Investigation of Compression and Reconstruction Methods for Solar Radiation Spectra above, within, and below the Sea Ice in Polar Environments

HANGZHOU WANG, LIWEN NAN, XIAOPING WANG, AND JIUZHOU TU

Ocean College, Zhejiang University, Zhoushan, Zhejiang, China

YING CHEN

State Key Laboratory of Fluid Power and Mechatronic Systems, Zhejiang University, Hangzhou, and Ocean College, Zhejiang University, Zhoushan, Zhejiang, China

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ABSTRACT

To reduce the amount of data transmitted, while retaining essential information on the original signal after reconstruction, this study investigated three different compression and reconstruction methods for the solar irradiance signals at different levels of the Arctic sea ice. To achieve this, characteristics of solar irradiance at different depths of the Arctic sea ice were analyzed; the solar irradiance profile was measured using a fiber optic spectrometer system at several ice packs from 11 to 25 August 2018, during the ninth Chinese National Arctic Research Expedition. Based on these measurements, three potential methods—fast Fourier transform (FFT), discrete wavelet transform (DWT), and Prony's method—were selected and applied for the compression and reconstruction of a group of 12 measurements from one profile. The results indicated that the FFT method was generally superior to the DWT method considering the corrected reconstruction rate R_c , under the same compression ratio C . The FFT method had the highest R_c for signals with higher roughness, whereas Prony's method was more suitable for smoother signals. To ensure that R_c was at least 80% and C was as large as possible, the compression and reconstruction method proposed in this study included the FFT method ($C = 6.4$) for the signals rougher than 1.5 and Prony's method employing the matrix pencil method ($C = 8$) for the smoother signals. The proposed method was successfully verified using an independent group of solar irradiance measurements from another profile in the Arctic sea ice.

1. Introduction

Solar radiation is an influential factor for the polar sea ice environment. Seasonal and annual variations of solar radiation play a significant role in governing the growth and retreat of sea ice in the polar region (Perovich et al. 2007; Serreze et al. 2007; Light et al. 2008). The availability of solar radiation also determines the development and community structure of the algae distributed within and under the sea ice (Arrigo et al. 2008; Boetius et al. 2013). Conversely, a change in the sea ice and its inclusions (e.g., ice algae) will alter the optical properties of sea ice, thereby reshaping the distribution of solar radiation in the sea ice environment.

The interaction between solar radiation and polar sea ice has attracted significant interest in the field of polar research. This is especially important, considering the changes in the Arctic sea ice during recent decades. These changes have manifested in the shrinking of the ice extent, reduction of ice thickness, and decrease in the age of the ice (Comiso 2002, 2006; Serreze et al. 2007). Advancing our knowledge of the solar radiation distribution in the Arctic sea ice, especially via long-term in situ observations, will improve our understanding of the issue and help predict variations in the sea ice. It will also facilitate estimations of the ice algae biomass and the primary production in the Arctic Ocean.

A kind of fiber optic spectrometry system was developed and refined to enable long-term in situ measurements of solar irradiance profiles in the Arctic sea ice. Multiple fiber probes were deployed at different depths of the sea ice, to collect and transmit the solar irradiance signal, using miniature spectrometers to detect the signal (Wang et al. 2014; Nan et al. 2017; Wang et al. 2019). The measured

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Corresponding author: Xiaoping Wang, xpwang@zju.edu.cn

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data were transmitted remotely via an Iridium modem (e.g., Iridium Satellite, LLC, Iridium 9602). Iridium communication is costly (more than \$1500 per megabyte); moreover, this issue is amplified in long-term time series applications, when the number of measurements within one profile (i.e., the amount of depth measured) and the spectral bands of one measurement increase. Various approaches should be investigated and adopted to reduce the amount of data that need to be transmitted remotely (i.e., to reduce the communication fee), while retaining the essential information of the original measurement.

Data compression and reconstruction methods are powerful and effective approaches for significantly reducing the data stored or transmitted, while keeping most of the useful information in the original data. These methods are used broadly in the field of audio signal processing, image signal processing, etc. (Li et al. 2011; Sandryhaila and Moura 2013; Zhu et al. 2018). The widely applied compression and reconstruction technologies that are currently used could be primarily divided into two classes: the Fourier-based methods and the complex exponential model-based methods (Bracewell 1986; Meurant 2012; Sayood 2012; Hu et al. 2013). The former mainly include such methods as fast Fourier transform (FFT) and discrete wavelet transform (DWT), while the latter primarily include Prony's method. These algorithms have been widely used for biomedical signal filtering, image data compression, seismic signal compression, and power quality analysis (Telagarapu et al. 2011; Reza et al. 2012; Fajardo et al. 2015; Rodríguez et al. 2018).

Based on the available literature, there are well-developed approaches and models [e.g., those utilizing artificial neural networks (ANN)] for predicting the solar radiation falling on the surface of Earth (Mummadisetty et al. 2015). However, to the best of our knowledge, there are no studies utilizing compression and reconstruction methods for the measurement of the solar radiation spectrum within polar sea ice. In this study, three methods, including FFT, DWT, and Prony's method, were applied to the solar irradiance signals, measured by the same type of a miniature spectrometer as that used in the fiber optic spectrometry system which was developed to enable in situ measurements of solar irradiance distribution in the Arctic sea ice environment over seasonal scales (Wang et al. 2019). The performance of each method was evaluated and compared to obtain the optimal solution for compressing and reconstructing the solar irradiance signals distributed at different levels of the Arctic sea ice.

