A Technique for Estimating Liquid Droplet Diameter and Liquid Water Content in Stratocumulus Clouds Using Radar and Lidar Measurements

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ABSTRACT: This paper describes a technique for estimating the liquid water content (LWC) and a characteristic particle diameter in stratocumulus clouds using radar and lidar observations. The uncertainty in LWC estimate from radar and lidar measurements is significantly reduced once the characteristic particle diameter is known. The technique is independent of the drop size distribution. It is applicable for a broad range of W-band reflectivity Z between ~30 and 0 dBZ and all values of lidar backscatter β observations. No partitioning of cloud or drizzle is required on the basis of an arbitrary threshold of Z as in prior studies. A method for estimating droplet diameter and LWC was derived from the electromagnetic simulations of radar and lidar observations. In situ stratocumulus cloud and drizzle probe spectra were input to the electromagnetic simulation. The retrieved droplet diameter and LWC were validated using in situ measurements from the southeastern Pacific Ocean. The retrieval method was applied to radar and lidar measurements from the northeastern Pacific. Uncertainty in the retrieved droplet diameter and LWC that are due to the measurement errors in radar and lidar backscatter measurements are 7% and 14%, respectively. The retrieved LWC was validated using the concurrent G-band radiometer estimates of the liquid water path.

KEYWORDS: Stratiform clouds; Cloud microphysics; Aircraft observations; Lidars/Lidar observations; Radars/Radar observations; Remote sensing

1. Introduction

Microphysical parameterization, namely, the effective radius of droplets and liquid water content (LWC) are used in modeling stratocumulus clouds for quantifying cloud–climate feedback. Accurate representation of microphysical parameterization using only the airborne in situ probe observations is incomplete as the observations of a small concentration of large droplets that represent drizzle in drop size distribution (DSD) spectra are not always detected. The presence of a few drizzle or large cloud droplets is ubiquitous in stratocumulus clouds (Fox and Illingworth 1997). In the present study, radar and lidar measurements are used for detecting bulk properties of cloud and drizzle droplets remotely. In addition, airborne in situ probes provide a direct estimate of droplet diameter and LWC, but their sampling volumes are extremely limited compared to radar and lidar sample volumes.

Radar reflectivity Z and lidar backscatter β of cloud and drizzle are generally proportional to one of the DSD moments. For example, Z is proportional to the sixth moment of the DSD for particle diameters in the Rayleigh-scattering regime, and β is proportional to the second moment of the DSD (Donovan and van Lammeren 2001). Radar attenuation A in the Rayleigh-scattering regime is proportional to the third moment of the DSD that is linearly related to the LWC (Meneghini et al. 1997; Vivekanandan et al. 1999). Larger droplets often minimally contribute to the LWC, but their larger sizes tend to dominate the radar reflectivity. Radar reflectivity and β are not linearly related to LWC. Therefore, LWC cannot be estimated accurately using radar or lidar measurements alone (Frisch et al. 1995; Khain et al. 2008; Donovan and van Lammeren 2001). Power-law-based Z–LWC methods are applicable only to the shallow clouds where no drizzle drops are present. On the other hand, lidar backscatter is dominated by a large concentration of small drops. Therefore, lidar and radar observations complement each other by representing small and large droplets in a DSD spectrum; in the present study, they are used to retrieve characteristic droplet diameters.

Measurements from Variability of American Monsoon Systems (VAMOS) Ocean–Cloud–Atmosphere–Land Study (VOCALS) (Wood et al. 2011) and Cloud Systems Evolution in the Trades (CSET; Albrecht et al. 2019) field campaigns were analyzed. Since detailed profiles of liquid droplet spectra were not collected during CSET, liquid droplet spectra collected by in situ cloud and drizzle probes on the NSF–NCAR C-130 aircraft during VAMOS in the southeastern Pacific Ocean were used as input to the simulations of radar and lidar observations. The simulated radar and lidar observations were used for developing a retrieval method for estimating cloud microphysical products, namely, characteristic particle diameter and LWC. The practical applicability of the retrieval method was demonstrated using the radar and lidar measurements from CSET. During the CSET field campaign, airborne measurements from the Gulfstream V High-Performance Instrumented Airborne Platform for Environmental Research (HIAPER) Cloud Radar (HCR; Vivekanandan et al. 2015) and the High Spectral Resolution Lidar (HSRL; Eloranta 2005) were made.
in the northeastern Pacific. The radar and lidar were designed to fly on the NSF–NCAR Gulfstream V (GV) HIAPER aircraft. HIAPER and GV refer to the same aircraft. The phrase GV is used most of the time in this paper, whereas the phrase HIAPER is associated with HCR. In addition, a millimeter-wave zenith pointing G-band radiometer (GVR; Pazmany 2007) recorded measurements for remotely estimating liquid water path (LWP). Observations during CSET were made by the HIAPER aircraft. There were no concurrent in situ measurements of particle size and LWC from a second aircraft during CSET. All of the remote sensors and in situ cloud physics probes were on GV. There was a 160 m blind zone between in situ probes and remote measurements. Since there were no coincident measurements from a second aircraft for comparing radar and lidar-based retrieval and in situ observations, the estimates of LWC and drop diameter by lidar and radar could not be validated. In this paper, a characteristic diameter, namely, radar–lidar-estimated diameter (RLED) is introduced. RLED is a function of radar and lidar backscatter observations, and it can be estimated directly from radar and lidar measurements without knowing about the mathematical form of the DSD.

