Estimating Four-Dimensional Internal Wave Spectrum in the Northern South China Sea

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ABSTRACT

Internal waves can transfer energy from large-scale to microscale processes; however, the spectra of these waves remain poorly known. A method that combines modal harmonic decomposition and maximum-likelihood method is proposed in this study to estimate four-dimensional internal wave spectrum using limited mooring observations. Using this method, a four-dimensional internal wave spectrum was obtained for the first time based on the mooring measurements collected during the South China Sea (SCS) Internal Wave Experiment in July 2014. The spectrum was then validated by comparing with the spectrum based on Fourier analysis and with the modified Garrett–Munk internal wave spectrum, respectively. The power of the internal wave spectrum decreased obviously with increasing frequency and wavenumber, with a falloff rate of ω^{-2} beyond tidal frequencies, and with falloff rates of k_h^{-2} and $k_z^{-2.5}$ for horizontal and vertical wavenumbers, respectively. In addition, at a fixed frequency and vertical wavenumber, the propagation direction and phase speed of internal waves can be obtained through the four-dimensional spectrum. In summary, we verified the feasibility of estimating four-dimensional internal wave spectrum using limited mooring observations in this study, and the method we proposed should be applicable to other regions where such mooring observations are available.

1. Introduction

Internal wave spectrum reflects energy distribution of internal wave field as a function of wave frequency

and wavenumber. Among processes with different scales, the wavenumber range of internal waves lies between large scale and turbulent scale, which serves as a continuum for energy transfer and eventually furnishes turbulent mixing in the ocean interior (Henye et al. 1986). Therefore, the parameterization of small-scale mixing is highly dependent on the performance of internal wave spectrum in different regions of the ocean (Polzin et al. 1995, 2014).

There are several “universal” internal wave spectral models. The first one is the Garrett–Munk (GM) spectrum

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based on the assumption that internal wave field is horizontally isotropic and vertically symmetric. The GM model has evolved over time, and now it has several incarnations. GM72 spectrum was initially introduced by [Garrett and Munk \(1972\)](#) and is considered as a milestone on studies of internal wave spectrum. However, it was built on the wintertime measurements at site “D” on the continental rise north of the Gulf Stream. GM72 spectrum cuts off sharply beyond some specified wavenumber as a top-hat model, which ignores much of small scales. GM75 spectrum ([Garrett and Munk 1975](#)) replaces the top hat used in GM72 spectrum with a tapered cutoff to conform to the even higher wavenumber data from [Millard \(1972\)](#). GM75 spectrum is much closer to the real ocean spectrum than GM72 spectrum, as suggested by [Garrett and Munk \(1975\)](#). [Cairns and Williams \(1976\)](#) provided a modification of GM75 spectrum, which is referred to as GM76 spectrum; it has a more rapid roll-off from the low-wavenumber plateau region to the high-wavenumber asymptote than GM75 spectrum. For simplicity, GM79 spectrum ([Munk 1981](#)) was introduced by replacing the wavenumber used in GM76 with vertical mode number. These GM spectra have certain universality for internal wave field in the world oceans, particularly for deep open oceans, and one of their important roles is to be used in mixing parameterization (e.g., [Henyey et al. 1986](#); [Polzin et al. 1995](#)).

With the existence of these GM spectra, one may ask how well these internal wave spectral models perform in the marginal seas. [Levine \(2002\)](#) presented a modification of GM79 spectrum, which mainly replaces the e -folding scale of buoyancy frequency with the Wentzel–Kramers–Brillouin (WKB) scaled thickness of the vertical waveguide; it compared well with the observations from the continental shelf. Another representative internal wave spectral model is the Internal Wave Experiment (IWEX) spectrum ([Müller et al. 1978](#)) based on a 42-day internal wave experiment in the Sargasso Sea, which considered contamination of internal waves by fine structures of temperature and currents and by noise in current measurements. Although this model improved adaptability, it is too complicated to use in practice when compared with the GM spectra, since it requires 12 parameters, which include anisotropic parameters, at each frequency. The background fields vary significantly from one marginal sea to another, and hence the parameters employed in internal wave spectrum should be adjusted accordingly.

The South China Sea (SCS) is the largest marginal sea of the Pacific, where performance of internal wave spectrum has also been investigated. There are complex topography and dynamic processes with different scales in the SCS, such as large-scale currents (e.g., [Wang et al. 2014, 2016](#)), mesoscale eddies (e.g., [Chen et al. 2011](#);

[Wang et al. 2012](#)), and so on, which may influence internal wave field and result in a unique internal wave spectrum. Although it is much easier to obtain internal wave frequency spectrum than wavenumber spectrum, studies on internal wave frequency spectrum in the SCS are still rare because of the requirement for high-temporal-resolution sampling. In the northern SCS, internal waves have geographical features. The ratio of total internal wave energy to the GM spectral energy ranges from 2 to 13 for different topographic regions, including the region west of the Luzon Strait, near the Dongsha Island, at the shelf break, and on the continental shelf ([Lien et al. 2005](#)). Both [Shang et al. \(2009\)](#) and [Xie et al. \(2010\)](#) pointed out that the internal wave frequency spectrum in the northern SCS is affected by the level of nonlinear interactions among internal waves.

