles, and (right) a comparison of sodar and UAV-based AAT-derived temperature and wind profiles.

show the location and nominal beam pattern of the sodar, with dotted horizontal lines indicating the minimum/maximum altitude at which sodar data were observed for each UAV overflight. The fixed black horizontal line shows a nominal upper limit for the device (300 m). White triangles at the base of each diagram indicate microphone locations. Black triangles and asterisks show temperature sensor and sonic anemometer locations, respectively.

The right panel of Fig. 6 shows the wind and temperature profiles for the sodar and AAT estimates are shown in the left panel. The first three subplots of the right panel (from left to right) show profiles as a function of height for wind components on the x , y , and z axes, respectively. The fourth subplot shows AAT estimates of virtual temperature versus observations taken on board the UAV by flying horizontal transects through the intervening medium within 20 min of the AAT overflight. The UAV measured thermodynamic temperature ($\pm 0.5^\circ\text{C}$), atmospheric pressure ($\pm 1.5\text{ hPa}$), and relative humidity ($\pm 5\%$) at a sampling rate of 0.2 Hz referenced to GPS using two sensors fitted below/aft the trailing edge of the UAV's wings.

The continuous black lines in each image depict the AAT estimate, whereas the black circles represent the sodar/UAV observations, with error bars representing their 1σ errors. The dotted red lines in each of the first three subplots show the vertical profile derived by interpolating between observations made on the ground and the UAV.

The interpolated profiles are used to provide insight into the value-added of the time delay information relative to that achievable without any AAT solution. This is most readily seen by comparing the results of the x -axis

AAT solution with that for the y axis. The latter, caused by the poorly conditioned out-of-plane geometry of the UAV and microphone array, tends to the value of the interpolated profile. Much more complex structures that deviate significantly from the interpolated profiles are present for the AAT-derived x and z components of wind velocity.

Figure 7 shows the differences between the AAT-derived and sodar observations for all 143 profiles as a function of height above the ground, with distributions for these comparisons shown in Fig. 8. The magnitude of the differences at each altitude are color coded in accordance with the bar on the right of each image in Fig. 7, the scales for which are $0\text{--}4\text{ m s}^{-1}$ for the x and y axes and $0\text{--}1\text{ m s}^{-1}$ for the z axis (note: a small number of AAT profiles included in this dataset offer questionable results, either because the signal processing algorithms performed poorly or the inversion appears to have failed; they are included, however, because they pass the statistical tests for rejection). The comparisons show the scatter of the AAT measurements, which arise from both observation error and atmospheric variation.

The vertical upper limit of the comparisons in Fig. 7 is bounded by the maximum height from which sodar echoes are obtained ($<300\text{ m}$) and the altitude of the UAV overflight (typically $>300\text{ m}$). Thus, in general, the upper limit of the comparisons is bounded by the altitude performance of the sodar. On a few occasions, however, the aircraft flew below 300 m, limiting the comparisons and giving the impression of poorer sodar altitude performance.

Figure 8 shows the mean difference for each vertical profile (as a function of profile number), with error bars

FIG. 7. Differences between UAV-based AAT-derived and sodar wind observations as a function of height and profile number. Images depict the components in (left) x , (center) y , and (right) z directions (change of scales between the images for v_x , v_y , and v_z). The altitude of the UAV overflights when the range was limited by the UAV flight as opposed to the sodar are shown as red dots.

depicting the standard deviation of the difference for that vertical profile. Profiles 1–80 are from day 1; profiles 81–143 are from day 2. There is a bias for v_x , v_y , v_z , and T of -0.5 , 0.7 , and 0.1 m s^{-1} , and 0.5°C , respectively, for which each dataset has a standard deviation of 0.8 , 1.5 , and 0.3 m s^{-1} , and 0.4°C , respectively. The 75th and 90th percentiles of the cumulative probability distributions for each variable are 1.1 and 1.5 m s^{-1} (for v_x), 1.9 and 2.7 m s^{-1} (for v_y), 0.4 and 0.5 m s^{-1} (for v_z), and 0.8° and 1.1°C (for T). These differences, which include errors in the sodar, the AAT, and any differences that exist in the small-scale wind field caused by space–time separation, compare favorably with previously observed verifications against radiosonde data (Strauch et al. 1984; May et al. 1989; May 1993; Tsuda et al. 1995; Kelley et al. 2007).

