On the Equilibration of Numerical Simulation of Internal Tide: A Case Study around the Hawaiian Ridge

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ABSTRACT

To investigate the equilibration of numerical simulation (ENS) of internal tide, a three-dimensional isopycnic coordinate internal tide model is applied to simulate the M_2 internal tide on idealized topography and around the Hawaiian Ridge. An idealized experiment is carried out on a Gaussian topography, and the temporal variations of the baroclinic velocity and the baroclinic energy flux are analyzed, then ENS is studied, and two criteria are presented. Moreover, the impacts of four parameters [horizontal and vertical eddy viscosity coefficients, bottom friction coefficient, and damping coefficient (to parameterize the nonhydrostatic processes in the model)] on ENS during numerical simulations, the baroclinic velocity, the baroclinic tidal energy, and the baroclinic energy flux are investigated. It appears that ENS for the M_2 internal tide is more sensitive to the horizontal eddy viscosity coefficient and the damping coefficient. To further examine the criteria of ENS, a numerical experiment is carried out to simulate the M_2 internal tidal constituent near the Hawaiian Ridge. The simulated surface tide shows good agreement with results from the Oregon State University tidal model and TOPEX/Poseidon (T/P) observations. The simulation results indicate that a 50 M_2 tidal period (25.88 days) run is capable of ensuring ENS for the M_2 internal tide in this case. In short, this paper presents a method and two criteria for examining ENS for internal tides for modelers.

1. Introduction

Internal tides are internal waves at tidal frequency and exist widely in the ocean. Barotropic tides, stratified fluids, and varying topographies are the three factors for internal tide generation. Some theoretical models have been developed to investigate the relationship between internal tides and the three factors. These models played

an important role in the study of the internal tidal generation mechanism. However, most of these theoretical models were restricted to two-dimensional linear problems or idealized weak topographies (Bell 1975; Thorpe 1992; MacCready and Pawlak 2001; Balmforth et al. 2002; Llewellyn Smith and Young 2002; St. Laurent and Garrett 2002; Khatiwala 2003). Moreover, some models use simplifications such as the assumption of constant buoyancy frequency or forcing the tides with only one constituent.

By contrast with analytical models, the numerical model can provide a more effective way for parameter exploration (Miao et al. 2011). There have been many

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numerical studies of internal tides and these studies can be divided into two categories: the idealized-topography-based studies and the practical-topography-based studies.

In the first category, the numerical models were usually used to study the generation mechanism and the features of the internal tides generated over an idealized topography. [Winters and D'Asaro \(1997\)](#) used a three-dimensional nonhydrostatic model to calculate nonlinear energy transfers under an idealized topography. Using the nonhydrostatic Massachusetts Institute of Technology General Circulation Model (MITgcm), many studies have been conducted to investigate the generation and energy conversion of the internal tide on idealized topographies ([Legg and Adcroft 2003](#); [Legg 2004a,b](#); [Legg and Huijts 2006](#); [Klymak et al. 2013](#); [Dunphy and Lamb 2014](#)). To deal with significant topographic variability, models adopting the sigma coordinate system [e.g., the Princeton Ocean Model (POM), the Regional Ocean Modeling System (ROMS)] were employed by some scholars ([Johnston and Merrifield 2003](#); [Di Lorenzo et al. 2006](#); [Mathur et al. 2014](#)). In addition, some models were also established to study specific problems, such as the relationship between the internal tide and the internal solitary wave ([Shaw and Chao 2006](#); [Shaw et al. 2009](#)), and the generation of the internal tide in an ocean with strong upper-ocean stratification ([Qian et al. 2010](#)).

In the second category, numerical models were applied to real sea areas to study the propagation and energy conversion of the internal tides. [Niwa and Hibiya \(2001\)](#) clarified the distribution of the M_2 internal tide in the Pacific Ocean using POM. Their inspiring works made the first step toward numerical modeling of global internal tides. [Simmons et al. \(2004\)](#) computed the energy flux out of barotropic and into the barotropic–baroclinic tides with the Hallberg Isopycnal Model (HIM). [Miao et al. \(2011\)](#) developed an isopycnal coordinate internal tide model to study multiconstituents of internal tides in the South China Sea. The description of layer interfaces in their model help us better understand the interfacial displacement of the internal tide. [Buijsman et al. \(2012, 2013\)](#) carried out a series of numerical experiments to examine the influence of double-ridge topography in Luzon Strait on the generation and the energy dissipation of the internal tides.

The Hawaiian Ridge has been studied intensively in the past decade by acoustic tomography, satellite altimetry, field observations, and numerical model simulations ([Zhao et al. 2011](#)). And there is some evidence showing the existence of strong internal tides in this region ([Dushaw et al. 1995](#); [Ray and Mitchum 1996, 1997](#); [Kang et al. 2000](#); [Zhao et al. 2010](#)). Many researchers

have investigated internal tides along the Hawaiian Ridge by using different kinds of numerical models. [Kang et al. \(2000\)](#) established a two-dimensional model to investigate the M_2 internal tide near Hawaii. [Carter et al. \(2008\)](#) employed POM to form an energy budget for the M_2 internal tide in a region of the Hawaiian Ridge. Recent work about internal tide simulation along the Hawaiian Ridge could be seen in [Rainville et al. \(2010\)](#), [Powell et al. \(2012\)](#), and so on. In their work, energy budget was focused on and energy flux and conversion were calculated.

The dynamic progress of generation–propagation–dissipation for internal tides is continuous in the real ocean. The main purpose for applying numerical models on internal tides is to simulate this dynamic process ([Alford et al. 2015](#)). However, before any analysis on the dynamic process begins, most numerical models will go through numerical processes that start from zero state (cold start) until they reach the statistically steady state. Model results derived before the equilibration of numerical simulation (ENS) of the numerical process may contain calculation errors due to computational instability or barotropic and baroclinic tidal spinup ([Kerry et al. 2014](#)). So, the numerical process is usually excluded ([Kerry et al. 2014](#); [Xu et al. 2016](#)) from model analysis. Actually, the equilibrium state in this paper referred to the numerical process rather than the dynamic process.

Nevertheless, after reviewing the previous works, it is found that ENS for internal tides is scarcely mentioned. Actually, ENS should be attained after a certain run time (the total calculation time for the model) to allow the internal tides to propagate throughout the domain ([Xu et al. 2016](#)) or to ensure the tidal spinup for barotropic and baroclinic tides ([Kerry et al. 2014](#)).

According to the descriptions in some of the above-mentioned research papers, we collected the run time for the M_2 internal tide simulations and list them in [Table 1](#). The run time for different works varies from 1.5 to 30.9 for the M_2 tidal period. There is no reason to believe that researchers with different goals apply the same run time. However, it is quite necessary to state the reasons why they should apply these specific lengths of numerical simulation time and to convince readers that these lengths of run time are long enough for their research goals. [Carter and Merrifield \(2007\)](#) noticed this problem and examined their model results after different tidal periods during the long integrations. In the modeling studies, the foundations to determine the run time of numerical simulations should be presented and the full energy budgets are expected to be analyzed to investigate whether the system had reached a statistically equilibrium state. However, some of the studies merely present the sketchy “quasi-steady state” by

TABLE 1. Different numerical models to simulate the M_2 internal tide and their run times.

