

Evaluation of Low-Cost, Automated Lake Ice Thickness Measurements

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ABSTRACT

Climate change is expected to decrease ice coverage and thickness globally while increasing the variability of ice coverage and thickness on midlatitude lakes. Ice thickness affects physical, biological, and chemical processes as well as safety conditions for scientists and the general public. Measurements of ice thickness that are both temporally frequent and spatially extensive remain a technical challenge. Here new observational methods using repurposed soil moisture sensors that facilitate high spatial–temporal sampling of ice thickness are field tested on Lake Mendota in Wisconsin during the winter 2015/16 season. Spatial variability in ice thickness was high, with differences of 10 cm of ice column thickness over 1.05 km of horizontal distance. When observational data were compared with manual measurements and model output from both the Freshwater Lake (FLake) model and General Lake Model (GLM), ice thickness from sensors matches manual measurements, whereas GLM and FLake results showed a thinner and thicker ice layer, respectively. The FLake-modeled ice column temperature effectively remained at 0°C, not matching observations. We also show that daily ice dynamics follows the expected linear function of ice thickness growth/melt, improving confidence in sensor accuracy under field conditions. We have demonstrated a new method that allows low-cost and high-frequency measurements of ice thickness, which will be needed both to advance winter limnology and to improve on-ice safety.

1. Introduction

Across the Northern Hemisphere, lakes have been experiencing a shortening of ice duration as well as increased interannual variability of ice coverage (Magnuson et al. 2000). Reduction of ice coverage and thickness are markers of ongoing climate change that is being observed in lakes (Adrian et al. 2009). An overall trend of increasing water temperature has been observed at these lakes (O'Reilly et al. 2015; Schmid et al. 2014;

Schneider and Hook 2010) and decreasing average surface shortwave albedo, which in concert with shorter ice duration acts to reinforce climate warming (Kirillin et al. 2012). A reduction in ice coverage and thickness will also decrease aquatic productivity during the growing season (O'Reilly et al. 2015) due to carry-over effects (Hampton et al. 2017). Chemically, shorter winters can alter the pH balance of lakes via air temperature–pH coupling (Koinig et al. 1998) and increase nutrient inputs (Jeppesen et al. 2009). Our ability to predict ecosystem function is limited by gaps in our understanding of under-ice ecological and physical conditions (Hampton et al. 2017). Further, for

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winter use of lakes by people, this reduction in ice coverage and thickness translates to an increased public safety risk for on-ice recreational activities because the majority of deaths are attributed to thin lake ice (Barss 2006). Better ice thickness measurements would allow advancements of modeling of lake ice phenology, including ice-on and ice-off dates, and forecasting of safety conditions to the general public.

At present, high-frequency observations of ice presence and thickness are difficult or costly. Current options to measure lake ice thickness include using subice column radiation (Bolsenga 1978), heated wires (Ramseier and Weaver 1975), sonar data (Brown and Duguay 2011; Melling et al. 1995), above-ice X- or Ku-band radar (Dugan et al. 2016; Gunn et al. 2015), or ice modeling that is based on satellite remote sensing optical data (Liston and Hall 1995; Wang et al. 2010). All of these methods require additional information or assumptions of local ice conditions, either ice temperature or presence/absence of ice strata layers, as layers of liquid water within the icepack is a source of error in many methods. Manual measurements of ice from drilled holes suffers from limited spatial and temporal sampling while they can also be potentially dangerous (Sleator 1995).

Our recent study presents a method of measuring in situ ice depth for a drastically reduced cost. The method of Whitaker et al. (2016) uses repurposed soil moisture sensors that have an approximate cost of \$600 each to quantify the phase change of water. These sensors are able to provide a novel method for high-frequency observations, similar to higher-cost in situ buoys (Polashenski et al. 2011), or subsurface sonar sensors that can cost \$18,000 each. By being cost effective, these sensors allow multiple spatial measurements and provides much needed information on physical conditions under ice (Hampton et al. 2017). These ice condition observations are needed by the community to ultimately improve predictions of ice phenology.