The goal of this paper is to present a technique for retrieving RLED and LWC in stratocumulus clouds. The proposed technique does not require an assumption about the shape of the DSD and is applicable over a broad range of $Z$ between $-30$ and $0 \text{ dBZ}$ and all values of $\beta$. No partitioning of cloud and drizzle is required on the basis of an arbitrary threshold of $Z$ as in Fielding et al. (2015). Past attempts at retrieval of LWC and measurements of droplet size using remote observations are described in section 2. Electromagnetic simulations of radar and lidar observations of DSD spectra were used for developing a method for retrieving RLED and LWC. Simulations of radar and lidar observations require scattering cross sections of water droplets. Section 3 describes scattering cross sections of water droplets as a function of diameter at radar and lidar frequencies. These cross sections are used for simulating radar and lidar observations of marine stratocumulus cloud composed of cloud and drizzle droplets. Section 4 describes the rigorous electromagnetic simulation of radar and lidar observations. Relations between radar and lidar observations and between RLED and LWC were derived using the simulated observations. The RLED is a function of $Z$ and $\beta$. A method for retrieving RLED from lidar and radar observations is presented in section 5. Accuracy of LWC retrieval using $Z$ is improved when RLED is known. Section 6 presents a technique for estimating LWC using the retrieved RLED and measured $Z$. The retrieval method was validated using in situ observations of a few representative stratocumulus cloud profiles from the VOCALS dataset. These cloud profiles cover low, medium, and high values of radar and lidar measurements. Section 7 presents RLED and LWC estimated from CSET radar and lidar measurements using the retrieval technique described in sections 5 and 6. The retrieved LWC was validated using the concurrent G-band radiometer-based LWP estimation. Section 8 presents a summary and conclusions. The appendix contains definitions of many of the acronyms and variables used in this paper.

2. Past attempts at liquid water content and droplet size retrieval

Radar, solar irradiance, microwave, millimeter-wave radiometers, and lidar observations are often used for retrieving microphysics of clouds, namely, droplet diameter and liquid water content (Meneghini et al. 1997; Vivekanandan et al. 1999, 2001; Ellis and Vivekanandan 2011). In the absence of drizzle, a characteristic cloud droplet radius can be estimated using an empirical relation (Dong and Mace 2003) between $Z$ and droplet radius. However, the empirical relation is not valid for a cloud droplet effective radius $>10 \mu m$ as the measurement volume may contain drizzle. Fielding et al. (2015) used a threshold of $-17 \text{ dBZ}$ for separating the cloud and drizzle in precipitating clouds as was originally proposed by Frisch et al. (1995). They used an optimal estimation method constrained by radar, lidar, and zenith radiances, and also assumed lognormal or gamma DSDs for estimating particle diameter and LWC. The combination of radar and lidar measurements permits more accurate microphysical characterization (Donovan and van Lammeren 2001; O’Connor et al. 2005). However, most of the techniques used for retrieving cloud microphysical properties from remote sensing observations assume a DSD and artificially partition the droplets into cloud and drizzle drops based on $Z$.

Droplet diameter and LWC are constrained by the observation vector (radar, lidar, radiometer), but assumptions such as lognormal or gamma drop size distributions are needed for the forward model. Most of the prior retrieval methods assume lognormal distribution with fixed total number concentrations and standard deviation of DSD spectra. Since the total number concentrations and standard deviation of DSD spectra vary over a broad range (Miles et al. 2000), retrieval methods based on DSD are susceptible to uncertainty in estimated particle diameter and LWC (Huang et al. 2012). When radar reflectivity alone is used for estimating microphysics, a reflectivity threshold is commonly used for identifying cloud and drizzle regions (Frisch et al. 1995; Wood et al. 2012). Power-law relations that are tuned to cloud and drizzle regions are used for retrieving cloud droplet diameter and LWC (Khain et al. 2008), but the power-law relation between LWC and $Z$ is not uniformly valid for various geographical regions.

Irradiance and microwave radiometer-based liquid water path estimations can be used as constraints for reducing the uncertainty in power-law based reflectivity estimates of particle diameter and LWC. Giangrande et al. (2010) describe Doppler moment-based retrieval method using Mie scattering for vertical profiling W-band radars. Using Doppler radar spectra, cloud and drizzle reflectivity were portioned in a radar sample volume and then used for estimating particle diameter and LWC (Luke et al. 2010; Luke and Kollias 2013). Ambient wind correction is needed for estimating the intrinsic Doppler spectra of cloud droplets and drizzle droplets. Under the right conditions, vertically pointing radar spectral data can be used to estimate the ambient wind if the drizzle and cloud droplet signatures are separated into so-called bimodal spectra. Doppler spectra estimated by an airborne radar are substantially broadened by platform motion (Sloss and Atlas 1968), mostly masking the bimodal spectra signatures. Reflectivity measurements
bounded by a surface reference technique, or coincident cloud radar and lidar measurements, have the potential for more accurately estimating the LWC and mean particle diameter (Meneghini et al. 2015; Matrosov 2010; Westbrook et al. 2010). Coincident observations provide the impetus for combined radar/lidar studies and retrievals of more accurate microphysical parameters, namely, characteristic particle diameter and LWC (Donovan and van Lammeren 2001; O’Connor et al. 2005). However, most of the above-mentioned retrieval methods assume a DSD such as modified gamma or lognormal.

In summary, the power-law based techniques for retrieving droplet diameter and LWC from remote sensing observations are limited by the following: (i) assumption of a mathematical form of DSD (Donovan and van Lammeren 2001; O’Connor et al. 2005), (ii) use of Z threshold for differentiating between cloud and drizzle returns (Frisch et al. 1995; Khain et al. 2008), and (iii) limiting the retrievals for samples with an effective radius less than 10 µm (Huang et al. 2012). The retrieval technique presented in this paper overcomes the above-mentioned shortcomings.

3. Radar and lidar backscattering cross sections

For developing the present retrieval method, coincident radar and lidar observations corresponding to LWC and characteristic particle diameter are necessary. But first we simulate radar and lidar measurements using in situ observations of known DSDs using scattering cross sections of water droplets. Scattering cross sections are also helpful for interpreting radar and lidar observations. Radar and lidar measurements of cloud liquid and precipitable water are related to scattering cross sections of the DSD and shape of the droplets as well as the dielectric constant of droplets (Battan 1973; Vivekanandan et al. 1991). Since only cloud and drizzle droplets are considered in this research work, particle shape is assumed spherical.