Model	Geometry	Reference	Run time
Nonhydrostatic model A	3D	Winters and D'Asaro (1997)	7T ^a
MITgcm	2D	Legg and Adcroft (2003)	5.9T
	3D	Legg (2004a,b)	3.1T
	2D	Legg and Huijts (2006)	11.2T
	2D	Buijsman et al. (2012)	7.4T
	2D, 3D	Buijsman et al. (2013)	30.9T
	3D	Dunphy and Lamb (2014)	30T
Nonhydrostatic model B	3D	Shaw and Chao (2006)	1.5T
	3D	Shaw et al. (2009)	5T
	2D	Qian et al. (2010)	5T
Two-layer model	2D	Kang et al. (2000)	9T
POM	3D	Niwa and Hibiya (2001)	19.3T
	3D	Carter et al. (2008)	18T
ROMS	3D	Kerry et al. (2014)	753.6T
	3D	Xu et al. (2016)	29T
Isopycnic coordinate model	3D	Miao et al. (2011)	30T

^a Term T indicates one period of the M_2 internal tide (approximately 12.42 h).

viewing the model results at the end of certain run times. We have not noticed works that present specific criteria for the equilibrium of their numerical simulations, so the alleged quasi-steady state is not convincing.

To fill this gap, a series of numerical experiments based on an isopycnic coordinate model (Miao et al. 2011) will be carried out to investigate ENS for the M_2 internal tide in this paper. The aim of this study is to present the criteria for ENS for internal tides. Moreover, we will try to apply the method to ideal and real numerical cases. Our work, hopefully, presents some new information and ideas to the modeling literature.

This paper is organized as follows: In section 2, the model description is introduced and the idealized numerical experiments are carried out to investigate ENS for the M_2 internal tide on a Gaussian topography and the influences of four parameters on ENS. In section 3, a numerical experiment is carried out around the Hawaiian Ridge to examine the criteria of ENS. Finally, the results are discussed and the present perspectives are concluded in section 4.

2. Model description and idealized experiments

The three-dimensional isopycnic coordinate internal tide model of Miao et al. (2011) is used in this study. The isopycnic system in the model can be understood by viewing the ocean interior as a stack of shallow fluid layers (Fig. 1), and each layer is characterized by a constant density similar to that in HIM (Simmons et al. 2004). The sequential numbers for each layer are 1, 2, $\dots$, $N(x, y)$ from top to bottom, where N is the total number of layers and depends on the topography. For the k th layer, η_k denotes the interfacial displacement. The density and

thickness of the k th layer are ρ_k and h_k , respectively. This model has been employed by Miao et al. (2011) to the simulation of the M_2 and K_1 internal tides in the South China Sea to examine its capability.

a. Model setup

The model domain is horizontally from $x = -100$ to 100 km in the east–west direction with a horizontal resolution of $\Delta x = 1$ km, and vertically from $z = -H = -3000$ m at the bottom to 0 m at the surface with 10 linearly distributed layers. A 10-cell-wide relaxing layer near the boundaries is used to absorb baroclinic waves by gradually relaxing the baroclinic velocities and displacements to zero (Rainville et al.

FIG. 1. Schematic of vertical discretization. Modified from Miao et al. (2011) with permission.

FIG. 2. Initial potential density and relevant buoyancy frequency.

2010). The initial potential density is assumed to be horizontally homogeneous (Niwa and Hibiya 2001; Simmons et al. 2004) and its vertical profile is shown in Fig. 2, which is computed using the profile at 20°N, 118°E from the *World Ocean Atlas 2005* (WOA05; Locarnini et al. 2006) dataset. A Gaussian ridge, uniform in the y direction, is placed at $x = 0$ km. The ridge height h is given by

$$h = h_0 \exp \left[\frac{-(x - x_0)^2}{2L^2} \right], \quad (1)$$

where h_0 and L are the maximum height and the scale width of the ridge, respectively. In this study, $h_0 = 200$ m and $L = 10$ km.

The tidal flow is forced by the oscillating barotropic M_2 tide. Flather conditions for the barotropic tide are applied to the open boundaries at $x = -100$ and 100 km. The velocity in the x direction is given by

$$U = U_0 \sin(\omega_{M_2} t), \quad (2)$$

where $\omega_{M_2} = 1.41 \times 10^{-4} \text{ s}^{-1}$ represents the M_2 tidal forcing frequency and $U_0 = 2 \text{ cm s}^{-1}$ (Legg and Huijts 2006) is the maximum tidal velocity on open boundaries. For simplicity, the horizontal and vertical eddy viscosity coefficients and the bottom friction coefficient are set to be zero in the idealized experiments (not zero in the practical experiment). To parameterize the non-hydrostatic processes in this model, a baroclinic linear damping term is added to the horizontal momentum

equations, which is similar to that adopted by Niwa and Hibiya (2001). The baroclinic linear damping coefficient γ is taken as 0.2 per day and the Coriolis force is included with $f = 8 \times 10^{-5} \text{ s}^{-1}$. In the present experiment, a time step of 2.24 s is used and the tidal model is run for 200 tidal periods.

b. Model results and evidences for ENS

There have been many works investigating the features, generations, and propagations of internal tides (Niwa and Hibiya 2001, 2004; Katsumata 2006; Jan et al. 2007; Song et al. 2010; Floor et al. 2011; Ponte and Cornuelle 2013; Wu et al. 2013; and references therein) that have been clearly understood by scientists. So, we are not trying to spell out the details of the dynamic process for internal tides. Instead, we will mainly focus on the temporal variations of the simulation results for M_2 internal tides and issues on ENS during the numerical process.

A series of panels (Fig. 3) shows snapshots of the baroclinic velocity field at the different run time. To obtain the baroclinic velocity, we have subtracted the barotropic velocity from the full velocity field. From the comparison among the insets of Fig. 3, it is easy to figure out the numerical process in which the M_2 internal tide propagates throughout the domain gradually. In Fig. 3a, waves emitted from top of the ridge and split into two nearly symmetric paths. A wave beam of positive u' over the ridge top propagates upward and then is reflected downward by the surface at $x = \pm 40$ km. Note that the baroclinic velocity is less than

FIG. 3. Snapshots of u' (m s^{-1}) for the run time equal to (a) 1, (b) 5, (c) 10, (d) 20, (e) 40, and (f) 60 M_2 tidal periods.

$1 \times 10^{-5} \text{ m s}^{-1}$ beyond $x = \pm 80 \text{ km}$, which indicates that the generated internal tide has not spread into this region yet. As shown in Fig. 3, the structure of the M_2 baroclinic velocity field becomes stable gradually as the simulation progresses. This shows that the simulated M_2 internal tide is reaching a steady state as the run time increases.