The review of Kirillin et al. (2012) notes that observational data of winter heat transport in a one-dimensional air–ice–water column are lacking. While ice-on and ice-off dates might match observations, with the lack of heat conduction observations, evaluation of winter processes within models is limited and ice thickness is largely unconstrained within models (Leppäranta and Wang 2008). Summer studies of the air–water column are filling knowledge gaps about energy exchange (Desai et al. 2009), and year-round and long-term lake–atmosphere energy flux datasets are becoming more common (Nordbo et al. 2011; Reed et al. 2018; Rouse et al. 2008). Without accompanying measurements of ice, winter processes are difficult to

access from only atmospheric observations. Given the lack of high-spatiotemporal-resolution measurements of lake ice thermodynamic processes, we evaluate measurements of lake ice thickness on the basis of the methods outlined in Whitaker et al. (2016) at Lake Mendota adjacent to Madison, Wisconsin, and compare these with two commonly used lake ice models and manual measurements. The main objectives of this study are 1) to evaluate the effectiveness of Whitaker et al. (2016) soil moisture sensor methods under field conditions, 2) to assess ice thickness measurements relative to manual and lake-modeling estimates, and 3) to quantify thermodynamic processes observable by the sensors within the ice column.

2. Methods

a. Ice thickness sensors and environmental observations

Ice thickness was measured using sensors that are based on the design of Whitaker et al. (2016), which used soil moisture sensors to measure the liquid-to-solid phase change of water. Following the protocol outlined in Whitaker et al. (2016), six sensor arrays were built. Four arrays used Decagon Devices, Inc. (now METER Group, Inc.), model Em50 dataloggers, each with five soil moisture and temperature sensors (Decagon Devices model 5TM). The individual sensor length was 10 cm, but the sensors had a larger measuring distance of 11.75 cm because of measurement of dielectric permittivity in three dimensions, below the end of the sensor, and thus were spaced on a wooden support according to the 11.75-cm measurement distance, as seen in Fig. 1a. This gave an individual sensor array of five 5TM sensors, with a total ice thickness measurement length of up to 58.75 cm. Two other sensor arrays were built using Onset Computer Corp. “HOBO” model H21-002 microstation loggers with four soil moisture probes each (Decagon Devices model 10HS probes). The 14.5-cm-long sensors had an individual measurement distance of 19.5 cm, and the total array measurement length is 78 cm of ice thickness. All of the arrays were deployed to be approximately flush with the top of the ice sheet.

Both the 5TM and the 10HS sensors report volumetric water content (VWC). Converting this measurement to ice depth required two calibration coefficients: sensor-specific sensitivity and an intercept for maximum depth that was dependent on the length of the sensor. When resting in open water the probes had a sensor-specific VWC reading a , and when the probes were fully covered in ice the sensor response is 0.

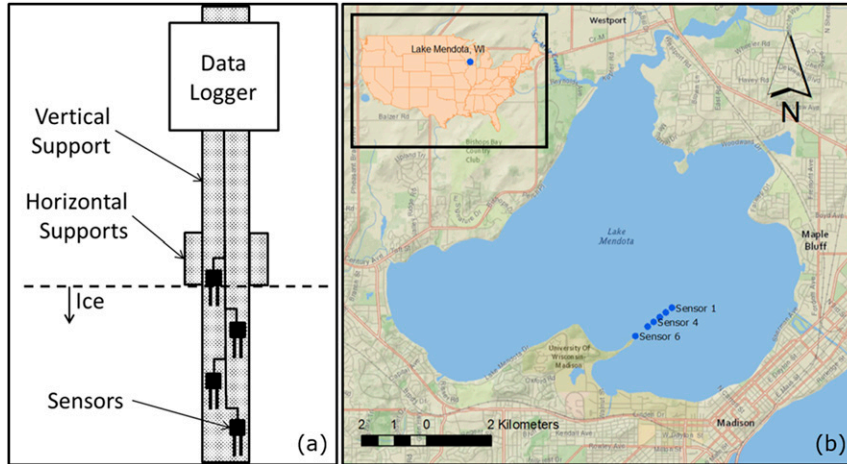


FIG. 1. (a) Schematic figure of one ice thickness sensor array. (b) Map of Lake Mendota, showing the location of the sensor arrays and the location of Lake Mendota (inset). The micrometeorological station was collocated with sensor 1.