2. Methods and materials

a. Spectrometry system overview

A brief overview of the fiber optic spectrometry system is repeated here, despite having been described

FIG. 1. Physical arrangement of the fiber optic spectrometry system.

previously (Fig. 1; Wang et al. 2019). The system was developed to enable in situ measurements of the solar irradiance distribution in the Arctic sea ice over seasonal scales. Eight fiber probes included in the system could be deployed at eight different levels of the sea ice, to collect and transmit the solar irradiance signals. Two miniature spectrometers (Hamamatsu Photonics K. K., C11009MA visible spectrometer and C11010MA near-infrared spectrometer) were used to measure the signals. Each spectrometer had 256 pixels, with the visible spectrometer having a spectral range of 340–780 nm, and the near-infrared spectrometer having a spectral range of 650–1050 nm. The combination of the two spectrometers could cover the spectral range of 350–1000 nm, which corresponded to 370 pixels for the spectrometers. The analog output of each spectrometer (i.e., the output corresponding to each pixel) was digitalized using a 16-bit analog-to-digital converter (ADC; Analog Devices Inc., AD7988-5). As a result, 740-byte data were generated for measuring one fiber probe, and 5920-byte data were generated for measuring all eight fiber probes. When the spectrometry system was operated multiple times a day and all the measurements were transmitted, the communication fee increased accordingly, which could restrict the wide application of this system. It is worth mentioning that these spectral measurements were primarily used to investigate the partitioning of solar radiation in the sea ice and the relevant optical properties of the sea ice (i.e., diffuse attenuation coefficient, extinction coefficient, and spectral transmittance, etc.). There is no need for extremely high accuracy of the measured data for a single band, but rather, the overall spectral shape of

FIG. 2. Schematic diagram of the device for measuring solar irradiance profile in the Arctic sea ice.

the measurement (e.g., where the peaks or valleys are located) is of interest. Hence, it is crucial to identify a simple and effective compression and reconstruction method—one that can allow minimal spectral measurement data to be transmitted remotely, without losing essential information, and, as importantly, to reconstruct the transmitted data (i.e., the compressed data) to reveal important features.

b. Measurement of solar irradiance profile in the Arctic sea ice

The variation of the solar irradiance at different levels of the Arctic sea ice poses significant challenges for the selection and development of the compression and reconstruction method. To understand typical characteristics of solar irradiance in the Arctic sea ice, we used the same type of a miniature spectrometer (i.e., C11009MA) equipped with a fiber probe that was built and calibrated to measure the solar irradiance distribution in several Arctic ice packs (thicker than 1 m). Measurements were taken from 11 to 25 August 2018, during the ninth Chinese National Arctic Research Expedition. The weather was cloudy, the humidity and temperature were higher than 95% and approximately 1°C separately during those days. The downwelling irradiance profile was obtained by gradually lowering the fiber probe down a 5-cm-diameter inclined auger hole (having a tilt angle of 45°) with a distance interval of 10 cm along the axis direction of the hole, in order to collect and transmit the solar irradiance signal which was detected by the spectrometer (Fig. 2). To monitor the potential intensity fluctuation of the incident solar radiation during the profiling measurement, a radiometrically calibrated hyperspectral irradiance sensor (TriOS Mess-und Datentechnik GmbH, RAMSES-ACC-VIS) was used and deployed approximately 1.2 m above the ice surface, to measure the downwelling irradiance.

In addition, 12 measurements from one profile collected at an ice pack with thickness of 1.7 m (no snow) on 25 August 2018 were used for the development of the compression and reconstruction method. Meanwhile, one profile measured at another ice pack with thickness of 1.55 m (covered by a thin layer of melting snow) on 13 August 2018, was randomly adopted to verify the performance of the developed method.

c. Selection and introduction of the compression and reconstruction methods

Generally, the characteristics of a signal are the most important factors when selecting appropriate compression and reconstruction methods. In this study, the measured solar irradiance was sampled pixel by pixel in equal time intervals and then digitized into discrete signals. Three types of methods are generally used for the compression and reconstruction of these aperiodic damped discrete signals. The first type is linear time–frequency analysis. The typical representatives are Fourier-based analysis and wavelet analysis, which are currently the most widely used methods in a variety of fields. The second type is the quadratic time–frequency analysis, which is typically represented by the Radon–Wigner transformation. However, this method has a limitation, as it is only applicable to specific signals such as chirp signals; hence, its applicability is primarily limited to medical and radar imaging. The third type is the parametric time–frequency analysis, which is typically represented by the Hilbert–Huang transformation and Prony-like methods. However, the former is widely used for denoising medical and seismic wave signals, but rarely used for signal compression and reconstruction, especially for a large compression ratio. In addition, a few complex algorithms, such as neural networks, can deal with these nonstationary discrete signals (Zhang and Benveniste 1992; Phooi and Ang 2006). However, these methods require a significantly larger sample size and a larger input of computing resources, which were not available or economical for this study. Therefore, to investigate solar irradiance signals, i.e., the compression and reconstruction of these signals, we selected and evaluated three methods: FFT method, DWT method, and Prony's method.

1) FFT METHOD

Any infinite signal x that is a periodic function of variable t can be expressed as the sum of a Fourier series:

$$x(t) = a_0 + 2 \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T} \right), \quad (1)$$

where a_n and b_n are the Fourier coefficients which can be calculated via Fourier transform, n is the wavenumber,

and T is the period of the signal. Based on Euler's formula,

$$e^{i\theta} = \cos\theta + i\sin\theta. \quad (2)$$

Equation (1) can be equivalently expressed as an infinite sum of exponentials as

$$x(t) = \sum_{n=-\infty}^{\infty} A_n e^{i2\pi nt/T}, \quad (3)$$

where $A_n = (1/T) \int_0^T x(t) e^{-i2\pi nt/T} dt$.