Energy flux $\mathbf{F}_E = \langle \mathbf{u}' p' \rangle$ is a fundamental quantity in internal wave energetics to identify energy sources, wave propagation, and energy sinks (Nash et al. 2005). We roughly estimated the depth-integrated energy flux in this section to show the numerical process during which the whole system reaches a relative steady state. The depth-integrated energy flux is defined as (Duda and Rainville 2008)

$$\mathbf{F}_E = \int_{-H}^0 \mathbf{u}' p' = \sum_{k=1}^N H_k \tilde{u}_k \tilde{P}_k, \quad (3)$$

where $\mathbf{u}'$ is the baroclinic velocity, p' is the perturbation pressure, H_k is the instantaneous thickness of the k th layer, $\tilde{u}_k$ is the baroclinic velocity in the k th layer, and $\tilde{P}_k$ is the perturbation pressure in the k th layer and is given by

$$\tilde{P}_k = P_k - \sum_{m=1}^N P_m H_m / \sum_{m=1}^N H_m, \quad k = 1, 2, \dots, N. \quad (4)$$

In the first part on the right of Eq. (4), P_k is the pressure in the k th layer and can be calculated by Eq. (5), and the second part is the depth-averaged pressure,

$$P = P_{\text{surf}} + \int_z^0 \Delta \rho(\hat{z}) g \eta(\hat{z}) d\hat{z}, \text{ namely:}$$

$$P_1 = 0, \quad P_k = \sum_{m=2}^k [(\rho_m - \rho_{m-1}) g \eta_m], \quad k = 2, \dots, N, \quad (5)$$

where ρ_m is the density in the m th layer and η_m is the displacements of the m th layer interface and can be calculated from the bottom to the surface as

$$\eta_N = H_N + h_N, \quad H_k = \eta_k - \eta_{k+1} + h_k, \quad k = N-1, \dots, 1, \quad (6)$$

where h_k is the thickness of the k th layer at rest.

The time average of the depth-integrated energy flux is given by

$$\overline{\mathbf{F}_E} = \frac{1}{\Delta T} \int_{t_0}^{t_0 + \Delta T} \mathbf{F}_E(t) dt, \quad (7)$$

where ΔT is taken to be one M_2 tidal period in the following analysis. In this experiment, no y variation is considered, so $\mathbf{F}_E$ is only a function of x in this experiment.

Figure 4 shows the distribution of $\overline{\mathbf{F}_E}$ as a function of x . Note that $\overline{\mathbf{F}_E}$ is positive on the east of the ridge and negative on the west of the ridge, which indicates that the internal tide generated on the ridge crest propagates from west to east on the east side of the ridge and from east to west on the other side. Another fact is that after one M_2 tidal period simulation, $\overline{\mathbf{F}_E}$ is nearly zero beyond

FIG. 4. Depth-integrated energy flux as a function of x for the run time equal to 1, 5, 10, 20, 40, and 60 M_2 tidal periods.

$x = \pm 80$ km, which is consistent with the conclusion drawn from Fig. 3a that the generated internal tide has not propagated to this region after one M_2 tidal period. Moreover, $\overline{F_E}$ changes rapidly within the tide generation region, extending 20 km on either side of the ridge crest (Floor et al. 2011) compared with the values outside the region. In the view of energy flux, the M_2 internal tide is on the stage of generation in the first tidal period and gradually reaches steady state as the model integral. A slight difference is found between the curve obtained with five tidal periods and those obtained with more than five tidal periods. This is evidence that according to the temporal variation of $\overline{F_E}$, ENS for the simulated M_2 internal tide may not reached within five M_2 tidal periods.

The simulation results indicate that in the progress of the numerical simulation for the M_2 internal tide, the

baroclinic velocity field and the time average of the depth-integrated energy flux gradually change with the time integral. It is necessary to study ENS before analyzing the features of internal tides.

c. Investigation of ENS for M_2 internal tide

In this section, ENS for the M_2 internal tide based on the experiment in section 2b is investigated. In the field of numerical simulation for the two-dimensional tides, ENS has been understood and applied to the numerical experiments by some scholars. For instance, Fang et al. (1999) carried out a numerical experiment to investigate the principal tidal constituents around the South China Sea. They presented a criterion of ENS as the computed values of ζ , u , and v that differed from the corresponding values one period before by less than 0.001 m, 0.001 m s $^{-1}$, and 0.001 m s $^{-1}$, respectively.

FIG. 5. (a) Temporal variation of DEI and (b) δ DEI.

FIG. 6. (a) Temporal variation of the BTE and (b) δE_{bc} before the 100th M_2 tidal period.

When it comes to the internal tide, the interfacial displacements are much stronger than the surface elevation, and they also draw concerns from scholars. Similar to the two-dimensional tides, after ENS for the internal tide is obtained, the interfacial elevations should change periodically. Thus, when ENS is reached in a single tidal forcing case, the differences between the computed values of interfacial elevations from two adjacent tidal periods ought to be relatively small. Moreover, the energy budget of the whole system should be balanced, which means the baroclinic tidal energy (BTE, including baroclinic kinetic energy and baroclinic potential energy) has no significant change.

We define the maximum difference of the interfacial elevations inside the domain between the p th and the $p - 1$ th tidal period $DEl(p)$ as

$$DEl(p) = \max_{i,j,k} (\eta_{i,j,k}^p - \eta_{i,j,k}^{p-1}), \quad p = 1, 2, \dots, M, \quad (8)$$

where i, j, k indicate the horizontal, zonal, and vertical indices of the domain, respectively. Term η_p is the displacements of interface after the p th tidal period, so $\eta_{i,j,k}^0 \equiv 0$, and M indicates the model run time whose unit is the M_2 tidal period. The temporal variations of DEl in the experiment are plotted in Fig. 5a. There exists an obvious downward trend before the 40th tidal period and a quasi-periodic oscillation near a small value after the 40th tidal period. To facilitate the analysis of ENS by DEl , the relative change rate of DEl is defined as

$$\delta DEl(t) = \left| \frac{DEl(t) - DEl(t-1)}{DEl(t-1)} \right|, \quad t = 2, \dots, M. \quad (9)$$

The temporal variation of δDEl calculated by Eq. (9) is plotted in Fig. 5b. The downward trend before the 40th tidal period and the quasi-periodic oscillation after it are

quite similar to the temporal variation in Fig. 5a. Based on the analysis of DEl and δDEl , we present the first criterion of ENS as follows: if δDEl of the experiment is less than 15% (the dotted line in Fig. 5b) after a specific moment, then ENS for the internal tide is obtained. According to this criterion, ENS for this experiment is obtained after 31 M_2 tidal periods.

BTE is defined as the sum of kinetic energy and potential energy contributed by the internal tide. In this model, the depth-integrated BTE is defined as

$$E_{bc} = \sum_{k=1}^N \left[\frac{1}{2} \rho_k H_k (\tilde{u}_k^2 + \tilde{v}_k^2) \right] + \sum_{k=2}^N \left[\frac{1}{2} (\rho_k - \rho_{k-1}) g \eta_k^2 \right], \quad (10)$$

where N is the total number of layers. The first term on the right side of Eq. (10) is the baroclinic kinetic energy, and the second term is the baroclinic potential energy. Same as the energy flux, the BTE is also averaged over one M_2 tidal period.

Figure 6a presents the temporal variation of the BTE before the 100th tidal period. The value of BTE after the first tidal period is 0.239 kJ, and it increases

TABLE 2. Overview of the cases and their nondimensional parameters.