Thus, following Whitaker et al. (2016), ice length is calculated using the following piecewise equation, where l_s is the model-specific sensor length (either 10 or 14.5 cm) and x is the water content measurement of sensor n within the array:

$$\text{Ice Length at Position } n(x) = \begin{cases} 0 & \text{if } x \geq l_s/a \\ -\left(\frac{l_s}{a}\right)x + l_s & \text{if } 0 < x < l_s/a \\ l & \text{if } x \leq 0 \end{cases} \quad (1)$$

To find ice depth, the individual ice length measurements were summed along each array per sensor per array. Here, l_m is the model-specific measurement length (11.75 and 19.5 cm for 5TM and 10HS sensors, respectively):

$$\text{Ice Depth}(x) = \begin{cases} \text{Ice Length}(1) & \text{if } x \leq l_m \\ \text{Ice Length}(2) + l_m & \text{if } l_m < x < 2l_m \\ \vdots & \\ \text{Ice Length}(n) + nl_m & \text{if } (n-1)l_m < x < nl_m \end{cases} \quad (2)$$

The sensor arrays, as shown in Fig. 1a, were deployed in early February of 2016 for 15 days, along a 1.05-km transect starting from a micrometeorological station placed in the deepest part of Lake Mendota (43.0995°N, 89.4045°W, 25-m depth) and ending at the lake shore (Fig. 1b). During the deployment, manual

measurements of ice thickness were made from drilled holes at each sensor location. Bulk ice temperature was collected using an average of each of the 5TM sensors that were fully encased in ice, since each sensor had surface-mounted thermistors. Snow cover on the ice surface was rare during the study period. The micrometeorological station collected four-component radiation data using a Kipp and Zonen B.V. model CNR4 net radiometer, and air temperature, relative humidity, wind speed, and surface latent and sensible heat fluxes were collected using a Campbell Scientific, Inc., model “IRGASON” at 2.5-m height above the ice surface. Environmental data were collected using a Campbell Scientific CR6 datalogger. Both the micrometeorological station and the sensor arrays observations were averaged every 30 min. For statistical analysis, daily means and range were used.

b. Winter lake modeling

Both the Freshwater Lake (FLake) model and the General Lake Model (GLM) were parameterized for Lake Mendota, at the model-identified specifications. To initialize the FLake model, version 2.0 (Kirillin et al. 2011; Martynov et al. 2010), solar radiation, air temperature, air humidity, wind speed, and cloudiness were entered into the model. Radiation, air temperature, humidity, and wind speeds were measured using the micrometeorological station, while incoming shortwave radiation data were used to model cloudiness on the basis of fraction of potential incoming shortwave radiation, calculated from latitude and longitude as well as day of year (Swift 1976). FLake model parameters were lake depth of 25 m, typical

wind fetch distance across the lake of 8 km, and thermal boundary conditions of 5°C at a sediment depth of 1 m (Snorheim et al. 2017).

The GLM, version 2.2 (Hipsey et al. 2014), was initialized and modeled using short- and longwave radiation, air temperature and relative humidity, wind speed, and the amount of precipitation. Model parameters such as size and average wind fetch distance, lake depth, inflows and outflows, latitude and longitude, and elevation were also included in the model (Snorheim et al. 2017) as static input values. Daily precipitation data were taken from local observations (Reed et al. 2018). Both models were run on 30-min time steps, and then average daily ice depths were calculated.