In this study, the measured solar irradiance signal $x(t)$, that consists of 256 points, was sampled in equal space. Therefore, $x(t)$ can be replaced by $\{x(k)\}$, where $k = 0, 1, 2, \dots, N-1$, $N = 256$. Let $t = kT/N$, substitute t for A_n , then A_n can be replaced by $\mathbf{X}(j)$, as follows:

$$\mathbf{X}(j) = \frac{1}{N\Delta t} \sum_{k=0}^{N-1} x(t) e^{-i2\pi jk/N} \Delta t = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{-i2\pi jk/N}, \quad (4)$$

where $j = 0, 1, 2, \dots, N-1$. Let $\omega = e^{-i2\pi/N}$ which is the n -times primitive unit root, Eq. (4) can be expressed as

$$\mathbf{X}(j) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x(k) \omega^{jk}. \quad (5)$$

Equation (5) is also the discrete Fourier transform (DFT)-based representation of the measured spectral data. In practice, matrices are often used in numerical calculations, and $\mathbf{X}(j)$ can also be treated as an N -dimensional vector $\mathbf{X} = [X_0, \dots, X_{N-1}]$. According to unit root properties, Eq. (5) can be expressed as

$$\begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ \vdots \\ X_{N-1} \end{bmatrix} = \frac{1}{\sqrt{N}} \begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \dots & \omega^0 \\ \omega^0 & \omega^1 & \omega^2 & \dots & \omega^{N-1} \\ \omega^0 & \omega^2 & \omega^4 & \dots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{N-1} & \omega^{2(N-1)} & \dots & \omega^{(N-1)^2} \end{bmatrix} \times \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix}. \quad (6)$$

The complexity of the DFT algorithm is $O(N^2)$. To overcome the shortcoming of low calculation efficiency of the DFT method, a widely used FFT algorithm named the Cooley–Tukey FFT algorithm was adopted in this study to speed up the calculations. This algorithm can also reduce the complexity from $O(N^2)$ to $O(N \log N)$.

FIG. 3. Flowchart of data compression and reconstruction using the FFT method.

The Cooley–Tukey FFT algorithm allows a split of the summation interval into even and odd parts:

$$\mathbf{X}(j) = \sum_{k=0}^{N/2-1} x_k^{\text{even}} \omega^{2jk} + \omega^{jk} \sum_{k=0}^{N/2-1} x_k^{\text{odd}} \omega^{2jk}. \quad (7)$$

As a compression and reconstruction method, FFT can be applied to most of the nonstationary signals (including the solar irradiance signal investigated in this study). The practical implementation steps of the FFT method are shown in Fig. 3. First, $\mathbf{X}(j)$ was calculated according to Eq. (6), and the frequency domain information of the spectral signal was obtained. Second, we constructed a new N -dimensional vector $\mathbf{S}(j)$, whose elements contained the first l frequency components arranged in $\mathbf{X}(j)$, and the remaining elements of $\mathbf{S}(j)$ were supplemented with 0. Finally, we applied the inverse FFT (iFFT) to $\mathbf{S}(j)$ to obtain the reconstructed spectral signal $Y(t)$.

2) DWT METHOD

Like the Fourier transform, wavelet transform is also a linear transformation which works well for the compression and denoising of complex signals and images. However, the main difference between the two methods is that the wavelet transform is localized in both time and frequency, whereas the Fourier transform is only localized in frequency, because the basis function of the wavelet transform (called mother wavelet) is different from that of the Fourier transform (i.e., sine or cosine wave). The wavelet transform primarily aims to conduct a multiscale analysis of the signal by scaling and translating the mother wavelet, which is usually a kind of small orthogonal wavelet basis with a certain scale. As

FIG. 4. Example of multiscale analysis of the signal by translating and scaling wavelet basis using wavelet transform.

illustrated in Fig. 4, when the wavelet basis $\Psi(t)$ is stretched to $\Psi(t/2)$, it can be used to detect the general characteristics of the signal. However, when the $\Psi(t)$ is compressed to $\Psi(2t)$, it is better to detect the sharper features of the signal. In addition, when the $\Psi(t)$ is translated to another place [e.g., $\Psi(t - k)$, $\Psi(t - 2k)$], the time information of the signal can be observed. Since the mother wavelet is localized in both time and frequency, the wavelet transform can identify not only the existence but also the occurrence time of a certain signal frequency. Therefore, wavelet transform has the ability of dealing with signals having discontinuities and sharp peaks (involving the solar irradiance signal).

In this study, we adopted the DWT method rather than continuous wavelet transform, because it was superior in terms of the redundancy of information representation and calculation speed (Meurant 2012). In general, the relationship between discrete wavelet transform ($DW_{\psi,x}(j, k)$) and wavelet $\psi_{j,k}(t)$ can be expressed as

$$(DW_{\psi,x})(j, k) = \langle x(t), \psi_{j,k}(t) \rangle = \int_{-\infty}^{\infty} x(t) \psi_{j,k}(t) dt, \quad (8)$$

where the signal $x(t) \in L^2(R)$, and mother wavelet $\psi_{j,k}(t)$ is given by

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k), \quad (9)$$

where j is scale, and k is translation in time. Meanwhile, the admissibility condition is required for the wavelet:

FIG. 5. Flowchart of signal decomposition and reconstruction processes with the corresponding wavelet coefficient, based on the DWT method.