No.	Case	A_h	A_v	C_k	γ
1	R0	0	0	0	2.3148×10^{-6}
2	F1	1000	0	0	2.3148×10^{-6}
3	F2	2000	0	0	2.3148×10^{-6}
4	F3	0	0.05	0	2.3148×10^{-6}
5	F4	0	0	0.0025	2.3148×10^{-6}
6	B1	0	0	0	2.3148×10^{-5}
7	B2	0	0	0	2.3148×10^{-7}
Unit			$m^2 s^{-1}$		s^{-1}

FIG. 7. (a) Temporal variation of DEI and (b) δ DEI for group 1.

rapidly in the first 10 M_2 tidal periods. After the 10th M_2 tidal period, the BTE hardly changed and its value stabilizes around 1.304 kJ. Similar to δ DEI, we define the relative change rate of the BTE after the t th period as

$$\delta E_{bc}(t) = \left| \frac{\overline{E}_{bc}(t) - \overline{E}_{bc}(t-1)}{\overline{E}_{bc}(t-1)} \right|, \quad t = 2, \dots, M, \quad (11)$$

where $\overline{E}_{bc}$ is the averaged value of BTE. Figure 6b shows the temporal variation of δE_{bc} computed by Eq. (11). It is clear evidence for the balance of the energy budget that δE_{bc} decreases with the time integral. Based on the analysis of BTE and δE_{bc} , the second criterion of ENS for the M_2 internal tide will be presented as follows: in view of the energy budget, if δE_{bc} is less than 1% (the dotted line in Fig. 6b) after a specific moment, then ENS for the internal tide is obtained. According to this criterion,

ENS for this experiment is obtained after nine M_2 tidal periods.

When we try to figure out ENS for internal tides, we should consider both of the criteria in this section. So, in this case, ENS for the M_2 internal tide is obtained after 385 h (31 M_2 tidal periods) of model time. Thereafter, the simulated interfacial fluid motion shows a periodic oscillation and the baroclinic energy budget is balanced.

d. Effects of parameters on ENS

The effects of the dissipation parameters on the simulation results are well known and are discussed in textbooks on computational fluid dynamics. However, the effects of these parameters on ENS for internal tides are still unknown and should be discussed. Based on the two criteria proposed in section 2c, we are devoted to figuring out the effects of four parameters on ENS for the M_2 internal tide by means of four groups of idealized

FIG. 8. (a) Temporal variation of the BTE and (b) δE_{bc} before the 100th M_2 tidal period for group 1.

FIG. 9. As in Fig. 7, but for group 2.

experiments. In this paper, we pick seven out of them to show the effects of parameters on ENS. Any reader who may be interested in the other cases can refer to the supplementary file. All seven model cases are listed in Table 2. The four nondimensional parameters A_h , A_v , C_k , and γ represent the horizontal and vertical eddy viscosity coefficients, the bottom friction coefficient, and the linear baroclinic damping coefficient, respectively. The other parameters in the seven experiments are exactly the same as the experiment in section 2a. With case R0 being the control group, seven experiments can be divided into two groups: group 1 includes R0, F1, F2, F3, and F4; group 2 includes R0, B1, and B2.

The first three parameters— A_h , A_v , and C_k —to be investigated in group 1 are important dissipation parameters in the numerical simulation of internal tides. Since the governing equations are hydrostatic, non-hydrostatic processes are not considered in this model (Miao et al. 2011). The effect of γ , which is taken as 0.2

per day ($2.3148 \times 10^{-6} \text{ s}^{-1}$) in Niwa and Hibiya (2001), will be investigated in group 2.

Figure 7a shows the temporal variations of DEI in group 1. All five curves take on a downward trend before the 40th tidal period and then have quasi-periodic oscillations around a small value. Note that the curves for cases F3, F4, and R0 almost overlap with each other, whereas the differences between F1, F2, and R0 are obvious. Besides, sensitivity experiments (please refer to the supplementary file) for cases F3 and F4 (reduced by half or doubled) bring no differences (less than 0.001% after every tidal period) from the values of DEI. So, the effects of A_v and C_k on ENS for the internal tide can be ignored. In contrast, the values of DEI decrease as A_h increases ($R0 \rightarrow F1 \rightarrow F2$) until they reach the same level after the 40th tidal period, which indicates that the increase of A_h contributes to ENS for the M_2 internal tide. Temporal variations of δDEI in group 1 are plotted in Fig. 7b. Based on the first criterion in section 2c, ENS

FIG. 10. As in Fig. 8, but for group 2.

FIG. 11. (a)–(f) Snapshots of u' (m s^{-1}) for experiments 2–7 at the end of the 60th M_2 tidal period.

for experiments 1–5 are obtained after 31, 6, 5, 31, and 31 M_2 tidal periods, respectively.

The temporal variations of BTE in group 1 are calculated and plotted in Fig. 8a, which shows a similar trend as in Fig. 6a. The differences between the five curves are consistent with those in Fig. 7a, which confirms the conclusion stated above. With the increase of A_h ($R0 \rightarrow F1 \rightarrow F2$), the BTEs decrease evidently. Based on the temporal variations of δE_{bc} (Fig. 8b) and the second criterion in section 2c, ENS for experiments 1–5 are obtained after 9, 19, 33, 9, and 9 M_2 tidal periods, respectively. The two criteria and ENSs for the M_2 internal tide in experiments 1–5 can be obtained after 31,

19, 33, 31, and 31 M_2 tidal periods, respectively. Among the three parameters investigated in group 1, A_v and C_k have little effect on ENS, whereas the larger the value of A_h is, the easier it will be to obtain ENS and the smaller the value of BTE will be after ENS.

A similar method can be applied to investigate the effect of γ . Figure 9 shows the temporal variations of DEI and δDEI in group 2. Similar patterns as in Fig. 7 are found. As γ increases ($B2 \rightarrow R0 \rightarrow B1$), the values of DEI decrease in every tidal period until they are almost the same after the 40th M_2 tidal period. As shown in Fig. 10, similar patterns as in Fig. 8 indicate that the larger the value of γ is, the smaller the BTE will be, which is the

FIG. 12. Depth-integrated energy flux as a function of x for the seven experiments in section 2d.

FIG. 13. Bathymetry (m) and T/P tracks (with track number) near the Hawaiian Ridge.

same as the effect of A_h . As a result, increasing γ also contributes to ENS for the M_2 internal tide. Based on the two criteria, ENSs for the M_2 internal tide in B1 and B2 are obtained after 30 and 44 M_2 tidal periods, respectively.

In the next two paragraphs, the baroclinic velocity field and the depth-integrated energy flux after ENS will be presented to analyze the effects of the four parameters on the features of the M_2 internal tide. According to the analyses stated above, all the results are taken from the 60th M_2 tidal periods to ensure ENS.

Figure 11 shows the baroclinic velocity fields of experiments 2–7 at the end of the 60th tidal period. A comparison with the result of R0 (Fig. 3f) indicates that 1) A_h and γ have an obvious effect on the baroclinic velocity field: a larger A_h or γ leads to a smaller baroclinic velocity and a more blurred velocity field (Fig. 3f $\rightarrow$ Fig. 11a $\rightarrow$ Fig. 11b or Fig. 11e $\rightarrow$ Fig. 3f $\rightarrow$ Fig. 11f) and 2) A_v and C_k have little effect on the value and structure of the baroclinic velocity.

Figure 12 compares the depth-integrated energy flux for all the experiments after the model runs 60 M_2 tidal periods. Trends for the seven curves are similar,

especially in the tide generation region ($[-20\text{ km}, 20\text{ km}]$). Note that the curves for cases F3 and F4 are almost overlapping with that for case R0. Figure 12 indicates that A_v and C_k have little effect on the depth-integrated energy flux, whereas larger A_h and γ lead to a smaller energy flux after ENS.