3. Results

Over the 15-day observational period, VWC was recorded by the sensors. Utilizing Eqs. (1) and (2), VWC (shown in Fig. 2a) was converted to ice length (measurement length) per sensor and the total ice length per sensor array as shown in Fig. 2b. By summing the total ice length at each sensor, an observation of total ice depth at the sensor array is formed (Fig. 2c). The temporal record of ice depth during the initial refreeze period of 7 days after sensors were deployed from all six observation locations is shown in Fig. 2d, which in turn can provide a full ice depth record of the transect.

After ice refroze around the sensors, the equilibrium value of ice depth at each location was recorded and was compared with manual measurements (Table 1). Average manually measured ice depth was 27.8 cm; 7 days later, sensors recorded an average ice depth of 32.8 cm and ice depths were statistically similar to each other between the two observation methods (paired t test; $p = 0.02$).

Over the experimental period, daily ice thickness measurement values were lower than FLake model output and higher than GLM output but were within range of the average of all model output (t test; $p < 0.01$) as well as the average of all manual measurements (t test; $p < 0.001$), as shown in Fig. 3a. Measurements of bulk ice temperature were significantly colder than FLake-modeled ice temperature (t test; $p > 0.05$), as shown in Fig. 3b. The GLM does not output ice column temperature, and therefore statistical comparisons were not done for this model. Since only the four arrays using 5TM soil moisture probes measured ice column temperature, not all measurement locations had bulk ice temperature observations.