Most studies on internal wave wavenumber spectrum in the northern SCS are based on images of seismic reflection and synthetic aperture radar (SAR). Using seismic measurements with a horizontal resolution of 6.5 m, [Dong et al. \(2009\)](#) presented a 1D horizontal wavenumber spectrum in the northeastern SCS, which was generally consistent with GM76 spectrum. Using SAR images, [Liu et al. \(2005\)](#) provided 2D horizontal wavenumber spectra and examined wave propagation, wavelength, and amplitude. Images of seismic reflection and SAR provide an alternative technique to consider horizontal wavenumber spectrum of internal wave simply; however, the information in frequency and vertical wavenumber domains are missing.

To an extent, these studies have undoubtedly improved our understanding of internal wave spectrum in global ocean, and specifically some marginal seas. However, they mostly provided 1D or 2D spectrum, such as 1D frequency spectrum, 1D horizontal wavenumber spectrum, or 2D horizontal wavenumber spectrum, and failed to provide a 4D spectrum and hence a detailed picture of internal wave field. In this paper, we provide a method to estimate 4D internal wave spectrum. Using this method, a 4D internal wave spectrum was obtained for the first time based on the mooring measurements collected during the SCS Internal Wave Experiment in July 2014. The paper is organized as follows. The field experiment and the method used to derive the 4D internal wave spectrum are given in [section 2](#). Validation and analysis of the 4D internal wave spectrum are presented in [section 3](#), followed by a summary in [section 4](#).

2. Data and method

a. SCS Internal Wave Experiment

To investigate the internal wave feature in the northern SCS, the SCS Internal Wave Experiment was conducted

FIG. 1. Locations of the four moorings, superimposed with bathymetry of the northern SCS from ETOPO1 (<https://www.ngdc.noaa.gov/mgg/global/global.html>).

near Dongsha Island during 24–28 July 2014. Four bottom-anchored moorings were deployed around (21°N, 118°E), with three of them (A1, A2, and A4) forming a triangle and the remaining one (A3) at the center of the triangle (Fig. 1). The horizontal spacing between two adjacent

moorings ranged from 2.9 to 8.2 km. Water depths of moorings A1–A4 were 984, 943, 999, and 959 m, respectively. Each mooring was equipped with a full-depth thermistor chain consisting of six CTDs and 53–54 thermometers mounted at different depths (Table 1).

TABLE 1. Detailed configuration of the moorings deployed in the northern SCS. The thermistor chain consists of six Sea-Bird 37SM CTDs and 53–54 thermometers. The double asterisks (**) mean that the CTDs only provide point measurements. The thermometers were mounted between two adjacent CTDs with a depth interval of 10 or 20 m.

Moorings	Latitude, longitude	Water depth (m)	Instrument	Instrument depth (m)	Depth interval (m)	Sampling interval (s)	Sampling period
A1	21°06.682'N, 117°48.182'E	984	CTDs	50, 100, 200, 300, 520, 960	**	6	1000 LST 24 Jul to
			Thermistor chain	50–300	10	6	0500 LST 28 Jul 2014
				300–960	20		
A2	21°03.026'N, 117°48.814'E	943	CTDs	2, 102, 200, 400, 672, 900	**	6	1116 LST 24 Jul to
			Thermistor chain	2–200	10	6	0759 LST 28 Jul 2014
				200–900	20		
A3	21°04.672'N, 117°50.578'E	999	CTDs	40, 100, 190, 300, 500, 944	**	6	1400 LST 24 Jul to
			Thermistor chain	40–300	10	6	0900 LST 28 Jul 2014
				300–944	20		
A4	21°04.862'N, 117°52.517'E	959	CTDs	2, 55, 155, 255, 465, 905	**	6	1417 LST 24 Jul to
			Thermistor chain	2–255	10	6	1111 LST 28 Jul 2014
				255–905	20

FIG. 2. (a)–(d) Temperature information at the four moorings. The black, dashed vertical lines indicate the moments of 0534, 0517, 0504, and 0448 LST 27 Jul 2014, respectively.

The depth interval was 10 or 20 m, and the sampling interval was 6 s, which was adequate to capture high-frequency internal waves, since the maximum buoyancy frequency around the thermocline was $2 \times 10^{-2} \text{ s}^{-1}$. More information on these moorings is given in Table 1.