3. Numerical simulation for M_2 internal tide around the Hawaiian Ridge

The isopycnic coordinate internal tide model introduced in section 2 is applied to simulate the M_2 internal tide around the Hawaiian Ridge in this section. As mentioned in the introduction, many researchers (Dushaw et al. 1995; Ray and Mitchum 1996, 1997; Kang et al. 2000; Zhao et al. 2010; Zhao et al. 2011) have investigated the generation and propagation processes of internal tides in the Hawaiian Ridge. So, rather than carrying out in-depth explorations on the M_2 internal tide, we will localize our research to an objective level to focus on the ENS for M_2 internal tides in this region. The preliminary analysis on the M_2 internal tide is provided as a numerical example on how to apply the criteria to a

FIG. 14. (a) Initial potential density, (b) relevant buoyancy frequency, and (c) layer thickness. Note that the potential density and buoyancy frequencies below 3500 m vary slowly with depth and hence are not shown here.

real case. We hope this will present some new information and ideas to the modeling literature.

a. Model setup

The model bathymetry (Fig. 13) is obtained from the ETOPO1 dataset (Amante and Eakins 2009) with a 0.1°

horizontal resolution over a region extending from 10°N , 180°W to 40°N , 150°W . The initial potential density is set to be horizontally homogeneous and is computed using the profile at 40°N , 150°W obtained from the WOA05 dataset. There are 15 uneven vertical layers, varying from 25 m at the top to 500 m at the bottom

FIG. 15. Temporal variations of DEI for the second through fifteenth layers.

TABLE 3. Statistics for the number of periods for ENS for all interfaces.

Layer	2	3	4	5	6	7	8
No. of periods	21	42	40	38	37	38	46
Layer	9	10	11	12	13	14	15
No. of periods	44	41	39	40	40	45	42

(Fig. 14). The tidal forcing is applied to the four lateral boundaries with Flather boundary conditions using barotropic tidal currents and elevations of the principal lunar semidiurnal tide (M_2) that is obtained from the TPXO7.2 global tide model (Egbert and Erofeeva 2002). The model is run for $100M_2$ tidal periods with a time step of 41.4 s (within the limitations of Courant–Friedrichs–Lewy condition). The nondimensional bottom coefficient κ is taken as $0.0025 \text{ m}^2 \text{ s}^{-1}$, $A_h = 3000 \text{ m}^2 \text{ s}^{-1}$, $A_v = 0.001 \text{ m}^2 \text{ s}^{-1}$, and $\gamma = 2.3148 \times 10^{-6} \text{ s}^{-1}$. The Coriolis frequency is taken to be the local values.

b. Results and preliminary analysis

1) TEMPORAL VARIATION OF DEI

Based on Eq. (9), the values of DEI and δDEI for all tidal periods in the practical experiment are calculated. To better understand the interfacial displacement of each layer, the DEI of each layer is plotted as a panel in Fig. 15. The temporal variations of DEI in all the panels of Fig. 15 clearly indicate that ENS in the practical experiment is obtained gradually before the 50th M_2 tidal period. Based on the first criterion, we counted the number of periods for ENS for all the interfaces in the experiment and listed them in Table 3. Based on this table, ENS of the whole system can be reached after $46M_2$ tidal periods.

2) TEMPORAL VARIATION OF BTE

We calculated the BTE for every tidal period and plotted the temporal variations of BTE and δE_{bc} in Figs. 16a and 16b, respectively. As shown in Fig. 16, the value of BTE keeps increasing until it reaches the maximum value of $1.55 \times 10^{13} \text{ kJ}$ and the value of δE_{bc} decreases from about 2% to 0% (except for the oscillation up to 0.06%). Based on the second criterion in section 2c, ENS for the M_2 internal tide can be obtained within only four M_2 tidal periods in light of the BTE.

On the basis of the previous two subsections, ENS for the M_2 internal tide in this section is obtained after 46 tidal periods. The simulation results will be analyzed preliminarily in the next subsections to examine the feasibility of the model and the criteria for ENS in practical experiment.

3) SURFACE TIDE

Before focusing on the internal tide, it is necessary to analyze the simulated results of surface tide. The simulated surface tide after the 50th M_2 tidal period is analyzed. After ENS for M_2 internal tides, the surface elevations during the last four tidal periods are used for harmonic analysis, and tidal harmonic constants are plotted in Fig. 17a. Compared to the cotidal charts from the Oregon State University (OSU) tidal model (Fig. 17b), the simulated results can capture the main features of the M_2 constituent around the Hawaiian Ridge: The M_2 constituent propagates from the northeast to the southwest, which is well known to researchers (Zhao et al. 2011). This is evidence to prove the feasibility and ENS of the model. The mean absolute differences (MADs) of amplitude and phase between the simulated results and the OSU model results are 1.1 cm and 5.5° , respectively. Further inspection shows that the

FIG. 16. Temporal variation of (a) BTE and (b) its relative change rate δE_{bc} .

FIG. 17. Cotidal charts for the M_2 tidal constituent from simulation results after 50 M_2 tidal periods and the OSU tidal model.

major difference between the simulated results and the OSU model results is small-scale fluctuations of coamplitude and cophase lines along the Hawaiian Ridge. These fluctuations are actually the surface response to the M_2 internal tide (Kang et al. 2000), from which it is easy to trace back the generation sources and propagation pattern of internal tide.

To better understand the differences, we calculate the root-mean-square difference (RMSD) E over a tidal period between the model and the OSU tidal model (Cummins and Oey 1997), which is given by

$$E = \sqrt{\frac{1}{2}(A_o^2 + A_m^2) - A_o A_m \cos(\phi_o - \phi_m)}, \quad (12)$$

where A is amplitude and ϕ is the phase for the M_2 constituent, and subscripts o and m refer to the OSU model and the model results, respectively. The largest RMSD difference in the domain is about 4.6 cm.

The overall agreement between the simulated results and the observations of the surface tide could also be found from Fig. 18, which shows the amplitude and phase along TOPEX/Poseidon (T/P) tracks 097, 173,

FIG. 18. Amplitude and phase along T/P tracks 097, 173, 249, 071, and 147.

FIG. 19. Amplitude of interfacial displacement with a resting depth of 500 m at the end of the (a) 1, (b) 3, (c) 50, and (d) 80 M_2 tidal periods.

249, 071, and 147. In Fig. 19, the barotropic results are comparable with Fan et al. (2011). Small-scale fluctuations could be found superposed on the large-scale barotropic waves, implying the modulation of the internal tide again. As shown in Fig. 18, the internal tide could modulate the surface amplitude by about 5 cm, which is comparable to the results of Ray and Mitchum (1996).

4) TEMPORAL VARIATIONS OF INTERFACIAL FEATURES FOR INTERNAL TIDE

Interfacial displacement at 500 m (the bottom of the seventh layer) and zonal baroclinic velocity at 25 m (the bottom of the first layer) are shown in Figs. 19 and 20, respectively. At the end of the first period, a trough-to-crest interfacial disturbance of about 60 m (Fig. 19a) and a zonal baroclinic velocity of about 10 cm s^{-1} (Fig. 20a), which are driven by the interaction between barotropic tide and rough topography, are generated along the Hawaiian Ridge. Figures 19 and 20 show the generation and propagation processes of the M_2 internal constituent, from which it is easy to find three main sources of the M_2

internal constituent, that is, Kauai Channel (KC), Nihoa Island (NI), and French Frigate Shoals (FFS), which are consistent with the results of Rudnick et al. (2003). The generated internal tides along the Hawaiian Ridge propagate mainly toward the northeast and southwest. On the other side, the pattern and the magnitude for interfacial displacement and zonal baroclinic velocity after the 50th period are almost identical to those after the 80th period, which indicates that ENS for the M_2 internal tide can be obtained before the 50th M_2 tidal period. In addition, it should be noticed that at the other small rough topographies (to the southwest and northwest of the Hawaiian Ridge), there are also internal tides generated.