The mean wind speed averaged over each vertical profile (co-observed by both sodar and AAT) is shown as a function of profile number in Fig. 9. The left image shows the wind speed determined using sodar data and the right image from AAT data. As the integration period for the signal returns of the sodar is 10 min, as opposed to 20 s for an aircraft overflight, groups of wind speed estimates for profiles/overflights appear fixed over time in the left-hand image.

Plotting differences between AAT and sodar as a function of wind speed shows systematic biases in the AAT solutions. These biases are strongly influenced by wind speed in the direction of each axis (Fig. 10, left) and weakly influenced by wind speed blowing orthogonal to each axis (Fig. 10, right): larger differences correspond to higher wind speeds. Such biases are consistent with

FIG. 8. Histograms of differences between (left) AAT wind and (center) sodar profiles, and (right) mean differences for each profile vs profile number/time. The leftmost set of histograms show the differences in wind for the x , y , and z components (pink, blue, and light brown, respectively). The rightmost histogram shows the differences in temperature. Error bars in the right-hand image depict the standard deviation of the AAT–sodar difference for each profile.

FIG. 9. Mean wind speed observed over the profile as a function of profile number as determined by the (left) sodar and (right) AAT. Profiles 1–80 are from day 1, and profiles 81–143 from day 2.

the effects of inadequately modeled and out-of-plane refraction.

If corrections derived from linear regression fits are applied to the raw data by simply taking the input and subtracting the bias from the regression (red lines in Fig. 10) for the given wind speed value, then the systematic biases reduce to $< 0.05 \text{ m s}^{-1}$ and $< 0.05^\circ\text{C}$ for v_x , v_y , and v_z , and T , with standard deviations of 0.6, 1.0, and 0.2 m s^{-1} , and 0.4°C , respectively (as sodar wind speed observations would not be available to compute regression corrections, AAT estimates were used). However, because the dataset is so small, it

is not clear these corrections are valid beyond this dataset.

The left-hand image of Fig. 11 shows the mean profile differences between AAT and sodar as a function of UAV overflight and the accuracy of sodar observations, color coded in accordance with the scale on the right-hand side of each image. The sodar measurement uncertainty increases with height from a mean of 0.3 m s^{-1} at 60 m to 0.7 m s^{-1} at 200 m. This results from a number of factors (the SNR of the sodar observations generally decrease with range from the sodar, the scattering volume increases, and larger vertical motions that are heterogeneous affect retrieval of horizontal wind). Although somewhat inconclusive, Fig. 11 indicates that the accuracy of the horizontal wind estimates may diminish when the altitude of the UAV overflight is greater than 600 m, which accords with previous simulations (Rogers and Finn 2013c). However, as the ray path geometry changes in its favor, the vertical velocity estimates of wind appear to improve for these higher-altitude flights.

If data for which only UAV overflight altitudes $< 600 \text{ m}$ and sodar data uncertainties < 0.5 are used—and based on these datasets linear regression corrections are applied—then, while the biases in the differences between the AAT and the independent instruments remain below 0.05 m s^{-1} , the rms values fall to 0.3, 0.5, and 0.2 m s^{-1} for each wind component. The distributions of the differences are shown in the right-hand image of Fig. 11.

As the sodar integrates its echo returns over a period of 10 min—and thus derives a mean of the vector sum of

FIG. 10. (left) Differences between AAT and sodar wind speed as a function of sodar-observed wind speed in the direction of the wind estimate and (right) differences for v_x and v_z as a function of their out-of-plane components (as expected, v_x is influenced only by v_y —not v_z —and v_z is unaffected).

FIG. 11. (left) Mean profile differences between AAT and sodar vs UAV overflight height and sodar accuracy, and (right) a histogram of differences for UAV overflight altitudes < 600 m and sodar data uncertainties < 0.5 m s^{-1} . Differences in wind speed for the x (pink), y (blue), and z (light brown) components.

the wind field as a function of height—a more accurate comparison of it to AAT may be obtained by comparing the vector-summed AAT profiles with the corresponding sodar data. Figure 12 (left) shows the mean difference between the sodar wind component data and the vector sum of any AAT overflight profiles that fall within the requisite 10-min time window of the sodar observation. The distribution of the differences is shown in Fig. 12 (right). The differences between for the integrated profiles as a function of height are shown in Fig. 13. The biases are -0.2 , 0.0 , and 0.1 m s^{-1} for each wind component, with standard deviations of 0.5 , 1.0 , and 0.2 m s^{-1} . A summary of the various AAT–sodar comparison statistics are shown in Table 1.