The temperature information at the four moorings is shown in Fig. 2, from which some internal solitary waves can be identified clearly in the upper layer. The A waves (consisting of rank-ordered packets) and B waves (usually having a single large wave) both arrived diurnally, which was attributable to variability in the barotropic tidal currents in the Luzon Strait (e.g., Alford et al. 2010; Ramp et al. 2010; Huang et al. 2014). Although the

soliton structure in time is likely to appear in frequency spectra as a series of harmonics extending out to high frequencies, the discrete arrivals of the coherent solitons are not well captured by the spectra (because they throw out phase information). Thus, the spectral results to be discussed below did not include the internal solitary waves. Generally, internal solitary waves can be effectively analyzed by tracking the arrival times of specific peaks at different moorings. For example, on 27 July 2014, an internal solitary wave was observed in sequence by mooring A4 at 0448 LST, by mooring A3 at 0504 LST, and by mooring A1 at 0534 LST, which implies a propagation path from A4 to A3 to A1, namely, from

FIG. 3. Density variation (units: kg m^{-3}) relative to the time-mean state derived from the six CTDs [(from top to bottom) numbered as CTD1 to CTD6] at mooring A1. The black curves are density variation based on the density calculated from temperature and salinity using the *seawater* toolbox. The cyan and orange curves are density variation induced by both temperature and salinity ($-\rho_0\alpha\Delta T + \rho_0\beta\Delta S$) and density variation induced only by temperature ($-\rho_0\alpha\Delta T$), respectively, based on a simple linear equation of state, with their RMS errors with respect to the black curves being labeled. Here, ρ_0 is time-mean density; α and β are thermal expansion coefficient and saline contraction coefficient, respectively; ΔT and ΔS are temperature variation and salinity variation relative to the time-mean state, respectively.

southeast to northwest. This result agrees with previous reports (e.g., Liu et al. 2004; Chang et al. 2006; Zhao and Alford 2006; Li et al. 2008; Huang et al. 2017).

Because of the lack of salinity measurements, the only feasible way is to use isothermal displacement as a proxy for isopycnal displacement in subsequent analysis of internal wave spectrum. This is reliable because the temperature-induced density variation ($-\rho_0\alpha\Delta T$), which was calculated based on a linear equation of state, accounted for most of the whole density variation caused by both temperature and salinity, only with a small root-mean-square (RMS) error of 0.01–0.18 kg m^{-3} (Fig. 3).

The isothermal displacement profiles $\xi(z, t)$ were derived from $\xi(z, t) = z(T, t) - \langle z(T) \rangle_t$ following Fjeldstad (1933) and Krauss (1966), where $z(T, t)$ is the depth of isothermal at different moments, and $\langle z(T) \rangle_t$ is the time mean of $z(T, t)$ for each mooring. To remove the influence of variable stratification on the vertical structure of internal waves, the isothermal displacement was WKB-scaled $\hat{\xi} = \xi \sqrt{N(z)/\bar{N}}$ and the depth coordinate was WKB-stretched $\hat{z} = \int_z^0 [N(z')/\bar{N}] dz'$ (e.g., Leaman and Sanford

1975; Althaus et al. 2003). Here, the caret denotes the WKB-scaled value. Because of the sparse salinity measurements, the buoyancy frequency profile $N(z)$ is a monthly-mean estimation in July 2014 based on the *World Ocean Atlas* (WOA) dataset (<https://www.nodc.noaa.gov/OC5/woa13/woa13data.html>), and $\bar{N}$ is the depth-averaged value of $N(z)$. This is reliable since the atlas-based buoyancy profile matches well with the CTD-derived buoyancy frequency data (Fig. 7a). The WKB scaling aims to convert the ocean to one of constant stratification in which vertical standing modes are sinusoids. Then each WKB-scaled isothermal displacement profile was linearly interpolated onto uniform 5-m vertical interval in the WKB-stretched depth coordinate. The isothermal displacements at the four moorings show a dominant diurnal cycle below 700 m and a dominant semidiurnal cycle above 700 m with their maximum amplitudes both exceeding 100 m (Fig. 4). Hereinafter, only the synchronous measurements from the four moorings were used to estimate 4D internal wave spectrum according to the requirement of method described next.

FIG. 4. WKB-scaled isothermal displacements at the four moorings varying with time and WKB-stretched depth.

b. Method

Because of the limited sampling locations in the horizontal (only four moorings), a method for estimating frequency–wavenumber spectrum called the maximum-likelihood method (MLM; [Capon 1969](#)) was used to estimate 4D internal wave spectrum, rather than the conventional spectrum estimation method based on direct Fourier transform. However, there will be a spatial aliasing if the original measurements are directly used in the MLM, because there are also signals with wavelength smaller than the array spacing. To solve this problem, the isothermal displacement was preprocessed by modal harmonic decomposition ([Buijsman et al. 2010](#)),

which separated different frequency/vertical mode components from the total field.

1) MODAL HARMONIC DECOMPOSITION

The modal harmonic decomposition consists of three steps. The first step is harmonic decomposition. WKB-scaled isothermal displacement during the whole period is fitted to $\hat{\xi}(\mathbf{x}, \hat{z}, t) = \sum_m A_m(\mathbf{x}, \hat{z}) \cos(\omega_m t) + B_m(\mathbf{x}, \hat{z}) \sin(\omega_m t)$ using Fourier transform. Here, $(\mathbf{x}, \hat{z})$ means the horizontal location and WKB-stretched depth of the isothermal displacement. The ω_m is the frequency set deriving from the Fourier transform. The corresponding amplitude and phase of isothermal displacement at

FIG. 5. Base-10 logarithm of amplitudes of WKB-scaled isothermal displacement with different frequencies after harmonic decomposition at the four moorings. The left-to-right vertical lines are Coriolis frequency f , tidal frequencies (including diurnal frequency D_1 , semidiurnal frequency D_2 , and their harmonics D_3 and D_4), and the depth-mean buoyancy frequency N .