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty, \quad (10)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, and ω is the frequency. The complexity of the DWT algorithm is $O(Nj)$.

In addition, a fast wavelet transformation technique named multiresolution analysis (MRA) was used to decompose the solar irradiance signal (Mallat 1989). Based on the MRA, the signal $x(t)$ can be quickly decomposed into several scales (i.e., six scales in this study; Fig. 5). At each scale, the signal can be decomposed into two new independent signals of low-frequency component cA_j (scale $j = 1-6$) and high-frequency component cD_j (scale $j = 1-6$) (Fig. 5) by separately passing through a low-pass filter g and a high-pass filter h with impulse response:

$$cA_j[t] = \sum_{k=-\infty}^{\infty} x(t)g(jt - k), \quad (11)$$

$$cD_j[t] = \sum_{k=-\infty}^{\infty} x(t)h(jt - k). \quad (12)$$

In general, the low-frequency component contains the most information from the original signal, while the high-frequency component contains the local details. Therefore, even if the high-frequency components are removed, most of the essential information of the original signal can still be retained.

To reduce the volume of data transmission (i.e., the volume of the data following the compression process) without affecting the integrity of the signal reconstruction, the high-frequency components (i.e., F_{cD_j} , $j=1-6$) were filtered to 0, and the low-frequency components (i.e., cA_6) were retained. Based on these parameters, a reconstructed signal $Y(t)$ could be obtained using the inverse discrete wavelet transform (iDWT).

3) PRONY'S METHOD USING MATRIX PENCIL

The modern Fourier analysis has a drawback in that it is often difficult to ensure a sufficiently high frequency resolution when this method is used to approximate a finite aperiodic signal. This issue can be addressed by Prony's method due to its reliance on autoregressive modeling. Prony's method approximates a sequence of equally spaced samples using a linear combination of p complex exponential functions with different amplitudes, damping factors, frequencies, and phase angles. Therefore, the discrete signal $x[n]$ can be represented by

$$x[n] = \sum_{k=1}^p A_k e^{j\theta_k} e^{(\alpha_k + j2\pi f_k)T_s(n-1)} = \sum_{k=1}^p h_k z_k^{(n-1)} \quad (13)$$

$$\mathbf{Y} = \begin{bmatrix} x[1] & x[2] & \cdots & x[p] & x[p+1] \\ x[2] & x[3] & \cdots & x[p+1] & x[p+2] \\ x[3] & x[4] & \cdots & x[p+2] & x[p+3] \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x[N-p] & x[N-p+1] & \cdots & x[N-1] & x[N] \end{bmatrix}_{(N-p) \times (p+1)}. \quad (14)$$

Another two matrixes $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are constructed by eliminating the last and the first column of $\mathbf{Y}$ separately:

$$\mathbf{Y}_1 = \begin{bmatrix} x[1] & x[2] & \cdots & x[p] \\ x[2] & x[3] & \cdots & x[p+1] \\ x[3] & x[4] & \cdots & x[p+2] \\ \vdots & \vdots & \ddots & \vdots \\ x[N-p] & x[N-p+1] & \cdots & x[N-1] \end{bmatrix}_{(N-p) \times p}, \quad (15)$$

$$\mathbf{Y}_2 = \begin{bmatrix} x[2] & \cdots & x[p] & x[p+1] \\ x[3] & \cdots & x[p+1] & x[p+2] \\ x[4] & \cdots & x[p+2] & x[p+3] \\ \vdots & \ddots & \vdots & \vdots \\ x[N-p+1] & \cdots & x[N-1] & x[N] \end{bmatrix}_{(N-p) \times p}. \quad (16)$$

where A_k is the initial amplitude, θ_k is the initial phase (radians), α_k is the damping factor (s^{-1}), f_k is the frequency (Hz), T_s is the sampling period (s), h_k is the time-independent component, and z_k is an exponential and time-dependent component.

The traditional Prony's method generally requires three steps to solve for h_k and z_k (Rodríguez et al. 2018):

- Step 1: Solve the linear prediction model to get the coefficients of the characteristic.
- Step 2: Calculate the roots of the characteristic formed from the linear prediction coefficients vector.
- Step 3: Solve the original set of linear equations to yield the estimates of the exponential amplitude and sinusoidal phase.

In this study, a kind of Prony's method called matrix pencil method was used to compress the measured solar irradiance data. This method solves the generalized eigenvalue problem we encountered that cannot be solved by most traditional methods (such as the least squares method and total least squares method). In this method, parameters h_k and z_k can be calculated by the following steps:

Step 1: A rectangular Hankel matrix $\mathbf{Y}$ is built based on the original signal $x[n]$:

Step 2: Obtain the values of z_k from the following formula:

$$z_k = \text{eigenvalues}(\mathbf{Y}_1^+ \mathbf{Y}_2), \quad (17)$$

where $\mathbf{Y}_1^+$ is defined as

$$\mathbf{Y}_1^+ = [\mathbf{Y}_1^H \mathbf{Y}_1]^{-1} \mathbf{Y}_1^H. \quad (18)$$

The damping factor α_k and frequency f_k can be computed as follows:

$$\alpha_k = \frac{\ln|z_k|}{T_s}, \quad (19)$$

$$f_k = \frac{\tan^{-1} \left[\frac{\text{Im}(z_k)}{\text{Re}(z_k)} \right]}{2\pi T_s}. \quad (20)$$