A sequence of AAT profiles may now be used to visualize atmospheric structures over time. Figure 14 shows a subset of images taken from a sequence of 2D profiles taken between about 1330 and 1400 local time on day 2 of the trial (note: the full set is available as a video from the authors on request; see the video in the online supplement.). The period corresponds roughly to profile numbers 100–120 in Fig. 7. In the original/full sequence, each profile is taken about 100 s apart over the 30-min period (20 s for an overflight, 80 s to turn around). The images in Fig. 14 are separated by approximately 5 min and run as follows: top left–top right, center left–center right, and lower left–lower right.

FIG. 12. Comparison between sodar and vector sum of AAT observations, uncorrected for (left) wind bias as a function of sodar profile number and (right) the distribution

FIG. 13. Differences between the vector sum of uncorrected AAT and sodar wind observations as a function of height and sodar profile number. The components in the x (left), y (center), and z (right) directions (color coding is as per Fig. 7).

As per Fig. 6, white streamlines show wind flow in the x – z plane and temperature is color coded according to the color bar on the right. Wind speeds in the x and z axes throughout this sequence were around 0.5 m s^{-1} , with the y -axis component around 1.5 – 2.0 m s^{-1} . The upper limit of the vertical axis is the maximum altitude at which a UAV overflight took place. However, as the UAV's altitude was not the same for each flight, the data in each image extends only as high as each individual overflight. It is noteworthy that—despite the varying overflight altitudes—between images the atmospheric structures are consistent over time relative to their estimated heights.

Based on an assessment of the synoptic data and knowledge of the local environment, we believe the sequence shows a thermal developing off some corrugated iron buildings and a concrete parking area, which locally disrupts a bay breeze from the east/right. Unfortunately, there were no UAV flights below 200 m (i.e., through the thermal) during the 30-min sequence of observations to provide independent temperature measurements against which the AAT estimates could be verified.

5. Concluding remarks

Two-dimensional, temporally averaged vertical atmospheric profiles are obtained by monitoring the natural sound of a UAV as it overflies an array of ground microphones; comparing the frequency-shifted received signal to that predicted using onboard measurements; generating time delay observations for the multiple intersecting ray paths that penetrate the intervening atmosphere; and converting the computed sound speed measurements to temperature and wind fields represented by RBFs using tomographic inversion. These profiles are compared to independent measurements taken on board the UAV and by a midrange sodar over a 2-day period.

It is difficult to conclusively establish the accuracy of the AAT estimates or the range of conditions over which AAT may be usefully employed. There are several reasons for this as follows:

- The test dataset is sparse, comprising data over a period of only 2 days: one with wind speeds that were low (0 – 3 m s^{-1}) and the other modest (3 – 8 m s^{-1}).
- Sodar and AAT observe fundamentally different atmospheric properties: sodar observes the time delay from a signal emitted and backscattered by a distributed set of temperature fluctuations generated by atmospheric turbulence that are assumed to travel with the wind, whereas AAT relates a one-way propagation delay of a signal to sound speed.
- The integration period of the sodar used in this experiment is 10 min, whereas the AAT overflight period is approximately 20 s. Thus, both provide

TABLE 1. Summary of AAT–sodar comparison statistics.

AAT minus sodar		Bias (m s^{-1} or $^{\circ}\text{C}$)	Std dev (m s^{-1} or $^{\circ}\text{C}$)
Raw data	—X	−0.5	0.8
	—Y	0.7	1.5
	—Z	0.1	0.3
	—T	0.5	0.5
Data corrected for wind-related bias	—X	<0.05	0.6
	—Y	<0.05	1.0
	—Z	<0.05	0.2
	—T	<0.05	0.4
Data corrected for bias and good observations only	—X	<0.05	0.3
	—Y	<0.05	0.5
	—Z	<0.05	0.2
	—T	<0.05	0.4
Vector sum data	—X	−0.2	0.5
	—Y	0.0	1.0
	—Z	0.1	0.2

FIG. 14. A sequence of 2D vertical temperature and wind velocity profiles derived using UAV-based AAT. The profiles were observed sequentially, approximately 100 s apart over a period of about 30 min. Symbols depict wind (a set of white streamlines) and the direction of wind flow (arrows). Temperature is coded in accordance with the scale on the right of each image.

vector sums of wind velocity over their respective (and very different) observation periods.