frequency ω_m are $\sqrt{A_m^2 + B_m^2}$ and $\arctan(B_m/A_m)$, respectively. Figure 5 shows the amplitudes $\sqrt{A_m^2 + B_m^2}$, which can easily distinguish the near-inertial waves, diurnal, semidiurnal internal tides and their harmonics. At the frequencies lower than 10 cpd, amplitudes were larger than $O(1)$ m; especially, large amplitudes of $O(10)$ m were mainly found at diurnal frequency below 700 m and at semidiurnal frequency within

nearly the whole water column. Amplitudes at frequencies between 10 and ~ 80 cpd were $O(0.1)$ m, and they were even smaller for frequencies higher than 100 cpd.

The second step is modal decomposition. The internal wave has the character of a standing wave in the vertical, while it propagates in the horizontal, since the ocean's boundaries in the vertical, bottom

FIG. 6. (a) The frequency–vertical wavenumber (ω – k_z) spectra of WKB-scaled vertical displacement at mooring A3. Positive k_z corresponds to upward-propagating phase, and negative k_z corresponds to downward-propagating phase. (b) The percentages of components with upward- and downward-propagating phase at each frequency.

and surface naturally confine the propagation of internal waves (Gerkema and Zimmerman 2008). This property is exploited in the method of vertical modal decomposition. To find out whether this method is suitable for our observation data, the frequency–vertical wavenumber spectrum of WKB-scaled vertical displacement was presented (Fig. 6), which indicated that although the component with downward-propagating phase is slightly larger than that with upward-propagating phase, especially for high-frequency waves, their differences are still small, not exceeding 0.08%. That is, the downward-propagating energy and upward-propagating energy are almost equal; namely, the assumption of vertically symmetric internal wave field is valid in our observation area and the modal decomposition method can be applied to our data. A similar conclusion can also be drawn from the four-quadrant frequency–vertical wavenumber spectrum of WKB-scaled horizontal baroclinic velocity (Fig. S1 in the online supplemental material).

Then the vertical modes $\psi(z)$ can be obtained by solving the hydrostatic Sturm–Liouville equation (Gill 1982):

$$\psi_n''(z) + \frac{N^2(z)}{c_n^2} \psi_n(z) = 0, \quad (1)$$

and

$$\psi_n(-H) = \psi_n(0) = 0. \quad (2)$$

Equation (2) assumes that vertical velocities at both sea surface and bottom are zero. The $N(z)$ is the monthly-mean buoyancy frequency profile in July 2014 based on the WOA dataset (Fig. 7a). The subscript n indicates the vertical mode number, corresponding to vertical wavenumber of $k_z^{(n)} = (n\pi/b)(N/N_0)$ (Munk 1981) in the WKB sense, where $b = 1300$ m, $N_0 = 5.2 \times 10^{-3} \text{ rad s}^{-1}$, and $\bar{N}$ is the depth-averaged value of $N(z)$. The phase speed of n th-mode internal waves with a frequency of ω can be determined using $c_p = (\omega/\sqrt{\omega^2 - f^2})c_n$. The $\psi_n(z)$ is the vertical structural function of n th-mode internal waves, which is sinusoid in the WKB-stretched depth coordinate.

In this study, the vertical structural functions of first 50 modes at each mooring were calculated based on its actual water depth, namely, 984, 943, 999, and 959 m

FIG. 7. (a) Buoyancy frequency profile in July 2014 based on the *WOA* dataset (the black curve), with the colored dots indicating CTD-derived data at the four moorings, and the colored triangles indicating actual depths at the four moorings. (b)–(e) Normalized vertical structural functions of the first five modes in a WKB-stretched depth coordinate for moorings A1–A4.

for mooring A1–A4, respectively. Results of the first five modes are shown in a WKB-stretched coordinate (Figs. 7b–e). The resulting vertical wavenumbers of n th-mode internal waves at moorings A1, A2, and A4 only have slight differences (within a factor of 1.01–1.06) from that at mooring A3. To make the horizontal wavenumber calculation (described below) simple, the vertical wavenumbers at the four moorings were uniformly set to the vertical wavenumber obtained at mooring A3.

The third step is modal harmonic decomposition. Amplitudes for the internal wave components within the internal wave frequency band $f < \omega_m < \bar{N}$ are fitted to $A_m(\mathbf{x}, \hat{z}) = \sum_n A_{mn}(\mathbf{x}) \psi_n(\hat{z})$ and $B_m(\mathbf{x}, \hat{z}) = \sum_n B_{mn}(\mathbf{x}) \psi_n(\hat{z})$ using the first 50 modes by least squares method, respectively.