Step 3: Obtain the values of h_k by solving the equation:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_p^0 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{p-1} & z_2^{p-1} & \cdots & z_p^{p-1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[p] \end{bmatrix}. \quad (21)$$

Amplitude A_k and phase θ_k can be computed as follows:

$$A_k = |h_k|, \quad (22)$$

$$\theta_k = \tan^{-1} \left[\frac{\text{Im}(h_k)}{\text{Re}(h_k)} \right]. \quad (23)$$

The main advantage of this method is that it is a natural transformation for impulse response and can be expressed as a function based on damped sinusoids. The complexity of the Prony's algorithm is $O(p^{5/3} \log^2 N)$. Following the compression process, the data that need to be transmitted are the calculated parameters h_k and z_k . Correspondingly, the signal can be reconstructed based on these transmitted parameters and Eq. (13).

d. Criteria for evaluating method performance

To investigate the optimal solution for compressing and reconstructing the spectral measurements based on the three methods described above (i.e., the FFT, DWT, and Prony's method), we must select the relevant criteria to evaluate and compare the performance of each method. Two parameters of compression ratio and reconstruction precision were chosen as the evaluation criteria. As these are opposing parameters, i.e., one parameter becomes better when the other one becomes worse, balancing these is important.

The compression ratio is a quality index to evaluate the compression efficiency of a data compression method. It could be defined as the volume ratio of the original signal and the signal after compression; it is expressed as

$$C = \frac{\text{Byte}(\text{Ori})}{\text{Byte}(\text{Com})} = \frac{512}{\text{Byte}(\text{Com})}, \quad (24)$$

where C is the compression ratio, and $\text{Byte}(\text{Ori})$ is the bytes of the original signal; its value was 512 bytes in this study. $\text{Byte}(\text{Com})$ is the bytes of the compressed signal, which is different for different methods.

For the FFT method, the compression ratio mainly depends on the number of signal frequency components l retained after the compression processes. Each frequency component is a complex number, which is

composed of a real part and an imaginary part. Each part requires 2 bytes for storage. Meanwhile, one extra byte is needed to record the position of each reserved point in the frequency domain. Therefore, the total number of bytes of the compressed data is

$$\text{Byte}(\text{Com})_{\text{FFT}} = (2 + 2 + 1)l = 5l. \quad (25)$$

Additionally, according to the characteristics of the unit root, the frequency domain data are distributed symmetrically. This means that we only need to transmit half of the compressed data. Therefore, the compression ratio of the FFT method can be calculated by

$$C = \frac{512}{\text{Byte}(\text{Com})_{\text{FFT}}/2} = \frac{204.8}{l}, \quad (26)$$

where l is separately assigned to be 128, 64, 32, 16, 8, and 4, for comparison.

The compression ratio of the DWT method depends on the number of decomposition scale j , and the data following the compression, which is only the low-frequency component of each scale after decomposition. As with the FFT method, the data that need to be transmitted consist of real and imaginary parts, with each part occupying 2 bytes. Thus, the compression ratio of the DWT method can be calculated as follows:

$$C = \frac{512}{(2+2)p_j} = \frac{128}{p_j}, \quad (27)$$

where p_j is the number of data to be transmitted, which is separately assigned to be 132, 70, 39, 24, 16, and 12, corresponding to the scale j from 1 to 6.

Finally, for Prony's method, the compression ratio is primarily determined based on the number of order (i.e., the number of complex exponential functions) p and the two parameters z_k and h_k . As each of the two parameters occupies 2 bytes, the compression ratio can be calculated as follows:

$$C = \frac{512}{p \times (2+2)} = \frac{128}{p}, \quad (28)$$

where p is assigned to be 128, 64, 32, 16, 8, and 4 in this study.

The reconstruction precision represents how close the reconstructed signal $\hat{x}[n]$ is to the original signal $x[n]$. This was evaluated via the normalized root-mean-square error $R[n]$:

$$R[n] = 1 - \frac{\|x[n] - \hat{x}[n]\|}{\|x[n] - \bar{x}\|}, \quad (29)$$

FIG. 6. Intensity of solar irradiance as a function of spectrometer pixel above, within, and under the Arctic sea ice. The number in the legend indicates the depth of the spectral measurement in the sea ice.

where $\bar{x}$ is the mean of the original signal. In this study, if $R[n]$ had a value of no less than 80%, it was considered that the original signal $x[n]$ was successfully reconstructed. When the total number of correctly reconstructed signals n_c was no smaller than 205 ($256 \times 80\% \approx 205$), we considered that the reconstructed signals were in good agreement with the 256 original signals. Therefore, the overall reconstruction

precision (i.e., the corrected reconstruction rate) R_c could be evaluated by the ratio of n_c and 256:

$$R_c = \frac{n_c}{256} \times 100\%. \quad (30)$$

In addition, although the general shape of the solar irradiance in the Arctic sea ice is very similar, the solar spectrum in the upper position of the sea ice has more significant peaks and valleys than that in the lower position of the sea ice (Wang et al. 2019). These characteristics may also influence the performance of the compression and reconstruction methods. Therefore, another parameter of roughness of the solar spectrum was used in this study, to facilitate the selection of the appropriate method for the signals collected from different depths of the sea ice. Roughness of the signal R_a is defined as

$$R_a = \frac{\text{std}(x[n+1] - x[n])^2}{\text{std}(t[n+1] - t[n])^2}, \quad (31)$$

where $t[n]$ is the smoothed curve of $x[n]$ based on three-point sliding filtering, $n = 1, \dots, 255$. The bigger R_a is,

FIG. 7. Original and reconstructed solar irradiance signal as a function of spectrometer pixel for five different signal roughness at six compression ratios, based on the FFT method.