Figure 21 is the snapshot at the end of the 50th M_2 tidal period for the interfacial disturbance amplitude of each layer in the log form. The trough-to-crest amplitude could reach over 30 m near the three main sources (KC, NI, and FFS). According to LeBlond and Mysak (1978), the magnitude of the surface manifestation of the M_2 internal tide is approximately $\Delta\rho/\rho_0$ (where $\Delta\rho$ is the difference of density and ρ_0 is the reference density) times the

FIG. 20. Zonal baroclinic velocity (m s^{-1}) at 100 m at the end of the (a) 1, (b) 3, (c) 50, and (d) 80 M_2 tidal periods.

interfacial amplitude. Then we can estimate that the magnitude of the surface manifestation of the internal tide is about 5 cm, which shows good agreement with the observations. In addition, the simulated amplitude agrees well with [Martin et al. \(2006\)](#), where a trough-to-crest amplitude as large as 60 m is observed. In the horizontal direction, the M_2 internal constituent mainly propagates northeastward and southwestward, perpendicular to the northwest-southeast ridge. There exist two beams indicating generation directions, which are the southward beam from NI and the northeastward beam from FFS. These two propagation directions are highly consistent with [Zhao et al.'s \(2011\)](#) research. In the vertical direction, the influence area of the M_2 internal tide in the first three layers is smaller than that in other layers, while the patterns below the fourth layer are similar, indicating that the internal tide may propagate further at a deeper depth.

4. Summary

For the numerical simulation of internal tides, ENS is the prerequisite before any analysis. Nonetheless, the

criteria to examine ENS are not presented by scholars. In this paper, we investigate ENS for internal tides on idealized topography and near the Hawaiian Ridge. Moreover, two criteria are provided to check the status of ENS for M_2 internal tides that should suit for different numerical models and cases. The idealized experiment is carried out on a Gaussian topography to investigate ENS for the M_2 internal tide. The first criterion of ENS for the M_2 internal tide is presented based on the temporal variations of simulated interfacial elevations. Then temporal variation of the baroclinic tidal energy is analyzed and the second criterion is put forward in the view of the baroclinic energy budget. ENS for the M_2 internal tide will be obtained after both criteria are reached.

Then we design a series of experiments to study the effects of four different parameters (A_h , A_v , C_k , and γ) on ENS for the M_2 internal tide. The applicability of the criteria is also examined in these experiments. Simulation results indicate that only two parameters— A_h and γ —have obvious effects on ENS for the M_2 internal tide. Moreover, with the two parameters

FIG. 21. Log of the interfacial disturbance amplitude (m) at each layer (the number in each panel denotes the bottom of the corresponding layer, e.g., panel 1 denotes the interfacial amplitude at the bottom of the first layer). Propagation direction of the FFS and NI beams (black lines) are also shown here.

increasing, the values of BTE and baroclinic velocity tend to be smaller.

Finally, a numerical simulation is carried out to investigate the M_2 internal tide around the Hawaiian Ridge and to examine the criteria presented above. Analyses based on the criteria indicate that ENS for the M_2 internal tide around the Hawaiian Ridge can be obtained after the 50th M_2 tidal period. As for the surface tide, simulated results show the overall characteristic of the M_2 constituent near the Hawaiian Ridge. The MADs of amplitude and phase between simulated results and OSU model results are 1.1 cm and 5.5° , respectively. The simulated results also show good agreement with the T/P observations. The temporal variations of interfacial displacement and zonal baroclinic velocity also confirm the applicability of the criteria of ENS. Moreover, we present three-dimensional distributions of interfacial disturbance amplitudes at the end of the article to show agreement with other studies.

The results of idealized and practical experiments emphasize the necessity to consider ENS before analyzing the features of simulated internal tides. The run time before ENS for internal tides depends on the model domain, configuration of bathymetry, stratification, and strength of dissipation. So, in this paper, we do not intend to present general formulas to determine the length of run time. Instead, we provide a strategy and two

criteria to judge the state of internal tide simulation and to ensure ENS before analyses. Nevertheless, this paper is just a preliminary research based on one tidal constituent (M_2). Further studies with multiconstituents or using different tidal models need to be accomplished.

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REFERENCES

- Alford, M. H., and Coauthors, 2015: The formation and fate of internal waves in the South China Sea. *Nature*, **521**, 65–69, doi:[10.1038/nature14399](https://doi.org/10.1038/nature14399).
- Amante, C., and B. W. Eakins, 2009: ETOPO1 1 arc-minute global relief model: Procedures, data sources and analysis. NOAA Tech. Memo. NESDIS NGDC-24, 19 pp., doi:[10.7289/V5C8276M](https://doi.org/10.7289/V5C8276M). [Available online at https://docs.lib.noaa.gov/noaa_documents/NESDIS/TM_NESDIS_NGDC/TM_NESDIS_NGDC_24.pdf.]