The amplitudes $\sqrt{A_{mn}^2 + B_{mn}^2}$ of isothermal displacement with different frequencies and mode numbers are shown in Fig. 8. The signal was dominantly distributed in tidal frequencies for the first 20 modes. As a result, the isothermal displacement of the n th mode with frequency ω_m at location $(\mathbf{x}, \hat{z})$ can be reconstructed as

$$\begin{aligned} \hat{\zeta}_{mn}(\mathbf{x}, \hat{z}, t) = & \psi_n(\hat{z}) A_{mn}(\mathbf{x}) \cos(\omega_m t) \\ & + \psi_n(\hat{z}) B_{mn}(\mathbf{x}) \sin(\omega_m t). \end{aligned}$$

2) MAXIMUM-LIKELIHOOD METHOD

Given an observation array consisting of M sensors (not in a straight line), the i th sensor is situated at the

FIG. 8. Base-10 logarithm of amplitude of WKB-scaled isothermal displacement with different frequencies and different mode numbers (or vertical wavenumbers) after modal harmonic decomposition at the four moorings. The left-to-right vertical lines are Coriolis frequency f , tidal frequencies (including diurnal frequency D_1 , semidiurnal frequency D_2 , and their harmonics D_3 and D_4), and the depth-mean buoyancy frequency N .

vector location $\mathbf{x}_i$ about an arbitrary origin. Data series of each sensor are divided into L segments, with L larger than the number of sensors M but not too large to ensure enough frequency resolution; we set $L = M + 1$ in this study. The cross-power spectrum between the i th and j th sensors at frequency ω is determined by $G_{ij}(\omega) = 1/L \sum_{l=1}^L S_{il}(\omega) S_{jl}^*(\omega)$, where $S_{il}(\omega)$ is Fourier transform of the data from the i th sensor and the l th segment, and the superscript $*$ denotes complex conjugate. The directional information of the input signal is contained

in this cross spectrum. However, this cross spectrum contains noise, and we must find a way to extract useful signal (directional information) from it.

The high-resolution frequency–wavenumber spectrum is defined as follows, according to Capon (1969) and Tokimatsu et al. (1992):

$$P(\omega, \mathbf{k}) = \sum_{i=1}^M \sum_{j=1}^M A_i^*(\omega, \mathbf{k}) A_j(\omega, \mathbf{k}) G_{ij}(\omega) \exp[i\mathbf{k} \cdot (\mathbf{x}_i - \mathbf{x}_j)], \quad (3)$$

FIG. 9. (a) Horizontal wavenumber spectra of the synthetic signals with $\Delta\theta = 0.25^\circ$ and a noise level of 1%. The black square shows $k_{0x} = 2\pi \times 10^{-5} \text{ rad m}^{-1}$ and $k_{0y} = -2\pi \times 10^{-4} \text{ rad m}^{-1}$, with $\arctan(k_{0y}/k_{0x}) = -84.29^\circ$; the black arrow indicates the spectral peak. (b)–(d) Performance of test cases: (b) peak width, (c) spectral integration along θ , and (d) ratio of the real peak to the spurious peak in the opposite direction with a noise level of 1% and different angular resolutions. (e) Performance of test cases (peak width) with $\Delta\theta = 0.25^\circ$ and different noise levels.

where $\mathbf{k}$ is the 3D wavenumber vector (in rad m^{-1}), and the weights

$$A_i(\omega, \mathbf{k}) = \frac{\sum_{j=1}^M q_{ij}(\omega, \mathbf{k})}{\sum_{i=1}^M \sum_{j=1}^M q_{ij}(\omega, \mathbf{k})},$$

$q_{ij}(\omega, \mathbf{k})$ is the inverse of matrix $\{\exp[i\mathbf{k} \cdot (\mathbf{x}_i - \mathbf{x}_j)]G_{ij}(\omega)\}$. The MLM can be regarded as a filter in wavenumber

space, with the wavenumber response being expressed as $B(\omega, \mathbf{k}, \mathbf{k}_0) = \sum_{j=1}^M A_j(\omega, \mathbf{k}_0) \exp[i(\mathbf{k} - \mathbf{k}_0) \cdot \mathbf{x}_j]$, whose shape and sidelobe structure are not only determined by array geometry but also different for each wavenumber $\mathbf{k}_0$ at which an estimate is obtained. The MLM can pass traveling wave with a wavenumber of $\mathbf{k}_0$ without being distorted and maximally reject traveling waves with wavenumbers other than $\mathbf{k}_0$. Therefore, the MLM is useful for the estimation of frequency–wavenumber spectrum when the incoherent noise power is relatively small compared to

the power of the traveling waves. In addition, to get reliable direction information, the minimum signal wavelength should be larger than the minimum array spacing; otherwise, a white azimuthal spectrum will appear because of spatial aliasing, and the maximum signal wavelength is about 6–7 times (Sato et al. 2001) or 8 times (Wu and Huang 2015) the maximum array spacing.

The MLM has a higher wavenumber resolution compared to the conventional spectrum estimation based on Fourier analysis. Generally, it is used to derive spectrum in (ω, k_x, k_y) domain when location differences of the sensors exist only in the horizontal. For example, it has been widely used in seismology (e.g., Kocaoğlu and Firtana 2011; Leong and Aung 2012) and oceanography (e.g., Drennan et al. 2003; Young and Babanin 2006; Donelan et al. 2015).