- AAT and sodar observations are affected differently by the effects of refraction, which is known to be imperfectly modeled in both systems.
- AAT offers temperature and wind velocity data well beyond the range of the sodar, making independent

validation of the profiles above 250 m and below 50 m impossible.

- The out-of-plane component of wind velocity is poorly observed by the AAT (and hence unreliable). Nevertheless, this setup does provide validation regarding the value-add of the derived time delay observations relative to the meteorological

data observed by the UAV and ground-based sensors.

- The flat trajectory of the UAV's flight path relative to the ground microphones provides poorly conditioned equations for the tomographic inversion. The condition of these equations can be substantially improved if the UAV is able to climb rapidly near the first and last sensors in the microphone array.

A future experiment will be designed to overcome some of the limitations of this measurement campaign. A scanning Doppler lidar will take velocity measurements in the vertical plane, and the microphones will be aligned in the x - y plane such that the AAT is able to observe wind speeds in 3D. Temperature measurements will be compared with thermistor measurements carried by other UAVs, which will fly concurrently with the Aerosonde. Nevertheless, for the sparse dataset and low-to-moderate wind speeds ($<8 \text{ ms}^{-1}$) experienced during the trial, the following is concluded:

- The AAT wind vector profiles and sodar measurements closely match one another.
- AAT temperature estimates also closely match measurements taken on board the UAV.
- Both wind and temperature errors were consistent with comparisons based on LES data.
- Uncorrected AAT estimates appear to suffer a wind-speed-dependent bias relative to sodar measurements; this is consistent with poorly modeled refraction.
- Errors introduced into the AAT inversions are independent of the UAV's direction of travel.
- Improvements are not derived by employing a sloped flight path over the length of the ground array of microphones (but this tactic does not harm the results either).
- AAT permits visualization of atmospheric profiles over 600-m baselines and up to 1200 m, although the accuracy of the horizontal wind velocity estimates appear to diminish when the altitude of the UAV overflight is greater than 600 m, and although vertical wind velocity estimates appear to improve slightly for these higher flights.

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APPENDIX

Atmospheric Model Used for AAT

Atmospheric state may be expressed as a linear combination of two independent sets of RBFs $\Delta T(\mathbf{X})$ and $\Delta \mathbf{V}(\mathbf{X})$ (Wiens and Behrens 2009; Rogers and Finn 2013c; Aster et al. 2013), where the RBFs are 2D or 3D Gaussian functions of the form $\phi(r) = e^{-kr^2}$, r is the distance from the RBF center $\mathbf{X}_{cj}$, and k is a scaling factor. If the RBFs are distributed evenly, the temperature and wind fields may be approximated by

$$\Delta T(\mathbf{X}) = \sum_{j=1}^{N_r} A_{Tj} e^{-\|\mathbf{kX} - \mathbf{X}_{cj}\|^2} \quad \text{and} \quad (\text{A1a})$$

$$\Delta \mathbf{V}(\mathbf{X}) = \sum_{j=1}^{N_r} \mathbf{A}_j e^{-\|\mathbf{kX} - \mathbf{X}_{cj}\|^2}, \quad (\text{A1b})$$

where N_r represents the number of RBFs used; A_{Tj} is the temperature coefficient for RBF(j); and $\mathbf{A}_j = [\mathbf{A}_{jx}, \mathbf{A}_{jy}, \mathbf{A}_{jz}]$ is the wind coefficient vector for RBF(j) in the x , y , and z directions. These equations may be expressed in matrix notation, $\mathbf{F}(\mathbf{X}) = \Phi(\mathbf{X})\mathbf{A}$, where $\mathbf{F}(\mathbf{X}) = [\Delta T(\mathbf{X}), \Delta \mathbf{V}_x, \Delta \mathbf{V}_y, \Delta \mathbf{V}_z]^T$ is a 4×1 column vector of temperature and wind speed component deviations (x , y , z), $\mathbf{A}$ is a $[4N_r \times 1]$ column vector of parameter coefficients, and $\Phi(\mathbf{X})$ is a $[4N_r \times 4]$ matrix of RBFs.