Normalized backscatter and extinction cross sections (σ_D and σ_e, respectively), and the ratio between σ_e and σ_D, are shown in Fig. 1 for W band (3.2-mm wavelength) as a function of particle diameter. Scattering cross sections were computed using the Mie theory (Mie 1908). The normalization factor is the optical cross section and it is equal to πD^2/4 for a particle of diameter D. The Rayleigh-scattering approximation is valid for D < 300 µm at the W-band wavelength. From the results shown in Fig. 1, the Rayleigh-scattering cross section as a function of D can be expressed as (1.63 × 10^{-13})D^6 µm^2, where diameter D is in micrometers. The σ_D increases with increasing D until it peaks near 1000 µm. For D > 300 µm, Mie scattering sets in, and the σ_D reaches a minimum at 1600-µm diameter. The σ_e increases with particle diameter and is equal to 2 for diameter > 1000 µm (Van de Hulst 1957). In the Rayleigh-scattering regime, the ratio of σ_e to σ_D monotonically decreases and asymptotically reaches a value of 300 for D > 3000 µm.

The above-described scattering cross sections were used for simulating radar observations. For computing lidar observations of water droplets, an improved Mie-scattering algorithm (Wiscombe 1980; Du 2004) was used because the diameter of water droplets is large compared to the lidar wavelength and the imaginary part of the refractive index of water is very small. Scattering cross sections σ_D and σ_e for liquid droplets at 0.532-µm lidar transmitted wavelength λ are shown in Figs. 2a and 2b. The σ_D in the Rayleigh regime for D < 0.05 µm. In Fig. 2a, σ_D varies by many orders of magnitude for D > 2.5 µm (size parameter πD/λ > 15) even for a small change in D (O’Connor et al. 2004). Since the refractive index for water at this wavelength is 1.33 – j(1.88 × 10^{-3}), extinction at this wavelength is primarily due to scattering. Extinction is the sum of scattering and absorption losses. The absorption loss is negligible because of the small value of the imaginary part of the refractive index at a wavelength of 0.532 µm. The imaginary part of the refractive index is almost zero (1.88 × 10^{-3}). At larger values of size parameter, σ_D decreases with numerous peaks. Large fluctuations in the numerical results are due to the low-absorbing nature of water at the 0.532-µm wavelength. Convergence in Mie scattering is obtained by ensemble averaging of the Mie-scattering cross section. It was uniformly averaged over a small interval of particle diameters centered on each particle diameter. For 0.15 < D < 2.0 µm, an averaging interval of 0.05 µm was used; for D > 2.0 µm an averaging interval of 0.5 µm was used. Each averaging interval was subdivided into more than 100 subintervals for averaging. The averaging intervals were selected to minimize smoothing over a large diameter interval. Figure 2b shows averaged σ_D, σ_e, and ratio of σ_e to σ_D as a function of the particle diameter. Oscillations in normalized σ_D are significantly reduced by averaging. The averaged σ_D asymptotically converged by averaging over subintervals.

From the results shown in Fig. 2b, the Rayleigh approximation for lidar σ_D is 13.05D^2 µm^2, where diameter D is in micrometers and is valid for D < 0.05 µm. For 5 < D < 100 µm, the σ_D is approximately proportional to D^2. The σ_e is well behaved as a function of particle diameter; it steadily increases.

![Fig. 1. W-band normalized extinction and backscatter cross sections, and the ratio between extinction and backscatter cross sections.](image-url)
as the diameter increases and asymptotically reaches a value of 2 for $D > 2 \mu m$ (Van de Hulst 1957). The ratio of $\sigma_r$ to $\sigma_D$ is defined as the lidar ratio (O'Connor et al. 2004). Because both $\sigma_D$ and $\sigma_r$ are proportional to $D^2$, the lidar ratio is relatively constant at $-20$ for cloud particle diameter $10 < D < 100 \mu m$. The lidar $\sigma_D$ is larger than the radar $\sigma_D$ by a factor of $10^3$ to $10^4$ for $10 < D < 1000 \mu m$. Hence the lidar is more sensitive in detecting small cloud droplets, but the lidar signal is susceptible to larger extinction that limits penetration of lidar signal beyond thick cloud layers.

4. Electromagnetic simulations of lidar and radar observations

In situ measurements of droplet spectra and corresponding simulated radar and lidar measurements are helpful for developing and validating a new retrieval method based on radar and lidar measurements. For this purpose, an electromagnetic scattering model was developed for simulating W-band radar reflectivity and lidar backscatter observations of liquid droplet spectra of marine stratocumulus clouds. Inputs to the electromagnetic scattering model are DSD and scattering cross sections. Since detailed profiles of DSD spectra were not collected in CSET, liquid droplet spectra collected in situ cloud [Cloud Droplet Probe (CDP)] and drizzle (PMS 2D-C) probes on the NSF–NCAR C-130 aircraft during the VOCALS were used for the simulation of radar and lidar observations.

Sizing calibration of the CDP [Droplet Measurement Technologies (DMT) Longmont, Colorado] was done using glass beads of known sizes, before and after the VOCALS field deployment. The manufacturer’s nominal value of 0.240 mm$^2$ was used for the probe sample area. Image analysis of the Fast 2D-C probe (a 25-\( \mu \)m-resolution PMS 2D-C probe, refitted prior to VOCALS with a DMT 64-photodiode array board) was done using the NCAR System for Optical Array Probe Data Analysis (SODA; A. Bansemer 2020, personal communication) tool. This software contains extensive algorithms to exclude streakers, shattered particles using the Field et al. (2006) interarrival time–based shattered particle removal, and the Korolev (2007) size correction to out-of-focus particles. The electronics response time of this probe has been determined by Hayman et al. (2016), hence the name Fast 2D-C, and sizing was verified by a spinning disk with particle images of a range of sizes.

The quality-controlled DSD concentrations of a total of 93 bins $[n(D)]$ from 30 bins of cloud probe data nominally between 1.0 and 50.0 \( \mu m \) and 63 bins of drizzle probe data nominally between 50.0 and 1600.0 \( \mu m \) were used in the simulation. In this paper, the terms cloud and drizzle refer to the droplets measured by cloud and drizzle probes. The DSD of droplets of radius $r$ is in $dN/d\log r$ format and is in units of number of particles per meter cubed. This format is a standard way of plotting DSDs, in which 1) one needs to have a log scale because of a wide range of $r$ values and 2) bins have different bin widths. The bins were normalized with respect to their corresponding widths. One advantage is that the area under the curve (from one $r$ value to another $r$ value) is proportional to the number of particles in that size range (Pitari et al. 2016).