Before being used to derive the 4D internal wave spectrum, the MLM was tested first by examining the spectrum of a given synthetic signal. The signal was generated in the form of $\zeta(x, y, z, t) = A \sin(k_{0z}z) \sin(k_{0x}x + k_{0y}y - \omega_0 t)$ plus a random noise, which is consistent with the characteristics of vertical standing waves. Here, signal amplitude A was set to 10 m, (x_i, y_i) ($i = 1, 2, 3, 4$) represented the same array locations as the four moorings A1–A4, and z_i ($i = 1, 2, 3, 4$) was set to the same depth value that needs to satisfy $\sin(k_{0z}z) \neq 0$. Wavenumbers were set to $k_{0x} = 2\pi \times 10^{-5} \text{ rad m}^{-1}$, $k_{0y} = -2\pi \times 10^{-4} \text{ rad m}^{-1}$, and $k_{0z} = 2\pi \times 10^{-2} \text{ rad m}^{-1}$. As a result, the signal has a horizontal wavelength ~ 10 km, which is larger than the minimum array spacing (2.9 km). The frequency was calculated using $\omega_0 = \sqrt{[(k_{0x}^2 + k_{0y}^2)N^2 + k_{0z}^2 f^2]/(k_{0x}^2 + k_{0y}^2 + k_{0z}^2)}$ with $N = 4 \times 10^{-2} \text{ rad s}^{-1}$ and $f = 5.2 \times 10^{-5} \text{ rad s}^{-1}$. Using the MLM, spectrum $P(\omega_0, k_h \cos\theta, k_h \sin\theta, k_{0z})$ can be estimated, where $k_h = \sqrt{k_{0x}^2 + k_{0y}^2}$ and θ ranges from 0° to 360° with an interval of $\Delta\theta = 0.25^\circ$. The spectral result showed a significant peak at the given wavenumbers (k_{0x}, k_{0y}) , indicating the signal propagates along -84.25° , with a phase speed of 0.64 m s^{-1} and a wavelength of 9.95 km (Fig. 9a). The cases with different angular resolutions showed that as $\Delta\theta$ value decreases, the spectral integration along θ only experiences a slight change (Fig. 9c), but the directional accuracy will be largely improved (Fig. 9b) and the distinction between the real peak and spurious peak in the opposite direction becomes larger when $\Delta\theta$ is smaller than 0.5° (Fig. 9d). However, considering that a too small $\Delta\theta$ value will lead to a substantial increase in computational cost, a value of $\Delta\theta = 0.25^\circ$ is chosen here. Besides, the cases with different random noise levels showed similar patterns except for a slight discrepancy in spectral level (Fig. 9e). Furthermore, by changing values of (k_{0x}, k_{0y}) to $k_{0x} = -5.34 \times 10^{-5} \text{ rad m}^{-1}$, $k_{0y} = -3.14 \times 10^{-5} \text{ rad m}^{-1}$

FIG. 10. Horizontal wavenumber spectra of the synthetic signals (plus a random noise). The magenta square shows $k_{0x} = -5.34 \times 10^{-5} \text{ rad m}^{-1}$ and $k_{0y} = -3.14 \times 10^{-5} \text{ rad m}^{-1}$, with $\arctan(k_{0y}/k_{0x}) = -149.53^\circ$. The magenta arrow indicates the spectral peak.

(Fig. 10), we found the MLM still worked well when the signal had a horizontal wavelength ~ 100 km, exceeding 8 times the maximum array spacing (8.2 km). These test cases indicate the MLM is capable of producing the valid horizontal wavenumber spectrum for signal in the form of $A \sin(k_z z) \sin(k_x x + k_y y - \omega t)$.

In the step of modal harmonic decomposition, the WKB-scaled isothermal displacements were separated into components with different frequencies and vertical wavenumbers. For the component at ω_m and $k_z^{(n)}$, its reconstructed displacements $\hat{\zeta}_{mn}(\mathbf{x}_i, z_i, t)$ ($i = 1, 2, 3, 4$) plus a random noise ($\sim 1\%$ of the signal amplitude) was used in the MLM to obtain its horizontal wavenumber spectrum $S(k_x^{(mn)}, k_y^{(mn)}, k_z^{(n)}, \omega_m)$ or azimuthal spectrum $S(\theta, k_h^{(mn)}, k_z^{(n)}, \omega_m)$. Here, $k_x^{(mn)} = k_h^{(mn)} \cos(\theta)$, $k_y^{(mn)} = k_h^{(mn)} \sin(\theta)$, with θ ranging from 0° to 360° with an interval of 0.25° and horizontal wavenumber being set to $k_h^{(mn)} = k_z^{(n)} \sqrt{(\omega_m^2 - f^2)/(N^2 - \omega_m^2)}$ according to the dispersion relationship of internal waves. The z_i ($i = 1, 2, 3, 4$) was set to the same depth, the spectra in all layers were estimated and then depth averaged. Only the components with frequency greater than or equal to 2 cpd were used to obtain their horizontal wavenumber spectra because our data lasting only about four days were not long enough to effectively analyze the signal of diurnal frequency after being divided into five segments (as mentioned above). Combining the horizontal wavenumber spectra for all components, the 4D internal wave spectrum $S(k_x, k_y, k_z, \omega)$ or $S(\theta, k_h, k_z, \omega)$ can be obtained finally. However, it is notable that for the component with a horizontal wavelength smaller than the minimum array spacing, the MLM may give a white azimuthal spectrum because of the aliasing caused by sparse array spacing. In such a case, only the integral of spectral