- Balmforth, N., G. Ierley, and W. Young, 2002: Tidal conversion by subcritical topography. *J. Phys. Oceanogr.*, **32**, 2900–2914, doi:[10.1175/1520-0485\(2002\)032<2900:TCBST>2.0.CO;2](https://doi.org/10.1175/1520-0485(2002)032<2900:TCBST>2.0.CO;2).
- Bell, T., 1975: Topographically generated internal waves in the open ocean. *J. Geophys. Res.*, **80**, 320–327, doi:[10.1029/JC080i003p00320](https://doi.org/10.1029/JC080i003p00320).
- Buijsman, M. C., S. Legg, and J. Klymak, 2012: Double-ridge internal tide interference and its effect on dissipation in Luzon Strait. *J. Phys. Oceanogr.*, **42**, 1337–1356, doi:[10.1175/JPO-D-11-0210.1](https://doi.org/10.1175/JPO-D-11-0210.1).
- , and Coauthors, 2013: Three-dimensional double-ridge internal tide resonance in Luzon Strait. *J. Phys. Oceanogr.*, **44**, 850–869, doi:[10.1175/JPO-D-13-024.1](https://doi.org/10.1175/JPO-D-13-024.1).
- Carter, G. S., and M. A. Merrifield, 2007: Open boundary conditions for regional tidal simulations. *Ocean Modell.*, **18**, 194–209, doi:[10.1016/j.ocemod.2007.04.003](https://doi.org/10.1016/j.ocemod.2007.04.003).
- , and Coauthors, 2008: Energetics of M_2 barotropic-to-baroclinic tidal conversion at the Hawaiian Islands. *J. Phys. Oceanogr.*, **38**, 2205–2223, doi:[10.1175/2008JPO3860.1](https://doi.org/10.1175/2008JPO3860.1).
- Cummins, P. F., and L.-Y. Oey, 1997: Simulation of barotropic and baroclinic tides off northern British Columbia. *J. Phys. Oceanogr.*, **27**, 762–781, doi:[10.1175/1520-0485\(1997\)027<0762:SOBAPT>2.0.CO;2](https://doi.org/10.1175/1520-0485(1997)027<0762:SOBAPT>2.0.CO;2).
- Di Lorenzo, E., W. R. Young, and S. L. Smith, 2006: Numerical and analytical estimates of M_2 tidal conversion at steep oceanic ridges. *J. Phys. Oceanogr.*, **36**, 1072–1084, doi:[10.1175/JPO2880.1](https://doi.org/10.1175/JPO2880.1).
- Duda, T. F., and L. Rainville, 2008: Diurnal and semidiurnal internal tide energy flux at a continental slope in the South China Sea. *J. Geophys. Res.*, **113**, C03025, doi:[10.1029/2007JC004418](https://doi.org/10.1029/2007JC004418).
- Dunphy, M., and K. G. Lamb, 2014: Focusing and vertical mode scattering of the first mode internal tide by mesoscale eddy interaction. *J. Geophys. Res. Oceans*, **119**, 523–536, doi:[10.1002/2013JC009293](https://doi.org/10.1002/2013JC009293).
- Dushaw, B. D., B. M. Howe, B. D. Cornuelle, P. F. Worcester, and D. S. Luther, 1995: Barotropic and baroclinic tides in the central North Pacific Ocean determined from long-range reciprocal acoustic transmissions. *J. Phys. Oceanogr.*, **25**, 631–647, doi:[10.1175/1520-0485\(1995\)025<0631:BABTIT>2.0.CO;2](https://doi.org/10.1175/1520-0485(1995)025<0631:BABTIT>2.0.CO;2).
- Egbert, G. D., and S. Y. Erofeeva, 2002: Efficient inverse modeling of barotropic ocean tides. *J. Atmos. Oceanic Technol.*, **19**, 183–204, doi:[10.1175/1520-0426\(2002\)019<0183:EIMOBO>2.0.CO;2](https://doi.org/10.1175/1520-0426(2002)019<0183:EIMOBO>2.0.CO;2).
- Fan, L.-L., B. Wang, and X.-Q. Lv, 2011: Cotidal charts near Hawaii derived from TOPEX/Poseidon altimetry data. *J. Atmos. Oceanic Technol.*, **28**, 606–614, doi:[10.1175/2010JTECHO809.1](https://doi.org/10.1175/2010JTECHO809.1).
- Fang, G., Y.-K. Kwok, K. Yu, and Y. Zhu, 1999: Numerical simulation of principal tidal constituents in the South China Sea, Gulf of Tonkin and Gulf of Thailand. *Cont. Shelf Res.*, **19**, 845–869, doi:[10.1016/S0278-4343\(99\)00002-3](https://doi.org/10.1016/S0278-4343(99)00002-3).
- Floor, J. W., F. Auclair, and P. Marsaleix, 2011: Energy transfers in internal tide generation, propagation and dissipation in the deep ocean. *Ocean Modell.*, **38**, 22–40, doi:[10.1016/j.ocemod.2011.01.009](https://doi.org/10.1016/j.ocemod.2011.01.009).
- Jan, S., C.-S. Chern, J. Wang, and S.-Y. Chao, 2007: Generation of diurnal K_1 internal tide in the Luzon Strait and its influence on surface tide in the South China Sea. *J. Geophys. Res.*, **112**, C06019, doi:[10.1029/2006JC004003](https://doi.org/10.1029/2006JC004003).
- Johnston, T., and M. A. Merrifield, 2003: Internal tide scattering at seamounts, ridges, and islands. *J. Geophys. Res.*, **108**, 3180, doi:[10.1029/2002JC001528](https://doi.org/10.1029/2002JC001528).
- Kang, S., M. Foreman, W. Crawford, and J. Cherniawsky, 2000: Numerical modeling of internal tide generation along the Hawaiian Ridge. *J. Phys. Oceanogr.*, **30**, 1083–1098, doi:[10.1175/1520-0485\(2000\)030<1083:NMOITG>2.0.CO;2](https://doi.org/10.1175/1520-0485(2000)030<1083:NMOITG>2.0.CO;2).
- Katsumata, K., 2006: Two- and three-dimensional numerical models of internal tide generation at a continental slope. *Ocean Modell.*, **12**, 32–45, doi:[10.1016/j.ocemod.2005.03.001](https://doi.org/10.1016/j.ocemod.2005.03.001).
- Kerry, C. G., B. S. Powell, and G. S. Carter, 2014: The impact of subtidal circulation on internal tide generation and propagation in the Philippine Sea. *J. Phys. Oceanogr.*, **44**, 1386–1405, doi:[10.1175/JPO-D-13-0142.1](https://doi.org/10.1175/JPO-D-13-0142.1).
- Khatiwala, S., 2003: Generation of internal tides in an ocean of finite depth: Analytical and numerical calculations. *Deep-Sea Res. I*, **50**, 3–21, doi:[10.1016/S0967-0637\(02\)00132-2](https://doi.org/10.1016/S0967-0637(02)00132-2).
- Klymak, J. M., M. Buijsman, S. A. Legg, and R. Pinkel, 2013: Parameterizing surface and internal tide scattering and breaking on supercritical topography: The one- and two-ridge cases. *J. Phys. Oceanogr.*, **43**, 1380–1397, doi:[10.1175/JPO-D-12-061.1](https://doi.org/10.1175/JPO-D-12-061.1).
- LeBlond, P., and L. Mysak, 1978: *Waves in the Ocean*. Elsevier Oceanography Series, Vol. 20, Elsevier, 602 pp.
- Legg, S., 2004a: Internal tides generated on a corrugated continental slope. Part I: Cross-slope barotropic forcing. *J. Phys. Oceanogr.*, **34**, 156–173, doi:[10.1175/1520-0485\(2004\)034<0156:ITGOAC>2.0.CO;2](https://doi.org/10.1175/1520-0485(2004)034<0156:ITGOAC>2.0.CO;2).
- , 2004b: Internal tides generated on a corrugated continental slope. Part II: Along-slope barotropic forcing. *J. Phys. Oceanogr.*, **34**, 1824–1838, doi:[10.1175/1520-0485\(2004\)034<1824:ITGOAC>2.0.CO;2](https://doi.org/10.1175/1520-0485(2004)034<1824:ITGOAC>2.0.CO;2).
- , and A. Adcroft, 2003: Internal wave breaking at concave and convex continental slopes. *J. Phys. Oceanogr.*, **33**, 2224–2246, doi:[10.1175/1520-0485\(2003\)033<2224:IWBACA>2.0.CO;2](https://doi.org/10.1175/1520-0485(2003)033<2224:IWBACA>2.0.CO;2).
- , and K. M. Huijts, 2006: Preliminary simulations of internal waves and mixing generated by finite amplitude tidal flow over isolated topography. *Deep-Sea Res. II*, **53**, 140–156, doi:[10.1016/j.dsr2.2005.09.014](https://doi.org/10.1016/j.dsr2.2005.09.014).
- Llewellyn Smith, S. G., and W. Young, 2002: Conversion of the barotropic tide. *J. Phys. Oceanogr.*, **32**, 1554–1566, doi:[10.1175/1520-0485\(2002\)032<1554:COTBT>2.0.CO;2](https://doi.org/10.1175/1520-0485(2002)032<1554:COTBT>2.0.CO;2).
- Locarnini, R. A., A. V. Mishonov, J. I. Antonov, T. P. Boyer, and H. E. Garcia, 2006: *Temperature*. Vol. 1, *World Ocean Atlas 2005*, NOAA Atlas NESDIS 61, 182 pp.
- MacCready, P., and G. Pawlak, 2001: Stratified flow along a corrugated slope: Separation drag and wave drag. *J. Phys. Oceanogr.*, **31**, 2824–2839, doi:[10.1175/1520-0485\(2001\)031<2824:SFAACS>2.0.CO;2](https://doi.org/10.1175/1520-0485(2001)031<2824:SFAACS>2.0.CO;2).
- Martin, J. P., D. L. Rudnick, and R. Pinkel, 2006: Spatially broad observations of internal waves in the upper ocean at the Hawaiian Ridge. *J. Phys. Oceanogr.*, **36**, 1085–1103, doi:[10.1175/JPO2881.1](https://doi.org/10.1175/JPO2881.1).
- Mathur, M., G. S. Carter, and T. Peacock, 2014: Topographic scattering of the low-mode internal tide in the deep ocean. *J. Geophys. Res. Oceans*, **119**, 2165–2182, doi:[10.1002/2013JC009152](https://doi.org/10.1002/2013JC009152).
- Miao, C., H. Chen, and X. Lu, 2011: An isopycnic-coordinate internal tide model and its application to the South China Sea. *Chin. J. Oceanol. Limnol.*, **29**, 1339–1356, doi:[10.1007/s00343-011-1023-5](https://doi.org/10.1007/s00343-011-1023-5).
- Nash, J. D., M. H. Alford, and E. Kunze, 2005: Estimating internal wave energy fluxes in the ocean. *J. Atmos. Oceanic Technol.*, **22**, 1551–1570, doi:[10.1175/JTECH1784.1](https://doi.org/10.1175/JTECH1784.1).
- Niwa, Y., and T. Hibiya, 2001: Numerical study of the spatial distribution of the M_2 internal tide in the Pacific Ocean. *J. Geophys. Res.*, **106**, 22 441–22 449, doi:[10.1029/2000JC000770](https://doi.org/10.1029/2000JC000770).