FIG. 8. Corrected reconstruction rate R_c as a function of signal roughness for six compression ratios, based on the FFT method.

the rougher the spectrum is (i.e., including more significant peaks and valleys).

3. Results and discussion

a. Typical characteristics of solar irradiance in the Arctic sea ice

In the process of analyzing the spectral measurements collected from 11 to 25 August 2018, during the ninth Chinese National Arctic Research Expedition, we corrected the influence of any fluctuations in the incident solar radiation for each profile by referencing the corresponding spectral irradiance outputted by the RAMSES-ACC-VIS to the values obtained in the first measurement. Subsequently, we calibrated the wavelength of the spectrometer λ using a five-order polynomial function provided by the vendor, as follows:

$$\lambda(p_{ix}) = c_0 + c_1 p_{ix} + c_2 p_{ix}^2 + c_3 p_{ix}^3 + c_4 p_{ix}^4 + c_5 p_{ix}^5, \quad (32)$$

where p_{ix} is the pixel number of the spectrometer, and c_0 to c_5 are the fitted coefficients. The results showed that the spectral shape of the solar irradiance incident on the ice surface was highly similar for each profile during the 14 days (solid line in Fig. 6), even though spectral intensity of the solar irradiance varied significantly, due to the different measurement times and weather conditions. It had the highest value at approximately 460 nm (i.e., pixel 72) and decreased at both sides, with shorter and longer wavelengths (solid line in Fig. 6). The valleys that appeared at the spectrum were due to the absorption of some substances (i.e., O_3 , O_2 , H_2O , CO_2 , etc.) during the transmission from the sun to the ice surface. For each profile, the general shape of the solar irradiance was remarkably similar for all the measurements (Fig. 6). However, the spectral intensity decreased

TABLE 1. Performance of the FFT method with respect to three demonstrative signal roughness. The boldface cells indicate R_c with a value no smaller than 80% and the corresponding compression ratio C .

Condition	R_c (%)	C
Roughest ($R_a = 2.035$)	97.66	1.6
	96.48	3.2
	93.36	6.4
	78.52	12.8
	73.05	25.6
	47.66	51.2
Normal ($R_a = 1.490$)	96.09	1.6
	87.89	3.2
	82.42	6.4
	64.84	12.8
	50.78	25.6
	31.25	51.2
Smoothest ($R_a = 1.401$)	98.83	1.6
	94.92	3.2
	91.41	6.4
	64.45	12.8
	59.77	25.6
	28.13	51.2

significantly as position deepened, with a much faster decrease in the near-infrared region than in the visible region. The solar spectrum became much smoother with the increasing depth of the sea ice. This can be explained by the attenuation characteristics of the sea ice (Granskog et al. 2015). In addition, even though intensity of the solar irradiance changes remarkably in different seasons, the general shape of the solar irradiance incident on the sea ice is highly similar throughout the year because the weather in the Arctic sea ice environment is predominantly cloudy and humid (Treffeisen et al. 2007). This characteristic lays a solid foundation for the proper functioning of the proposed compression and reconstruction method in the Arctic sea ice, potentially over seasonal scales.

b. Performance evaluation of the compression and reconstruction methods

From the 12 measurements collected at an ice pack on 25 August 2018, we found that the roughness of the solar irradiance signal R_a generally decreased with an increase in the depth of the sea ice, from 2.035 at the ice surface to 1.401 in the water (Fig. 7a). This phenomenon can be attributed to the fact that the absorption coefficient of the sea ice in the wavelength longer than roughly 650 nm (i.e., pixel 175 in Fig. 7a) was far bigger than that in the shorter wavelengths (i.e., the visible bands shorter than 650 nm). Meanwhile, the peaks and valleys in the solar spectrum on the ice surface which contributed significantly to the value of R_a were predominantly located in bands longer than roughly 650 nm and would be attenuated quickly as the

FIG. 9. Original and reconstructed solar irradiance signal as a function of spectrometer pixel for five different signal roughness at six compression ratios, based on the DWT method.

signal transmitting in the sea ice. Following the application of the three compression and reconstruction methods to these spectral measurements, the performance of each method was evaluated, and is discussed below.

1) FFT METHOD

The results showed that, when the compression ratio was smaller than 6.4, the reconstructed signal matched well with the original signal, which was observed for each signal with a different roughness (Figs. 7a–c). It can be concluded that the compression and reconstruction method based on the FFT method was to some extent, effective in our study when the compression ratio was small. However, the deviation of the reconstructed signal from the original signal increased with increasing compression ratios, especially for those bigger than 6.4 (Fig. 7). This was caused by the loss of frequency components that contained most local information of the original signal (e.g., peaks and valleys). The bigger the compression ratio was, the higher the amount of local information that was lost. As a result, the reconstructed signal became smoother and the discrepancy between the reconstructed signal and the original signal became bigger with bigger compression ratios (Figs. 7d–f).

For each compression ratio, the general trend of the corrected reconstruction rate R_c with roughness of the original signal R_a was considerably similar (Fig. 8). Although R_c fluctuated significantly for roughness smaller than roughly 1.5, it initially approximately decreased with increasing roughness and reached a minimum value at 1.666. However, R_c reached the maximum value at 2.035. The value of R_c was greater than 80% for all roughness, when the compression ratio was less than 3.2. However, it decreased significantly as

FIG. 10. Corrected reconstruction rate R_c as a function of signal roughness for six compression ratios, based on the DWT method.