FIG. 11. Comparison of 1D (a) frequency spectrum $S(\omega)$, (b) vertical wavenumber spectrum $S(k_z)$, and (c) horizontal wavenumber spectrum $S(k_h)$ based on different methods: Fourier analysis (blue curves; mean results of the four moorings), MLM (green curves), GM79 (magenta curves), and modified GM79 (red curves). Coriolis frequency, buoyancy frequency, and frequencies of D_1 and D_2 internal tides are marked at the top of (a). The gray dotted lines and bold black lines indicate the falloff rates of spectra based on GM79 and modified GM79/FFT analysis, respectively. The blue shaded regions indicate the 95% confidence intervals.

energy along θ , not the distribution of spectral energy along θ , makes sense.

3. Validation and analysis of results

a. Validation of 4D internal wave spectrum

Based on the Fourier analysis, the isothermal displacements (without modal harmonic decomposition) can be used to estimate 2D frequency–vertical wavenumber spectrum $S(k_z, \omega)_{\text{FFT}}$, as well as 1D frequency

spectrum $S(\omega)_{\text{FFT}}$ and 1D vertical wavenumber spectrum $S(k_z)_{\text{FFT}}$. The spectra based on Fourier analysis were referred to as FFT spectra. Taking the FFT spectra as the standards, we modified the GM79 spectra. Then the 4D internal wave spectrum was validated against the FFT spectra and modified GM79 spectra.

1) MODIFICATION OF GM79 SPECTRUM

Based on the assumption that the internal wave field is horizontally isotropic and vertically symmetric, the

FIG. 12. Comparison of 2D frequency-vertical wavenumber spectrum $S(k_z, \omega)$ based on four methods: (a) Fourier analysis (mean result of the four moorings), (b) MLM, (c) GM79, and (d) modified GM79. A base-10 logarithmic scale is used to present the spectral levels.

spectrum of vertical displacement is defined as $F_z(\omega, j) = b^2 N_0 N^{-1} (\omega^2 - f^2) \omega^{-2} E_{GM}(\omega, j)$ according to GM79 spectrum (Munk 1981). The $E_{GM}(\omega, j)$ is a dimensionless energy density that is determined by $E_{GM}(\omega, j) = E_0 B(\omega) H(j)$, where the modal dependence $H(j) = (j^t + j_*^t)^{-1} / \sum_{j=1}^{\infty} (j^t + j_*^t)^{-1}$ and the frequency dependence $B(\omega) = (2/\pi) (f/\omega \sqrt{\omega^2 - f^2})$ ($f \leq \omega \leq N$) need to satisfy $\sum_{j=1}^{\infty} H(j) = 1$ and $\int_f^N B(\omega) d\omega = 1$, respectively. Here, $E_0 = 6.3 \times 10^{-5}$ is a nondimensional spectral energy level, j is the vertical mode number, $j_* = 3$ is a mode scale number, $t = 2$ is the high wavenumber slope, and $b = 1300$ m is the e -folding scale of stratification.

Significant differences of energy level and slope between the FFT spectra and GM79 spectra mainly occurred in the vertical wavenumber spectrum (Figs. 11 and 12), based on which the modal dependence of GM79 was modified. Two changes were made. One involved changing the value of j_* from 3 to 1, based on the higher energy levels of low-mode components of the FFT spectra than those of GM79 spectra (Fig. 11b), with a smaller j_* indicating lower modes contributing relatively more to the total internal wave energy. Levine (2002) suggested that in the deep ocean j_* is typically found to be near 3, but in

shallow water mode one dominates. Yang and Yoo (1999) found $j_* = 1$ could fit the linear internal waves well in shallow water over the continental shelf region of the Mid-Atlantic Bight. And the northern SCS is indeed dominated by energetic internal tides, especially low-mode internal tides. For example, through numerical simulation Jan et al. (2008) indicated the dominance of low vertical modes of baroclinic tides in the northern SCS. The other involved changing the value of high vertical wavenumber slope t from 2.0 to 2.5, based on the shape of $S(k_z)_{FFT}$ (Fig. 11b). High-wavenumber falloff rate in a specific region is important for both internal wave spectrum itself and its associated processes, such as energy cascade. There were several studies focusing on this issue in different regions, as reviewed by Polzin and Lvov (2011). Here, we provide a value of vertical wavenumber slope as -2.5 , which indicates the specific behavior of internal wave field in the northern SCS. Not only the modified GM79 vertical wavenumber spectrum agrees well with the observed one (FFT estimation; Fig. 11b), but also the modified GM79 frequency-vertical wavenumber spectrum is consistent with the observed one, with a

FIG. 13. Frequency–wavenumber spectrum $S(k_h, k_z, \omega)$ of internal waves in the northern SCS. A base-10 logarithmic scale is used to present the spectral levels (colored dots).

significant improvement in reproducing the observed low spectral levels in the regions of high frequency and wavenumber (Fig. 12).