DSD shape approximations, such as lognormal or gamma distributions, are not assumed for approximating the in situ DSDs for preserving observed characteristics of the droplet spectra that may contain a few large droplets (Fox and Illingworth 1997). The discrete DSDs without curve fitting were directly used for simulating the radar and lidar observations.

Reflectivity $Z$ and attenuation $A$ are computed using the radar backscatter and extinction cross sections as

$$Z = \frac{\lambda^4}{[K^2/\pi]} \sum_{i=1}^{q1} \sigma_i(D)n(D) \text{ mm}^6 \text{ m}^{-3}$$

and

$$A = \frac{\lambda^4}{[K^2/\pi]} \sum_{i=1}^{q1} \sigma_i(D)n(D)$$

FIG. 2. (a) Lidar normalized backscatter and extinction cross sections, (b) averaged lidar normalized extinction, backscatter cross sections, and the ratio between extinction and backscatter cross sections.
\[
A = 10 \log_{10} \left\{ \exp \left[ 1000 \sum_{i=1}^{93} \sigma_v(D_i)n(D_i) \right] \right\} \text{ dB km}^{-1}, \tag{2}
\]

where \( \lambda \) is the radar wavelength in millimeters, \( |K|^2 \) is the dielectric factor, and \( \sigma_v \) and \( \sigma_a \) are in the units of millimeters squared. The value of \( |K|^2 \) is 0.686 for W band. Figure 3 shows a scatterplot between \( Z \) and \( A \) for two categories: (i) cloud droplets and (ii) drizzle droplets. Not surprisingly, the cloud droplets reflectivity \( Z \) is smaller than that of drizzle droplets. Reflectivity and \( A \) are proportional to the sixth and third moments of the DSD, respectively. Reflectivity varies over six orders of magnitude, whereas the attenuation varies between 0 and 4 dB km\(^{-1}\).

The following are two power-law fits of attenuation as a function of reflectivity for \( Z < -17 \text{ dBZ} \) and \( Z > -17 \text{ dBZ} \) based on the simulated data using Eqs. (1) and (2):

\[
A = 18.6 Z^{0.58} \text{ dB km}^{-1} \text{ and } A = 1.68 Z^{0.9} \text{ dB km}^{-1}, \tag{3a}
\]

\[
A = 5 \text{ dB km}^{-1} \text{ and } A = 0.03 Z^{1/3} \text{ dB km}^{-1}, \tag{3b}
\]

where \( A \) is one-way attenuation in decibels per kilometer and \( Z \) is in \( \text{mm}^6 \text{m}^{-3} \). Equations (3a) and (3b) correspond to the attenuations of cloud and drizzle, respectively. The multiplicative coefficients in the above two equations differ by more than a factor of 10. The larger multiplicative coefficient for the lower \( Z \) suggests the attenuation is a function of mean droplet diameter. For obtaining the intrinsic reflectivity from the attenuated measured reflectivity, these power-law equations are used in section 7c.

In situ aircraft measurements of 4650 1-Hz droplet spectra were organized into 83 ascending or descending flight profiles or events for simulating vertical-pointing radar backscatter profiles of stratuscumulus clouds. Reflectivity profiles as a function of event number are shown in Fig. 4a. The maximum depth of the vertical profile in the cloud is <500 m. On average, radar reflectivity decreases as altitude increases because of a decrease in average cloud droplet diameter (Wood et al. 2012).

Lidar backscatter and extinction were computed for the same 4650 DSD spectra using the lidar cross sections in Fig. 2. The backscatter \( \beta \) and extinction \( \alpha \) as a function of lidar cross sections and droplet spectra can be expressed as

\[
\beta = \sum_{i=1}^{93} \sigma_{\beta}(D_i)n(D_i) \text{ m}^{-1} \text{ sr}^{-1} \quad \text{and} \quad (4)
\]

\[
\alpha = \sum_{i=1}^{93} \sigma_{\alpha}(D_i)n(D_i) \text{ m}^{-1}, \tag{5}
\]

where \( \sigma_{\beta} \) is in meters squared per steradian and \( \sigma_{\alpha} \) is in meters squared. The lidar backscatter is shown as a function of event number in Fig. 4b. Lidar backscatter varies between \( 10^{-5} \) and \( 0.8 \times 10^{-4} \text{ m}^{-1} \text{ sr}^{-1} \). Lidar backscatter is larger at a higher altitude because of larger backscatter cross sections and large concentrations of cloud droplets. It is interesting to note that radar reflectivity varies over six orders of magnitude whereas lidar backscatter varies only over two orders of magnitude. This is mainly because the radar backscatter is in the Rayleigh-scattering regime and the lidar backscatter is in the Mie-scattering regime.

The scatterplot between HSRL extinction and backscatter in Fig. 5 shows a linear relation. The extinction is about 20 times the lidar backscatter (O’Connor et al. 2004; Ansmann et al. 2009). The larger extinction impedes the lidar signal from penetrating deeper into water clouds. The absorption loss is small as the imaginary part of the refractive index is negligible. At W band, Z and attenuation are relatively smaller than the corresponding lidar backscatter and attenuation. Hence, the W-band radar signal penetrates through most of the stratuscumulus clouds, but its sensitivity for detecting cloud droplets is lower than that of the lidar.

5. Characteristic particle diameter

Large drizzle droplets dominate reflectivity but, they minimally contribute to LWC. Cloud droplets contribute to LWC significantly. Figure 6 shows the scatterplot of \( Z \) versus LWC for VOCALS DSDs. For a specified LWC, radar reflectivity is lower for cloud droplets and larger for drizzle. Therefore, an accurate estimate of particle diameter will significantly reduce uncertainty in LWC estimates using reflectivity and characteristic particle diameter (Khain et al. 2008). When DSD is dominated by drizzle, \( Z \) will be larger for a given LWC. Also, when there are few drizzle drops, \( Z \) is lower for a specified LWC. Since the proposed technique uses the retrieved RLED and reflectivity, the uncertainty in the LWC estimation is reduced.