TABLE 2. Performance of the DWT method with respect to three demonstrative signal roughness. The boldface cells indicate R_c with a value no smaller than 80% and the corresponding compression ratio C .

Condition	R_c (%)	C
Roughest ($R_a = 2.035$)	98.05	1.0
	96.48	1.8
	78.13	3.3
	62.50	5.3
	60.55	8.0
	20.70	10.7
Normal ($R_a = 1.490$)	99.22	1.0
	98.83	1.8
	96.09	3.3
	86.72	5.3
	82.03	8.0
	41.80	10.7
Smoothest ($R_a = 1.401$)	98.44	1.0
	98.05	1.8
	96.48	3.3
	87.50	5.3
	84.77	8.0
	45.70	10.7

the compression ratio increased, from an average value of 82.87% at 6.4% to 30.38% at 51.2. Detailed information regarding the relationship between R_c and the compression ratio with respect to the roughness is

presented in Table 1. Hence, we can conclude that, based on the requirement for R_c to be at least 80% in this study, the compression and reconstruction methods based on the FFT method were not appropriate for our application, when the compression ratio exceeded 6.4.

2) DWT METHOD

For the DWT method, it was observed that the reconstructed signal was in good agreement with the original signal, for compression ratios less than 1.8 (Figs. 9a,b). However, when the compression ratio exceeded 3.3, the reconstructed signal clearly deviated from the original signal. Like the FFT method, this deviation became more significant with increasing compression ratios (Figs. 9c–f). The corrected reconstruction rate R_c was relatively stable with different roughness for compression ratios less than 1.8 (Fig. 10). However, it roughly decreased with increasing roughness for greater compression ratios (i.e., those greater than 3.3), although R_c was relatively stable for roughness less than 1.5. The detailed relationship between R_c and the compression ratio for the three demonstrative levels of roughness is presented in Table 2. From the perspective of the value of R_c under the same compression ratio, the performance of the DWT method was not comparable to that of the FFT method

FIG. 11. Original and reconstructed solar irradiance signal as a function of spectrometer pixel for five different signal roughness at six compression ratios, based on Prony's method using matrix pencil.

FIG. 12. Corrected reconstruction rate R_c as a function of signal roughness for six compression ratios, based on Prony's method using matrix pencil.

discussed above (Figs. 8 and 10). This is likely due to the significantly larger amount of information redundancy that exists in the DWT method, as compared with that in the FFT method; this would lead to a relatively smaller compression ratio with the DWT method.

3) PRONY'S METHOD USING MATRIX PENCIL

The results from this method indicated that the general shape of the reconstructed signal for each level of roughness was still in good agreement with that of the original signal for the compression ratio, even as high as 16 (Figs. 11a–e), which was not the case for the FFT and DWT methods (Figs. 7 and 9). The corrected reconstruction rate R_c was smaller than 80% for almost all compression ratios when the roughness was bigger than approximately 1.5 (phase II in Fig. 12), which indicated that Prony's method was not effective in our study when the roughness of the signal was large (i.e., greater than approximately 1.5). However, R_c was very stable and was not very sensitive to the compression ratio with a mean value of 81.03%, when the roughness was smaller than 1.5 and the compression ratio was lower than 8 (phase I in Fig. 12; Table 3). That may be attributed to the fact that Prony's method is a natural transformation for impulse responses, which makes it more suitable for reconstructing smoothed curves rather than curves with sharp peaks or valleys. Additionally, the distribution of the polylines shown in Fig. 12 was much denser than that shown in Fig. 8 (i.e., for the FFT method) and in Fig. 10 (i.e., for the DWT method). This indicates that the correlation between the polylines at different compression ratios for Prony's method was higher than that for the DWT and FFT methods, which also means that, to some extent, Prony's method was much more stable than the other two methods for this study.

TABLE 3. Performance of Prony's method using matrix pencil with respect to three demonstrative signal roughness. The boldface cells indicate R_c with a value no smaller than 80% and the corresponding compression ratio C .

Condition	R_c (%)	C
Roughest ($R_a = 2.035$)	73.83	1
	73.43	2
	70.31	4
	47.27	8
	41.41	16
	39.84	32
Normal ($R_a = 1.490$)	87.11	1
	84.77	2
	85.16	4
	84.37	8
	70.31	16
	57.81	32
Smoothest ($R_a = 1.401$)	86.72	1
	85.16	2
	85.55	4
	84.77	8
	75.12	16
	57.81	32

c. Performance comparison of the compression and reconstruction methods

As mentioned above, when selecting the appropriate compression and reconstruction methods for this study, the primary concern was the corrected reconstruction rate R_c , which was set at 80% to allow the reconstructed signals to reflect most of the essential information of the original signals. Once this condition was satisfied, the compression ratio C was selected to be as high as possible to reduce the amount of data following the compression process. Based on these two criteria, we compared the performance of the three methods used in this study, as shown in Fig. 13. The results denoted that the FFT method was superior to the other two methods (i.e., the DWT method and Prony's method) when the roughness of the original signal was above 1.5, because the obtained compression ratio was much bigger for the FFT method than for the other two methods (the circled points in Figs. 13a–c). However, Prony's method was the best of the three methods for roughness below 1.5, considering the acquired compression ratio (the circled points in Figs. 13d–f).