2) PERFORMANCE OF 4D INTERNAL WAVE SPECTRUM

The 2D frequency–vertical wavenumber spectrum $S(k_z, \omega)_{\text{MLM}}$ effectively reproduces the magnitude and shape of observed $S(k_z, \omega)_{\text{FFT}}$ and modified GM79 spectrum $S(k_z, \omega)_{\text{GM79*}}$, which decreases with increasing frequency and vertical wavenumber (Fig. 12). In addition, both 1D frequency spectrum $S(\omega)_{\text{MLM}}$ and vertical wavenumber spectrum $S(k_z)_{\text{MLM}}$ agree well with the observed ones and modified GM79 spectra, exhibiting the falloff rates of ω^{-2} and $k_z^{-2.5}$, respectively (Figs. 11a,b). Departure occurred at the tidal frequency between these spectra, which is not involved in the GM model. The 1D horizontal wavenumber spectrum $S(k_h)_{\text{MLM}}$ is also consistent with the modified GM79 spectrum $S(k_h)_{\text{GM79*}}$, exhibiting a falloff rate of k_h^{-2} (Fig. 11c). These agreements verify the validity of the 4D internal wave spectrum we obtained.

b. Analysis of 4D internal wave spectrum

By integrating the 4D internal wave spectrum $S(\theta, k_h, k_z, \omega)$ along θ , the 3D internal wave spectrum $S(k_h, k_z, \omega)$ can be obtained, which shows how internal wave spectrum varies with frequency and wavenumbers. This 3D spectrum decreases significantly with increasing

frequency, horizontal wavenumber, and vertical wavenumber (Fig. 13), with a falloff rate of ω^{-2} beyond tidal frequencies, and with falloff rates of k_h^{-2} and $k_z^{-2.5}$ for horizontal and vertical wavenumbers, respectively (Fig. 11).

The 4D internal wave spectrum $S(k_x, k_y, k_z, \omega)$ can also be projected to (k_x, k_y) domain for a fixed frequency and vertical wavenumber. Such spectrum can provide information about power, direction, and speed of propagating waves. The spectral peak indicates the horizontal wavenumber and the direction of incoming waves. The corresponding phase speed c and wavelength λ can be obtained by $c = \omega / \sqrt{k_x^2 + k_y^2}$ and $\lambda = 2\pi / \sqrt{k_x^2 + k_y^2}$, respectively. For example, spectrum for the first-mode semidiurnal internal waves indicated a propagation direction along 160° (Fig. 14a), which is consistent with the fact that M_2 internal tide was generated in the Luzon Strait and propagated westward into the northern SCS (e.g., Simmons et al. 2011; Zhao 2014). The phase speed c and wavelength λ are about 2.3 m s^{-1} and 101 km, respectively, which are close to the values reported by Buijsman et al. (2010). By comparing spectra of the same mode, the result suggests that the energy of first-mode internal wave was mainly distributed in the frequencies lower than 6 cpd and generally propagated in a zonal direction (Fig. 14b). When comparing spectra of the same frequency, the energy of M_2 internal tide was mainly contained in the first five modes, with the first three modes propagating in an almost zonal direction (Fig. 14c).

FIG. 14. Horizontal wavenumber spectra $S(k_x, k_y)$ for (a) first-mode semidiurnal internal waves; (b) first-mode internal waves with frequency varying from 2 to 10 cpd, corresponding to colored circles from the inner most outward; and (c) semidiurnal internal waves with mode number increasing from 1 to 10, corresponding to colored circles from the inner most outward. A base-10 logarithmic scale is used to present the spectral levels.

4. Summary

The contribution of this study is to provide a method for estimating 4D internal wave spectrum with limited mooring measurements, and examining the features of internal wave field. This method is made up of two parts. One is the modal harmonic decomposition, which separates the isothermal displacement into components with different frequencies and vertical modes/wavenumbers. The other is the MLM, via which the horizontal wavenumber spectrum or azimuthal spectrum of each separated component is obtained and finally the 4D internal wave spectrum can be derived.

Although this method has been showed to be effective based on both synthetic signals and mooring measurements in the SCS, it still has several limitations that need attention. First, in order to ensure that the vertical wavenumbers of n th-mode internal wave component at all moorings are approximately equal, the water depths of all moorings should be equal or extremely close. Second, the spectral results did not include the internal solitary waves, since the discrete arrivals of the coherent solitons are not well captured by the spectra (because they throw out phase information). Third, the moorings should be deployed at three locations at least and cannot

be arranged in a straight line, since the estimation of horizontal wavenumber spectrum needs direction information, which should be sampled by the different moorings. Fourth, the array spacing should be predesigned according to the smallest wavelength of interest, because the azimuthal information of internal waves with a horizontal wavelength shorter than the array spacing cannot be resolved because of spatial aliasing. Last, the sampling period should be long enough to distinguish the low-frequency internal wave components, since the time series of data will be divided into several segments and the number of divided segments should be larger than that of moorings when using the MLM.

This study will attract more attention and raise awareness of estimating 4D internal wave spectrum using limited mooring observations. It provides guidance for applying this method to other regions where such mooring observations are available.

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