- , and —, 2004: Three-dimensional numerical simulation of M_2 internal tides in the East China Sea. *J. Geophys. Res.*, **109**, C04027, doi:[10.1029/2003JC001923](https://doi.org/10.1029/2003JC001923).
- Ponte, A. L., and B. D. Cornuelle, 2013: Coastal numerical modelling of tides: Sensitivity to domain size and remotely generated internal tide. *Ocean Modell.*, **62**, 17–26, doi:[10.1016/j.ocemod.2012.11.007](https://doi.org/10.1016/j.ocemod.2012.11.007).
- Powell, B., I. Janeković, G. Carter, and M. Merrifield, 2012: Sensitivity of internal tide generation in Hawaii. *Geophys. Res. Lett.*, **39**, L10606, doi:[10.1029/2012GL051724](https://doi.org/10.1029/2012GL051724).
- Qian, H., P.-T. Shaw, and D. S. Ko, 2010: Generation of internal waves by barotropic tidal flow over a steep ridge. *Deep-Sea Res. I*, **57**, 1521–1531, doi:[10.1016/j.dsr.2010.09.001](https://doi.org/10.1016/j.dsr.2010.09.001).
- Rainville, L., T. S. Johnston, G. S. Carter, M. A. Merrifield, R. Pinkel, P. F. Worcester, and B. D. Dushaw, 2010: Interference pattern and propagation of the M_2 internal tide south of the Hawaiian Ridge. *J. Phys. Oceanogr.*, **40**, 311–325, doi:[10.1175/2009JPO4256.1](https://doi.org/10.1175/2009JPO4256.1).
- Ray, R. D., and G. T. Mitchum, 1996: Surface manifestation of internal tides generated near Hawaii. *Geophys. Res. Lett.*, **23**, 2101–2104, doi:[10.1029/96GL02050](https://doi.org/10.1029/96GL02050).
- , and —, 1997: Surface manifestation of internal tides in the deep ocean: Observations from altimetry and island gauges. *Prog. Oceanogr.*, **40**, 135–162, doi:[10.1016/S0079-6611\(97\)00025-6](https://doi.org/10.1016/S0079-6611(97)00025-6).
- Rudnick, D. L., and Coauthors, 2003: From tides to mixing along the Hawaiian Ridge. *Science*, **301**, 355–357, doi:[10.1126/science.1085837](https://doi.org/10.1126/science.1085837).
- Shaw, P.-T., and S.-Y. Chao, 2006: A nonhydrostatic primitive-equation model for studying small-scale processes: An object-oriented approach. *Cont. Shelf Res.*, **26**, 1416–1432, doi:[10.1016/j.csr.2006.01.018](https://doi.org/10.1016/j.csr.2006.01.018).
- , D. S. Ko, and S.-Y. Chao, 2009: Internal solitary waves induced by flow over a ridge: With applications to the northern South China Sea. *J. Geophys. Res.*, **114**, C02019, doi:[10.1029/2008JC005007](https://doi.org/10.1029/2008JC005007).
- Simmons, H. L., R. W. Hallberg, and B. K. Arbic, 2004: Internal wave generation in a global baroclinic tide model. *Deep-Sea Res. II*, **51**, 3043–3068, doi:[10.1016/j.dsr2.2004.09.015](https://doi.org/10.1016/j.dsr2.2004.09.015).
- Song, D., T. Pohlmann, X. Chen, and D. Wu, 2010: The role of sea water viscosity in modeling the vertical movement of internal tides. *Ocean Modell.*, **34**, 63–69, doi:[10.1016/j.ocemod.2010.04.005](https://doi.org/10.1016/j.ocemod.2010.04.005).
- St. Laurent, L., and C. Garrett, 2002: The role of internal tides in mixing the deep ocean. *J. Phys. Oceanogr.*, **32**, 2882–2899, doi:[10.1175/1520-0485\(2002\)032<2882:TROI>2.0.CO;2](https://doi.org/10.1175/1520-0485(2002)032<2882:TROI>2.0.CO;2).
- Thorpe, S., 1992: The generation of internal waves by flow over the rough topography of a continental slope. *Proc. Roy. Soc. London*, **439A**, 115–130, doi:[10.1098/rspa.1992.0137](https://doi.org/10.1098/rspa.1992.0137).
- Winters, K. B., and E. A. D'Asaro, 1997: Direct simulation of internal wave energy transfer. *J. Phys. Oceanogr.*, **27**, 1937–1945, doi:[10.1175/1520-0485\(1997\)027<1937:DSOIWE>2.0.CO;2](https://doi.org/10.1175/1520-0485(1997)027<1937:DSOIWE>2.0.CO;2).
- Wu, L., C. Miao, and W. Zhao, 2013: Patterns of K_1 and M_2 internal tides and their seasonal variations in the northern South China Sea. *J. Oceanogr.*, **69**, 481–494, doi:[10.1007/s10872-013-0183-7](https://doi.org/10.1007/s10872-013-0183-7).
- Xu, Z., K. Liu, B. Yin, Z. Zhao, Y. Wang, and Q. Li, 2016: Long-range propagation and associated variability of internal tides in the South China Sea. *J. Geophys. Res. Oceans*, **121**, 8268–8286, doi:[10.1002/2016JC012105](https://doi.org/10.1002/2016JC012105).
- Zhao, Z., M. H. Alford, J. A. MacKinnon, and R. Pinkel, 2010: Long-range propagation of the semidiurnal internal tide from the Hawaiian Ridge. *J. Phys. Oceanogr.*, **40**, 713–736, doi:[10.1175/2009JPO4207.1](https://doi.org/10.1175/2009JPO4207.1).
- , —, J. Girtton, T. S. Johnston, and G. Carter, 2011: Internal tides around the Hawaiian Ridge estimated from multisatellite altimetry. *J. Geophys. Res.*, **116**, C12039, doi:[10.1029/2011JC007045](https://doi.org/10.1029/2011JC007045).