Therefore, to simplify and facilitate the practical application, the FFT method was selected as the compression and reconstruction method for roughness of the signal above 1.5, and Prony's method using matrix pencil was selected for the smaller signal roughness (Table 4). It is worth mentioning that the exact compression ratio was still unclear, despite having selected a specific

FIG. 13. Corrected reconstruction rate R_c as a function of compression ratio for the three methods at six signal roughness R_a . The circled point in each panel indicates the biggest compression ratio of the three methods when R_c is set at 80%.

method for different signal roughness. We found that the obtained compression ratio of the FFT method ranged from 6.53 at roughness of 1.666 to 12.16 at roughness of 2.035, when R_c was set at 80% (Figs. 13a–c). Combined with the fact that the compression ratio of the FFT method was not random and was given by Eq. (26), this was set to be 6.4 in this study. Whereas, the obtained compression ratio for Prony’s method ranged from 8.12 at roughness of 1.456 to 11.95 at roughness of 1.401 (Figs. 13d–f). Since the theoretical value of the compression ratio was described by Eq. (28), the compression ratio of Prony’s method was 8. To sum this up, the compression and reconstruction methods selected in this study were the FFT-based method with the compression ratio of 6.4 (i.e., $l = 32$), when the roughness of the signal is above 1.5, and Prony’s method using matrix pencil, with the compression ratio of 8 (i.e., $p = 16$), for the smaller roughness values. It is worthy to mention that measurement noise was included in the spectral data used for the development of the three compression and reconstruction methods. From our previous study (Wang et al. 2017), the measurement noise which was dominated by the electronic noise of the spectrometer together with its driver electronics and the incident light shot noise, was less than approximately $1.56 \text{ mW m}^{-2} \text{ nm}^{-1}$. In view that the noise was small and primarily concentrated in the high-frequency band

(Liu et al. 2018), and the three proposed methods had the ability of noise suppression, the influence of the measurement noise could be negligible in this study.

d. Performance verification of the proposed compression and reconstruction methods

To verify the practical application and performance of the proposed compression and reconstruction methods, these methods were applied to a group of solar irradiance measurements (i.e., 15 measurements taken at 15 different depths of the sea ice) from one profile collected from an ice pack with a thickness of 1.55 m (covered by a thin layer of melting snow) on 13 August 2018. We observed that the general shape of the solar irradiance on the ice surface and within the sea ice (Fig. 14a) was in good agreement with the corresponding measurements on 25 August 2018 (i.e., those depicted in Fig. 6). The roughness of the solar irradiance signal decreased from

TABLE 4. The developed compression and reconstruction methods for different signal roughness conditions.

Condition	Method		
	FFT	DWT	Prony
$R_a \geq 1.50$	✓	×	×
$R_a < 1.50$	×	×	✓

FIG. 14. (a) Original and reconstructed solar irradiance signal as a function of spectrometer pixel for five signal roughness based on the FFT method ($R_a > 1.5$) and Prony's method using matrix pencil ($R_a < 1.5$), and (b) corrected reconstruction rate R_c as a function of signal roughness R_a . The five original curves in (a) are measured in the sea ice with depths of 0, 7.7, 15.3, 68.9, and 183.9 cm from the ice surface.

1.723 for the signal at the ice surface to 1.364 for the signal in the underlying ocean (Fig. 14a). Based on the compression and reconstruction method proposed above and the roughness of each signal, we applied the FFT method with a compression ratio of 6.4 and Prony's method using matrix pencil with a compression ratio of 8 for the signal roughness greater and less than 1.5, respectively. The results showed that the reconstructed signal matched well with the original signal for each measurement with different roughness. Moreover, the corrected reconstruction rate R_c was bigger than 80% for all roughness (i.e., all measurements), with a maximum of 86.41% at 1.512 and a minimum of 80.10% at 1.394 (Fig. 14b).

4. Conclusions

This study investigated compression and reconstruction methods for solar irradiance signals distributed at different depths of the Arctic sea ice. A fiber optic spectrometry system was developed and used to measure solar irradiance profile in the sea ice at several ice packs from 11 to 25 August 2018, during the ninth Chinese National Arctic Research Expedition. Based on the characteristics of the measured solar irradiance signal, three potential methods including FFT, DWT, and Prony's method, were applied to a group of 12 spectral measurements from one profile. To facilitate the evaluation and comparison of the performance of these methods, the two

parameters of compression ratio C and corrected reconstruction rate R_c were adopted and calculated as the key criteria. The study additionally introduced another parameter of roughness of the signal. The results showed that the reconstructed signal matched well with the original signal for all three methods when the compression ratio was small. However, considering the value of R_c under the same compression ratio conditions, the FFT method worked better than the DWT method. The FFT method was more appropriate for processing signals with larger roughness since R_c was bigger than that of the other two methods. In contrast, Prony's method was much more stable and more suitable for dealing with smoother signals. Based on the requirements that R_c should be at least 80% and the compression ratio should be as big as possible, we suggested that the optimal compression and reconstruction methods for our application were the FFT-based method for a compression ratio of 6.4 and the Prony's method using matrix pencil base for a compression ratio of 8, for the signal with roughness above and below 1.5, respectively. The proposed method was successfully verified by applying it to an independent group of 15 solar irradiance measurements from another profile. Future work will primarily focus on the seasonal field application of the developed algorithm in the Arctic sea ice and the investigation of other compression and reconstruction methods to further improve the compression ratio, while keeping the current corrected reconstruction rate configuration (i.e., R_c above 80%).

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