Characteristic particle diameter, such as median volume diameter (MVD), and effective diameter \( D_{eff} \) are used to
characterize the mathematical form of the DSD. MVD is widely used for the characterization of droplet diameter using the in situ cloud and drizzle probe observations. MVD separates a DSD into two halves with respect to $D^3$ as

$$\sum_{i=1}^{D_{i=MVD}} D_i^3 n(D_i) = \sum_{i=1}^{D_{i=MVD}} D_i^3 n(D_i).$$

For a remote sensing application using visible frequencies such as from a satellite, scattered radiation is proportional to the geometrical cross section or $D^2$. Hansen and Travis (1974) proposed a geometrical cross section weighted particle
diameter called the effective diameter $D_{\text{eff}}$, which is useful for remote sensing in the visible wavelength, and it is defined as

$$D_{\text{eff}} = \left[ \frac{\sum_{i=1}^{93} D_i^6 n(D_i)}{\sum_{i=1}^{93} D_i^2 n(D_i)} \right]^{\frac{1}{3}}.$$  \hspace{1cm} (7)

In the above equation, the summation over 93 bins includes 30 bins of cloud probe and 63 bins of drizzle probe data, as described in section 4. The $D_{\text{eff}}$ and MVDs cannot be directly estimated using radar and lidar observations. For cloud and drizzle droplets, the radar and lidar measurements are proportional to the sixth and second moments of the DSD. In the case of reflectivity and lidar backscatter measurements, it is desirable to choose a characteristic droplet diameter, namely, RLED, that is a function of those moments of the DSD. RLED can be directly retrieved independently of the mathematical form of DSD because it is a direct function of radar and lidar measurables as shown below:

$$\text{RLED} = \left[ \frac{\sum_{i=1}^{93} D_i^6 n(D_i)}{\sum_{i=1}^{93} D_i^2 n(D_i)} \right]^{\frac{1}{3}}.$$  \hspace{1cm} (8)

Two independent measurements, namely, radar reflectivity and lidar backscatter, allow the estimation of RLED without any assumptions of shape or closed-form mathematical description of DSD (e.g., exponential, gamma or lognormal functions). Radar-to-lidar ratio versus RLED in Fig. 7a shows monotonic relation as RLED is a proxy of radar-to-lidar reflectivity ratio. RLED is directly estimated using the ratio of radar and lidar backscatter simulations ($Z/\beta$) from Fig. 7a as

$$\text{RLED} = 9.12(\frac{Z}{\beta})^{0.25} \mu m.$$  \hspace{1cm} (9)

In the above equation, $Z$ is radar reflectivity ($\text{mm}^6 \text{m}^{-3}$) and $\beta$ is lidar backscatter ($\text{m}^{-1} \text{sr}^{-1}$). Equations (8) and (9) are equivalent except that the RLED is shown as a function of $Z$ and $\beta$ in Eq. (9). RLED varies between 20 and 160 $\mu m$ for 45-dB dynamic range in $Z/\beta$. RLED has several advantages. It can be estimated directly from radar and lidar measurements without knowing the mathematical form of the DSD. The inclusion of radar reflectivity in the RLED formulation emphasizes the large end of the size spectrum, which has been found to be significant in the detection of precipitation onset (Politovich 1989; Vivekanandan et al. 2001). A few large droplets significantly enhance $Z$. In a size distribution with both small and large particles (i.e., broad spectrum), the RLED
value is biased toward larger particle size. In general, RLED is greater than or equal to MVD and $D_{eff}$ as shown in Figs. 7b and 7c. MVD and $D_{eff}$ are related to RLED for a specified mathematical form of DSD. For a narrow DSD spectrum, MVD, $D_{eff}$, and RLED converge to the same value (Vivekanandan et al. 2001).

Summing over the volumes of all drops and multiplying by the density of water, the liquid water content was computed from DSD spectra using the following equation, where $D$ is in meters and $\rho_w$ is the density of water in grams per cubic centimeter:

$$LWC = \frac{\pi 10^{-6}}{6} \rho_w \sum_{i=1}^{91} D^3 n(D) \ \text{gm}^{-3}. \quad (10)$$

Vertical profiles of LWC and RLED in Fig. 8 show characteristics of stratocumulus cloud that typically exhibit decreasing particle size and increasing LWC. Figure 8a shows LWC profiles for the 83 events of radar and lidar observations shown in Fig. 4. LWC varies between 0.01 and 1.0 g m$^{-3}$ with larger values at higher altitudes. Figure 8b shows corresponding RLED, and it varies between 10 and 200 $\mu$m.

The corresponding simulated radar reflectivity versus lidar backscatter is shown in Fig. 9. Lack of any correlation and the independence between radar and lidar backscatter simulations suggest they have the potential for estimating two independent moments of the DSD, namely, RLED and LWC. There is no physical reason to expect such a correlation as the radar and lidar measurements are related to independent moments of the DSDs. In the next section, a method for retrieval of LWC using RLED and $Z$ is described.

### 6. Retrieval of liquid water content

A power-law relation such as $Z = aLWC^b$ is tuned for clouds with and without drizzle in Khain et al. (2008). As shown in Fig. 6, cloud droplets contribute to LWC significantly, whereas drizzle droplets dominate $Z$ and minimally contribute to LWC. Since the proposed technique makes use of the retrieved RLED along with reflectivity, the uncertainty in the LWC estimation is reduced.

Simulation results in Fig. 10 show a scatterplot between RLED and LWC for stratocumulus clouds. Even though cloud and drizzle droplets span a wide range of LWC, a larger number of cloud droplets cluster around higher LWC than drizzle droplets. Therefore, the use of RLED along with reflectivity can improve the accuracy of the LWC estimate. Based on this dependence of LWC on RLED, an algorithm for estimating LWC is formulated. The reflectivity is normalized by the RLED for obtaining a linear relation between LWC and the normalized reflectivity as shown below:

$$Z_{\text{norm}} = \frac{Z}{(0.53 \times \text{RLED})^{1/4}}. \quad (11)$$

In the above equation, $Z$ is in mm$^6$ m$^{-3}$ and the RLED is in millimeters. Figure 11 shows a scatterplot between the normalized radar reflectivity and the LWC. The LWC is retrieved as

$$LWC = 2.3 \times 10^{-6} Z_{\text{norm}}^{-0.004} \ \text{g m}^{-3}. \quad (12)$$

Equations (11) and (12) are used for retrieving RLED and LWC for all of the 4650 droplet spectra. Uncertainty in the retrieved RLED and LWC is caused by variation in DSD and
uncertainty in radar and backscatter measurements. Uncertainty due to variation in DSD was computed using 4650 droplet spectra. The root-mean-square errors (RMSE) for the retrieved RLED and LWC are 0.14 μm and 0.02 g m⁻³, respectively, due to the variation in the DSD. Measurement uncertainty in radar and lidar observations were applied to Eqs. (9) and (11) for estimating uncertainty in RLED and LWC (Hogan et al. 2005). Lidar backscatter and radar reflectivity are measured with 10% and 1-dB uncertainties, respectively. Uncertainty in the retrieved RLED and LWC due to the measurement errors in radar and lidar backscatter measurements is 7% and 14%, respectively.