We can improve the estimate of the wind and temperature profiles by taking additional measurements on the ground and at the UAV and using the above-mentioned equations to constrain the least squares adjustment by estimating the RBF coefficients, $\Delta T = \mathbf{R}\mathbf{W}_T$, $\Delta \mathbf{V}_x = \mathbf{R}\mathbf{W}_x$, $\Delta \mathbf{V}_y = \mathbf{R}\mathbf{W}_y$, $\Delta \mathbf{V}_z = \mathbf{R}\mathbf{W}_z$, where $R_{ij} = e^{-k\|\mathbf{X}_i - \mathbf{X}_{cj}\|^2}$, $i = 1, \dots, M_r$ (the number of temperature and wind speed measurements). These equations can then be combined into a single matrix relationship, $\mathbf{A}\mathbf{W} = \mathbf{b}$, where $\mathbf{b}$ is a column vector containing all travel time and direct measurements of temperature and wind speed deviations solved using regularized weighted least squares (Rogers and Finn 2013c; Finn and Rogers 2016c, 2017).

This inversion technique significantly reduces the number of model parameters such that the system is overdetermined and can be solved by least squares as the model parameters are now the RBF coefficients. The variance in the coefficients, which may be derived from the covariance matrix of the inversion (Tarantola 2005), also provides an estimate of the accuracy of the overall solution.

The observations in AAT are the propagation delays of the rays as they pass through the atmosphere. If the wind

and temperature fields are linearized about mean values T_0 and $\mathbf{V}_0$, the travel time, t_{pj} , for sound ray(j) is given by

$$t_{pj} = \frac{l_j}{c_0} \left(1 - \frac{\mathbf{V}_0 \cdot \mathbf{v}_{\text{ray}}}{c_0} \right) - \int_0^{l_j} \left\{ \frac{\Delta T[\mathbf{X}(l)]}{2T_0 c_0} + \frac{\Delta \mathbf{V}[\mathbf{X}(l)] \cdot \mathbf{v}_{\text{ray}}}{c_0^2} \right\} dl, \quad (\text{A2})$$

where l_j is the path distance, from $X_u(l) = [x_u(l), y_u(l), z_u(l)]$ is the location of the UAV to $X_r(l) = [x_r(l), y_r(l), z_r(l)]$ is the location of the receiver; c_0 is mean sound speed, $\mathbf{V}_0$ is mean wind speed vector over the volume; $\Delta T(\mathbf{X})$ is the (time averaged) temperature deviation at location $\mathbf{X}$; $\Delta \mathbf{V}(\mathbf{X}) = [\Delta \mathbf{V}_x, \Delta \mathbf{V}_y, \Delta \mathbf{V}_z]$ is the (time averaged) wind speed deviation at location $\mathbf{X}$; and dl is an integration length along the ray path.

For propagation in an inhomogeneous moving medium between a (moving) UAV and a stationary ground receiver, the frequency received by a ground sensor is [Eq. (5.68) in [Ostashev and Wilson 2016](#)]

$$f_r(t + t_p) = \frac{1 + \mathbf{n}_u(t) \cdot \mathbf{v}_u(t)/c_u(t)}{1 + \mathbf{n}_u(t) \cdot [\mathbf{v}_u(t) - u_u(t)]/c_u(t)} f_u(t), \quad (\text{A3})$$

where $\mathbf{n}_u(t)$ is the unit vector normal to the wave front of the wave emitted by the UAV, $\mathbf{v}_u(t)$ is the velocity of the medium at the UAV at time t , t_p is the propagation time, $u_u(t)$ is the velocity of the UAV emitter at t , $f_u(t)$ is the instantaneous source frequency at t , $f_r(t + t_p)$ is the instantaneous received frequency at $t + t_p$, and c_u is the speed of sound at the UAV.

Spectral analysis of the UAV signature shows it comprises strong narrowband tones superimposed onto a broadband random component, with almost all of the narrowband energy below 2 kHz ([Ferguson and Lo 2000](#)). These tones correspond closely to the engine firing rate, N_R (the number of engine rotations per second) and propeller blade rate (PBR) of the aircraft, such that $\text{PBR} = N_R N_B$, where N_B equals the number of blades on the propeller, with each of these sound sources (engine firing rate and propeller) generating its own set of harmonics.