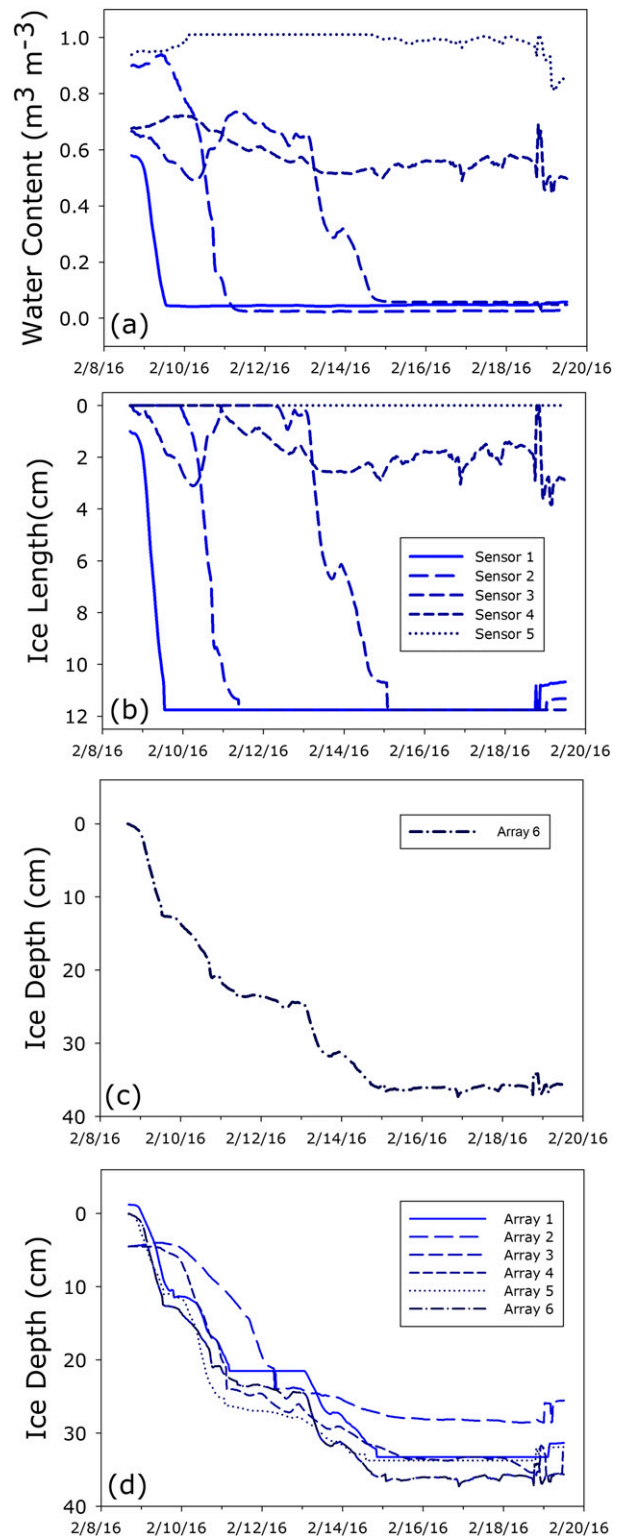


FIG. 2. (a) Water content reading from a single measurement location consisting of an array of five soil moisture sensors, and (b) the same five sensors converted to local ice length. Also shown are (c) the calculated ice depth of the selected measurement array and (d) the calculated ice depths from all six measurement locations.

TABLE 1. Manual ice thickness measurements compared with equilibrium ice thickness sensor values over the 1.05-km sampling transect.

Obs no.	Sensor ice thickness obs (cm)	Manual ice thickness obs (cm)	Distance from micrometeorological station (km)
1	33.25	27.9	0.0
2	28.5	27.3	0.175
3	36.0	27.9	0.35
4	36.0	27.9	0.525
5	33.75	27.3	0.7
6	29.25	26.7	1.05
Avg ice thickness	32.8	27.5	

Using data collected from the 5TM sensors, the diurnal range of ice column temperatures was compared with the daily mean ice temperature. Here we note a positive correlation ($R^2 = 0.60$, slope = 0.44, 95% confidence of slope from 0.15 to 0.72, F value = 0.008, and $n = 10$ days) in which days with colder mean ice temperatures showed larger daily fluctuations in ice temperatures as illustrated in Fig. 4a. In Fig. 4b the daily

mean ice thickness (cm) is correlated with the daily change in ice thickness (cm) ($R^2 = 0.72$, slope = -0.27 , 95% confidence of slope from -0.41 to -0.13 , F value = 0.002, and $n = 9$ days). As ice thickness increases, we show a corresponding decrease in the rate of ice growth, as the ice layer has slower rates of ice growth or melt.

4. Discussion

a. Improving ice observations

High spatial and temporal measurements of ice thickness is a challenge. Remote sensing of ice is an option (Bolsenga 1978; Brown and Duguay 2011; Gunn et al. 2015; Wang et al. 2010), but limitations and costs can reduce the effectiveness of such methods. Manual methods are limited in both space and time as well as having an increased human safety issue (Sleator 1995). In situ sensors show promise in expanding the observational capacities of winter lake ice work in the future. Using the methodology we demonstrated in Whitaker et al. (2016), sensors were successfully deployed for a 15-day period with six observations locations along a