The performance of the retrieval method was validated using in situ and remote observations. Measured stratocumulus profiles that cover low, medium, and high radar and lidar measurements were selected from the VOCALS in situ measurements. For these DSD profiles, RLED and LWC are known as they are calculated from the DSDs. Simulated radar and lidar observations are known for these profiles. Figure 12 shows DSD for two profiles (events 1 and 36), each averaged from cloud base to cloud top. Profile 1 shows a narrower cloud droplet size distribution, smaller mean cloud droplet diameter, and fewer drizzle drops; for simplicity, this is termed “non-precipitating,” even if it does contain a small extension into the drizzle drop range ($D > 50 \mu m$). In contrast, profile 36 has a...
wider cloud droplet size distribution, larger mean cloud droplet diameter, and a significant extent into the drizzle drop size range; for simplicity, this case is termed "precipitating." Figure 13 shows reflectivity, lidar backscatter profiles, radar-to-lidar ratio, and lidar ratio. Reflectivity decreases whereas, lidar backscatter increases as the altitude increases. Radar reflectivity varies between $-30$ and $10$ dBZ for these profiles. The dynamic range of lidar backscatter is three orders of magnitude less than in the radar reflectivity. Radar-to-lidar backscatter ratio decreases with the increase in altitude. Lidar ratio is between 19 and 20 sr for events 1, 18, and 74. These values suggest particles are mostly cloud droplets and radar reflectivity $>0$ dBZ. The radar reflectivity is $>0$ dBZ for event 36. The larger radar-to-lidar ratio for event 36 corresponds to Mie scattering from many large drizzle drops, which results in a lidar ratio $<18$ sr.

The method described in section 5 for estimating RLED and the estimation of LWC described in this section are used for retrieving RLED and LWC from radar and lidar simulated observations. The profiles of retrieved RLED vary between 15 and 130 $\mu$m, and LWC varies between 0.05 and 0.45 g m$^{-3}$. (see Fig. 14). The retrieved profiles show an overall decrease in RLED and an increase in LWC as the altitude increases. The differences between in situ and remotely retrieved RLED is less than 4 $\mu$m. The difference between in situ and remotely retrieved LWC is less than 0.10 g m$^{-3}$, and the difference increases to 0.2 g m$^{-3}$ for $Z > 0$ dBZ. The normalized differences between in situ and remotely retrieved RLED and LWC are less than 5% and 50%, respectively. Values were normalized with respect to in situ measurements. The normalized difference is the lowest for the narrow DSD of event one and the largest for the broader DSD of event 36 that has many large drizzle drops. The RMSEs for retrieved RLED for events 1, 18, and 74 are 0.23, 0.07, and 0.15 $\mu$m, respectively. The RMSE for retrieved LWC for events 1, 18, and 74 are 0.02, 0.02, and 0.03 g m$^{-3}$, respectively. The HCR reflectivity for events 1, 18 and 74 is $<0$ dBZ. For event 36 in which radar reflectivity is $>0$ dBZ, the RMSE of RLED is 1.38 $\mu$m and the RMSE of LWC is 0.10 g m$^{-3}$. The increased RMSE for event 36 is primarily due to Mie scattering in W-band reflectivity.

7. Application of the retrieval method to CSET observations

The HCR and HSRL collected concurrent radar and lidar observations for the first time in CSET field deployment during the summer of 2015. The main objective of CSET was to study the characteristics and evolution of stratocumulus clouds over the northeastern Pacific Ocean. These data were used to illustrate the validity of the LWC retrieval method presented in section 6.

a. HCR

The HCR (Vivekanandan et al. 2015) operated at a frequency of 94 GHz (3-mm wavelength). The radar is capable of
estimating radial winds and reflectivity to a range of 15 km with 19.2-m gate spacing. The HCR can be operated in scanning and staring modes. The dynamic range of HCR reflectivity is between $2^{35}$ and $30 \text{ dB}_Z$. Reflectivity can be estimated with a 1-dB accuracy. For ease of reference, some of the salient technical specifications of HCR are listed in Table 1. HCR sensitivity is $2^{30} \text{ dB}_Z$ at 1 km for a 0.256-μs transmitted pulse width when the received signal is corrected for noise and is averaged over 0.1 s.

Pulse-pair estimates are averaged over a 0.1-s interval to reduce fluctuations in the Doppler moment estimates, namely, reflectivity, mean velocity, and spectrum width. The along-track resolution is a function of aircraft speed and dwell time of the beam. The HCR beam’s footprint is 3 m at a range of 250 m and it increases to 180 m at a 15-km range. Since the aircraft traverses 140 m s$^{-1}$, the HCR beam’s footprint increases from 14 to 180 m as the range increases from 250 m to 15 km.

b. High Spectral Resolution Lidar

The capability of HCR is enhanced by the coordination with the HSRL. The HSRL is a micropulse lidar and it is safely deployable in populated urban areas and on an airborne platform. One of the unique features of the HSRL is simultaneous estimates of backscatter and extinction of the clouds and
aerosol. Measurements at 0.532 μm are of paramount importance to radiation transfer studies because they are in the visible spectrum. A telescope expands the 2-cm beam to 40 cm. The beam is offset by 4° in reference to local vertical for eliminating specular reflection from highly oriented ice particles and ocean surface. A microtransmitted pulse length of 50 ns resolves 7.5 m along the laser beam. The loss of transmitted signal sensitivity resulting from micropulse operation is compensated by a longer dwell time of the laser beam. Reflected photons are accumulated over 0.5-s duration as 4000 pulses are transmitted per second. When deployed aboard the GV platform, 0.5-s dwell time corresponds to the along-track resolution of 70 m.