This provides a rich set of linearly related frequencies f_h , where $h = 1, \dots, N_h$, N_h being the number of detectable harmonics on either the UAV (subscript u) or ground (subscript r). An estimate of composite frequencies, $f_u \approx f_{uh}/h$ and $f_r \approx f_{rh}/h$, can then be established, where the higher harmonics offer the advantage that the standard error is divided by the harmonic number, even though the SNR is typically less for these higher harmonics. Using least squares, composite frequencies may then be formed,

$$f_{u,r} = \sum_{h=1}^{N_{u,rh}} \sigma_{f_{u,r}}^2 f_{u,rh} / h \sigma_{f_{u,rh}}^2, \quad (\text{A4})$$

where $\sigma_{f_{u,r}}^2 = \left[\sum_{h=1}^{N_{u,rh}} \sigma_{f_{u,rh}}^{-2} \right]^{-1}$ are the variances of the estimates and $\sigma_{f_{u,rh}}$ are the lower bounds of the standard errors in the estimates $f_{u,rh}$ of each harmonic of f_u and f_r , given by the Cramer–Rao lower bound ([Rogers and Finn 2013b](#)).

The UAV location and velocity at each epoch and position of all ground microphones may be accurately measured (± 0.02 m). The sound field generated by the UAV may also be measured using microphones both on board the aircraft and on the ground, which allows for computation of $f_u(t)$ and $f_r(t)$ using Eq. (A4). Meteorological observations such as wind velocity, thermodynamic temperature, mixing ratio, and humidity (specific and relative) may be made on board the UAV, allowing the sound speed $c_u(t)$, unit vector normal to the wave front $\mathbf{n}_u(t)$, and wind velocity $\mathbf{v}_u(t)$ to be derived. Representing $f_r(t + t_p)$ as a Taylor series and using techniques developed by [Finn and Rogers \(2015, 2016c\)](#) and [Rogers and Finn \(2017\)](#), t_p may be determined assuming straight ray path propagation,

$$t_p = t_{p0} + [f_r^C(t + t_p) - f_r^M(t + t_{p0})] \left[\frac{\partial f_r(t + t_{p0})}{\partial t} \right]^{-1}, \quad (\text{A5})$$

where $f_r^M(t + t_{p0})$ is the measured value of f_r at $t_{p0} = l(t)/c(t)$, $f_r^C(t + t_p)$ is the value of $f_r(t + t_p)$ computed from Eq. (9), and $\partial f_r(t + t_{p0})/\partial t$ is the numerical derivative of f_r at t_{p0} .

Variations in the sound speed along the ray cause it to refract ([Ostashev et al. 2008](#)). Relative to the nominal straight-line approximation for a ray, these variations in sound speed cause a curvature in the ray and a deviation from the nominal travel time along it as a result of the difference in sound speed from its nominal value. A full computation of these combined effects depends upon ray path geometry and the atmospheric profiles through which the ray propagates and is beyond the scope of this paper.

An approximate approach is used ([Urlick 1983](#)) that is a compromise between physical accuracy and computational complexity. Sound speed variations are approximated by piecewise linear gradients under the assumption that rays travel through each layer as circular arcs, radius $r_c = (1/g_i)(c_i/\cos\theta_i)$, where g_i is the gradient in the layer, c_i is the sound speed at the entry point of the ray in layer i , and θ_i is the angle of the ray entering the layer measured with respect to the horizontal. For a known vertical step dz the ray's exit angle may be computed and the time spent traveling along the

ray in the layer dt_{pji} is given by integrating the path-length divided by sound speed. The distance traveled along the ray in each layer is $ds_{pji} = dt_{pji}c_i$. The launch angle of the ray is approximated to the straight-line path of the UAV–microphone geometry and the “true” delay computed by integrating along the refracted ray, that is, $t_{pj}^r = \sum_{i=1}^{N_i} dt_{pji}$, where N_i is the number of layers. The delay t_{pj} representing the unrefracted straight-line path from UAV to the ground microphone is then adjusted by the ratio of the refracted delay, t_{pj}^r , relative to the straight-line path between its start and end points divided by the mean sound speed.

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