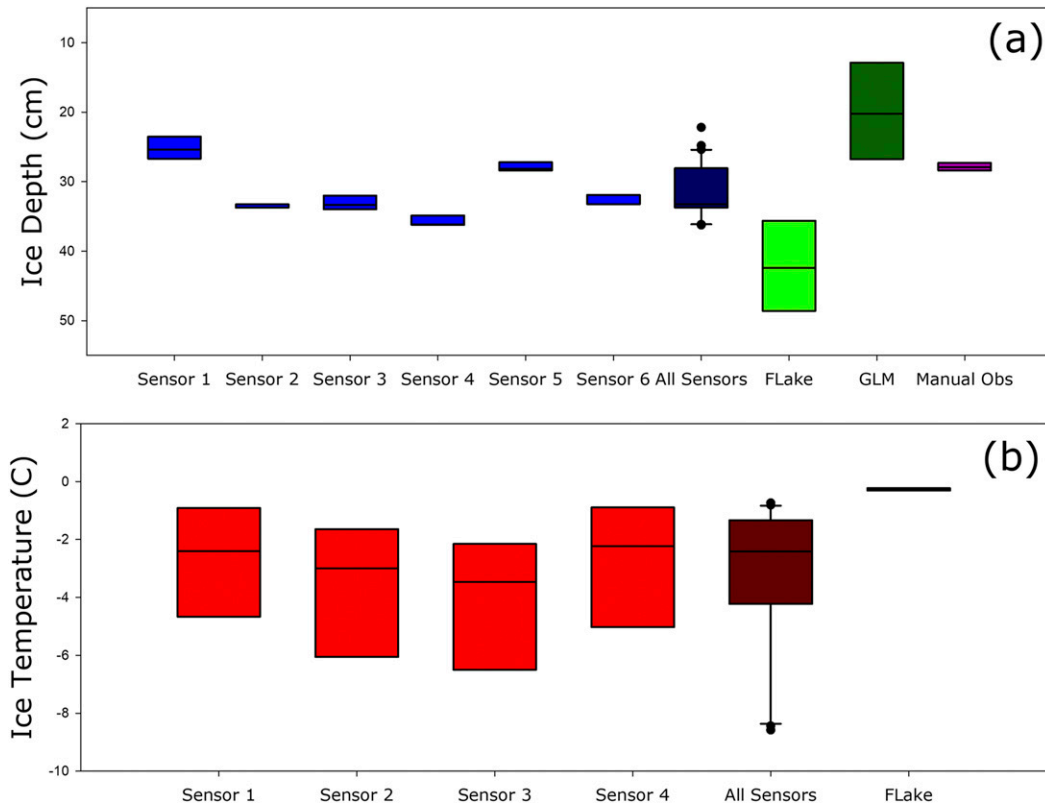


FIG. 3. (a) Comparison of daily ice thickness measurements from all six individual ice thickness arrays, all of the ice thickness sensor arrays grouped together, model output from FLake, model output from GLM, and the manual measurements. (b) Daily average ice column temperature from four arrays, all sensor arrays grouped together, and the FLake-modeled temperature at daily time scales.

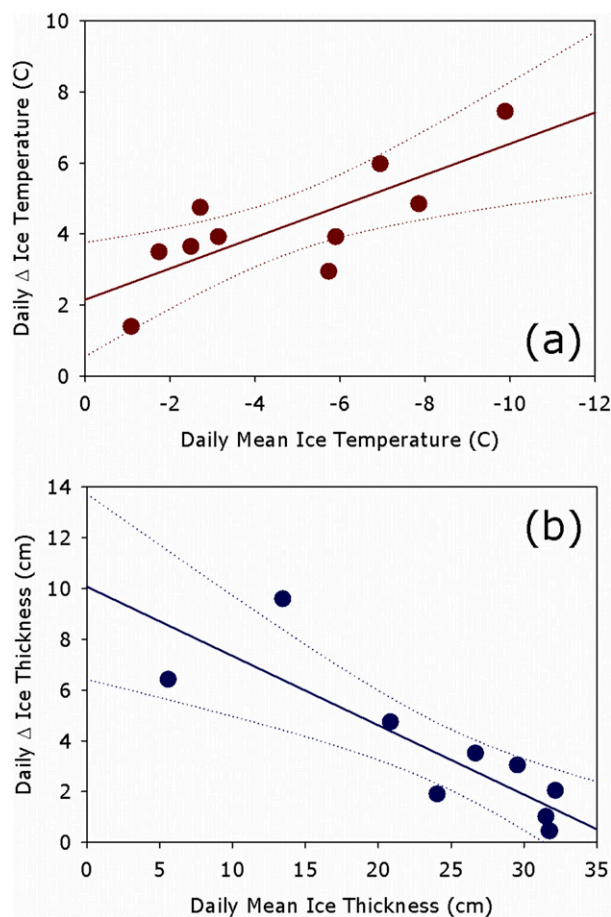


FIG. 4. (a) Amount of daily change in ice temperature observations as a function of daily mean ice temperature observations. (b) Amount of daily change in ice thickness observations as a function of daily mean ice thickness observations.

1.05-km transect, making one of the highest-spatiotemporal-resolution ice datasets. With predeployment calibration only needing two values, one depending on sensor length with no differences between sensors of the same model and a sensor-specific value that can be easily determined, applying these methods into a larger-scale study is not prohibited by labor costs.