Descriptions of an HSRL system and its measurement technique can be found in Eloranta (2005). A notch filter in the HSRL separates molecular scattering from aerosol and cloud particle backscatter based on their Doppler spectrum widths. Doppler spectrum width of air molecules is > 300 m s⁻¹ due to random thermal motion whereas, the Doppler spectrum width of aerosol and cloud particles is less than 3 m s⁻¹. The Doppler spectrum width of air molecules is 100 times as wide as that of the cloud and aerosol spectrum width. At the 0.532-μm operating wavelength of the HSRL, 300 m s⁻¹ corresponds to a Doppler spectrum width of 1 GHz. An iodine absorption cell maintained at a specific temperature and pressure with a bandpass filter width of 1.8 GHz is used for separating molecular scattering from aerosol backscatter signals. High spectral purity of the laser transmitted frequency is necessary for frequency-dependent discrimination of backscattered signals from air molecules and aerosol or droplets. The laser transmitted frequency is locked with iodine absorption spectra for ensuring frequency stability using an active feedback control loop. The technical specifications of the HSRL are listed in Table 2. Measurements of molecular backscatter enable estimation of extinction. The extinction is used for estimating attenuation-corrected lidar backscatter. The lidar is sensitive enough to collect measurements up to an optical depth of 2.2.

c. Retrieval of particle size and LWC using CSET measurements

The retrieval method described in sections 5 and 6 was applied to the observations collected during CSET project (Albrecht et al. 2019; Schwartz et al. 2019). Figures 15a and 15b show B-scan displays of Z and β, respectively. The data were collected in zenith-pointing mode from GV between 1915 and 1920 UTC 24 July 2015. The horizontal axis designates the time the aircraft traveled. The aircraft altitude was 100 m MSL. The 5-min duration of the observation corresponds to a 42-km swath. The maximum altitude of the stratocumulus clouds is less than 3 km, and they are all warm clouds. Reflectivity values range between −30 and 10 dBZ. The middle panel shows the attenuation-corrected lidar backscatter measurements. Lidar backscatter was corrected for attenuation using the HSRL extinction measurement as described in section 7b. The peak values of lidar measurements correspond to the cloud base. Below the cloud base, the increase in Z is due to the larger mean droplet size or drizzle. The bottom panel shows the attenuation-corrected HSRL reflectivity. Reflectivity is usually underestimated because of the attenuation of the radar signal as it propagates through cloud layers. Therefore, attenuated W-band radar measurements must be corrected for attenuation effects before retrieving particle size and LWC. The attenuation correction is based on the A–Z power-law relations in section 4. The HCR reflectivity was corrected for attenuation using the simple power-law Eq. (3) between reflectivity and attenuation (Hiltonfeld and Bordan 1954).

In addition to the radar and lidar observations, concurrent seven-channel GVR measurements were also collected in order to measure the LWP. Since there were no concurrent in situ measurements of particle size and LWC from a second aircraft during CSET, the LWP estimated by G-band radiometer was used for validation of the LWC retrieved radar and lidar. Radar and lidar-based retrieved LWC is in grams per meter cubed, and the G-band radiometer path-integrated LWC is in millimeters.
Intrinsic spatial resolutions of all three instruments, namely, HCR, HSRL, and GVR, are different. The HSRL footprint is 40 cm, the HCR footprint varies between 4 and 35 m, and the G-band radiometer footprint varies from 10 to 90 m. These various footprints were averaged over 0.5-s flight duration that corresponds to 70-m spatial resolution along the flight track. The spatial resolution along the aircraft track is averaged for keeping the same spatial resolution for individual sensors along the flight track, but the cross-track resolutions of sensors vary between 40 cm and 100 m as a function of instrument footprint.

Retrieved RLED and LWC are shown in Fig. 16. The regions of large lidar backscatter $\sim 10^{-2} \text{ m}^{-1} \text{ sr}^{-1}$ are consisting of droplets $< 20 \mu \text{m}$. HCR reflectivity of small cloud droplets is $<-10 \text{ dBZ}$ and the LWC above the cloud base in the regions of small cloud droplets is $> 0.2 \text{ g m}^{-3}$. The HCR values of reflectivity $> 10 \text{ dBZ}$ below the cloud base around 1917:30 and 1918:30 UTC correspond to small LWC $< 0.02 \text{ g m}^{-3}$ and RLED $> 500 \mu \text{m}$. The highest values of LWC are always at the cloud top and have similar values even in the weak cloud between the big cells. Interpretation of the meteorology of the highest values of LWC at the cloud top may require an analysis of microphysical and dynamical estimates, and this analysis is beyond the scope of the present study.

Observations during CSET were made only by a single airplane, that is, the Gulfstream V HIAPER aircraft. All of the remote sensors and in situ probes were on the HIAPER aircraft. There was a 160-m blind zone between in situ probes and remote measurements. The 42 km swath of radar and lidar data included observations of clouds at various stages of evolution. The GVR collected good measurements only on 24 July 2015, for the time period shown in this paper.