Further improvements in sensor design would continue to increase the cost effectiveness of in situ sensors. At temperatures approaching -20°C , alkaline battery current is reduced, and therefore increasing the battery capacities, using lithium batteries, or reducing the datalogger power draw would enable continuous measurement. Sensors that could be deployed before the start of lake freeze either from attaching to a solid structure (jetty, bridge support, or dock) or a float would allow ice observations for the entire winter seasons including conditions where human safety issues would not allow manual measurements. A sensor design that

further reduces heat conduction between the atmosphere and the ice layer would improve accuracy of measurements. Wood was used as the structure of the arrays because of its low thermal conductivity and lower-than-water density so that the sensors would float in open water; however, the sensors and wires were black, and the low albedo of the wires plus the high thermal conductivity of the wires created the possibility of energy being moved through the ice layer because of the sensors. Attempting to match ice albedo and thermal conduction would mitigate this issue in the future.

b. Measurement intercomparison

This new measurements method of high-resolution lake ice thickness observations showed good agreement with manual measurements; however, the in situ sensors and model output from both the FLake and GLM agree only when model results are averaged between both models. Differences between models are most likely due to differences in radiation inputs, net radiation, and atmospheric cloudiness for GLM and incoming short- and longwave radiation for FLake. Albedo is modeled internally in FLake, possibly the cause of the overestimation of lake ice thickness if albedo and hence solar energy input is too low. In the GLM, the atmospheric cloudiness parameter is a potential source of error, for similar reasons of high atmospheric energy inputs from a warmer, cloud-covered atmosphere. Lake ice melt and growth are primarily dependent on atmospheric conditions (Leppäranta and Wang 2008), and the conclusions of Kirillin et al. (2012) state that atmosphere–ice–water column measurements should be the focus of observational datasets going forward, with a focus on the atmosphere–lake interface.

Bulk ice temperatures and indirectly observed heat conduction is in line with expectations. FLake-modeled ice temperature output, which is near-zero degrees, and without the ability to model a vertical temperature gradient, removal of energy from the top of ice column into the lower atmosphere via longwave emissions translates directly into new lake ice formation. However, we see in the observations that the energy removal process is a function of ice thickness itself; thick ice grows and melts slower than thin ice. When ice is thicker, the daily growth/melt rate of ice is smaller, because there is a larger reservoir of thermodynamic energy present in the ice. The process of heat conduction through the ice column and the storage of energy within the ice column likely explains why the FLake model is overestimating ice thickness during the study period. Heat conduction, inferred from changes in daily average temperature, showed larger diurnal temperature swings during cold days, as expected.

Ice growth/melt was also a strong function of ice thickness.

5. Conclusions

Climate change is predicted to decrease the temporal duration and increase the variability of ice coverage and thickness for midlatitude lakes. With these changes comes increased uncertainty with physical, chemical, and biological lake processes both during the winter and carrying over to ice-free time periods. An improved measurement dataset of lake ice thickness allows for data–model comparison and model development focused on winter processes. We show an area where bulk ice column temperature observations and models diverge, when FLake-modeled ice column temperature effectively remained at 0°C as compared with the sensor's observations of realistic negative temperatures. However, modeled lake ice thickness, averaged between two models, is not drastically divergent to observations.

Besides valuable advancements to scientific understanding made possible by these results, the new methods show a potential for low-cost and repeated lake ice thickness measurements. Given the low cost of these sensors and our initial findings of limited variations in ice thickness across one transect, a complete study of lake ice thickness is now relatively efficient and cost effective. Additional work is needed to determine optimal spacing and placement and may be dependent on a number of factors unique to any lake (e.g., bathymetry, shoreline shading by vegetation, variations of groundwater or stream water inputs, and sediment heat fluxes). With projected increased variability of ice coverage and thickness, the dangers of wintertime lake scientific and recreational activities increase as well. Increased observational capacities of lake ice would lead toward improvements in operational ice thickness forecasting, something currently not attempted by government agencies due to inherent current technical challenges. With further advancements, human lives can be saved by ice thickness forecasts and public warning of dangerous safety conditions.

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