The GVR estimated LWP for every 6.5 s. HCR data were averaged over 0.5 s along the track for matching with the HSRL data collection interval. Radar and lidar retrieved LWC was integrated over the entire altitude for estimating the LWP. The radar and lidar retrieved LWP was validated using the concurrent G-band radiometer estimates of the LWP. A moving average of 13 ($=6.5/0.5$) data points along the aircraft track of HCR and HSRL retrieved LWP was performed for matching with the GVR measurement time interval. Figure 17 shows LWP estimates from radar and lidar observations and GVR. The GVR-based LWP includes the totality of cloud layers that were undetected by the radar and lidar. The temporal structure of GVR-estimated LWP agrees with the radar and lidar–estimated LWP. Particularly, the spatial location of peak values of GVR-estimated LWP coincide with the radar and lidar–based peak LWP. The RMSE between GVR-estimated LWP and radar and lidar–estimated LWP is 0.1 mm. Despite sampling volume differences among radar, lidar, and radiometer, and platform motion of 140 m s$^{-1}$, the agreement between independent estimates of radar, and lidar LWP and GVR LWP is noteworthy.
8. Summary and conclusions

A technique for RLED and LWC using coincident radar and lidar observation was presented. It is applicable for \( Z \) between \(-30\) and 0 dBZ and all values of lidar backscatter observations. For nonprecipitating optically thin clouds that were routinely observed during CSET campaign, the technique described in this paper is applicable to the data collected in nadir and zenith. The retrieval technique is not applicable to an optically thick cloud that prevents lidar detection of precipitable water when the aircraft is in the cloud, above or below the cloud. The retrieval method of LWC and RLED was developed using simulations of radar and lidar observations of stratocumulus DSD spectra. The DSD spectra were collected in the southeastern Pacific during the VOCALS. Retrieved LWC using radar and lidar observations were validated using CSET observations over the northeastern Pacific.

Mie-scattering cross sections of cloud and drizzle droplets for W band and visible wavelengths were presented. Scattering cross sections at W band are well behaved as a function of droplet size as the dielectric constant of liquid at W band has a significant imaginary component that corresponds to the absorption of the incident wave. However, at the visible wavelength, the imaginary component is negligible. As a result, large fluctuations of backscattering cross sections are exhibited. These large fluctuations in the Mie-scattering regime at visible wavelength were smoothed by ensemble averaging. The backscatter and extinction cross sections were used in simulating radar and lidar observations of stratocumulus cloud liquid and precipitable water from VOCALS. The simulated results were used for formulating equations for estimating RLED and LWC.

RLED is a function of \( Z \) and \( b \); it is directly estimated from radar and lidar observations without any assumption about DSD’s mathematical form. The inclusion of RLED along with reflectivity in the LWC retrieval equation broadened the applicability of the LWC retrieval for cloud liquid and precipitable water. Thus, the technique for estimating RLED and LWC presented in this paper does not require any assumption about DSD’s mathematical form. The root-mean-square errors (RMSE) for the retrieved RLED and LWC are 0.14 m and 0.02 g m\(^{-2}\), respectively, for the VOCALS dataset of 4650 spectra. For uncertainty of 10% and 1 dB in lidar, and radar measurements, the errors in RLED and LWC are 7% and 14%, respectively.

Uncertainty due to DSD variation in the LWC estimation is significantly reduced when reflectivity normalized by RLED is used. The accuracy of retrieval equations of RLED and LWC was validated using the DSD profiles in the VOCALS. The differences between in situ and remotely retrieved RLED is less than 4 \( \mu m \). The maximum difference between in situ and remotely retrieved LWC is less than 0.10 g m\(^{-3}\) for \( Z < 0\) dBZ, and the difference increases to 0.2 g m\(^{-3}\) for \( Z > 0\) dBZ. The LWC retrieval is not satisfactory because of Mie scattering in the W-band reflectivity.

The retrieval method was applied to CSET measurements. Despite sampling volume differences among radar, lidar, and radiometer, and platform motion, comparison between LWP of G-band radiometer and path-integrated radar and lidar-based
LWP shows maximum values of LWP and temporal features are in good agreement. The RMSE between GVR-estimated LWP and radar and lidar–estimated LWP is 0.1 mm. The difference in radar and lidar-based LWP might have been caused by the lack of sensitivity of radar and lidar in detecting tenuous stratocumulus clouds.

The proposed algorithm based on the electromagnetic simulations of lidar and radar observations of the stratocumulus cloud has the potential for estimating cloud microphysical parameters, namely, RLED and LWC. The comparison with GVR-estimated LWP demonstrates the radar and lidar measurements are capable of estimating particle size and LWC for climate science and cloud process studies dominated by cloud liquid. The retrieval technique presented in this paper does not require a mathematical form of DSD or use of reflectivity threshold for differentiating between nonprecipitating and precipitating backscatter returns. Since the proposed technique uses the retrieved RLED and reflectivity, the uncertainty in the LWC estimation is reduced.

Nonprecipitating optically thin clouds were routinely observed during CSET campaign. The retrieval techniques mentioned in the literature review cannot be applied to these clouds as they are fundamentally different than the stratocumulus clouds usually used for developing retrieval techniques (Wood et al. 2012; O’Connor et al. 2005). Due to the low liquid water content of these clouds, the HSRL does not suffer from attenuation, thereby enabling retrievals from our proposed technique. In addition, the seamlessness of our technique is applicable to both precipitating and nonprecipitating clouds without making any inherent assumptions is novel. The HCR and HSRL measurements, in conjunction with in situ HIAPER instruments and dropsondes, have the potential to increase our understanding of cloud and precipitation processes through accurate retrievals of cloud and precipitation macro and microphysical properties.

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APPENDIX

List of Acronyms

- A: Radar attenuation
- CDP: Cloud Droplet Probe
- EOL: Earth Observing Laboratory
- GV: Gulfstream V
- GVR: G-band radiometer
- HCR: HIAPER Cloud Radar
- HIAPER: High-Performance Instrumented Airborne Platform for Environmental Research
- HSRL: High Spectral Resolution Lidar
- $|K|^2$: Dielectric factor
- LWC: Liquid water content
- LWP: Liquid water path
- MSL: Mean sea level
- NCAR: National Center for Atmospheric Research
- NSF: National Science Foundation
- PMS 2D-C: Particle Measuring Systems 2D-Cloud probe
- RLED: Radar–lidar-estimated diameter
- Z: Radar reflectivity
- $\alpha$: Lidar extinction
- $\beta$: Lidar backscatter
- $\lambda_o$: Transmitted wavelength
- $\sigma_D$: Backscatter cross section
- $\sigma_e$: Extinction cross section

REFERENCES

Albrecht, B., and Coauthors, 2019: Cloud System Evolution in the Trades (CSET): Following the evolution of boundary layer cloud systems with the NSF–NCAR GV. *Bull. Amer. Meteor. Soc.*, 100, 93–121, [https://doi.org/10.1175/BAMS-D-17-0180.1](https://doi.org/10.1175/BAMS-D-17-0180.1).


