Estimating Nearshore Waves by Assimilating Buoy Directional Spectrum Data in SWAN

DAVID T. WALKER^a AND KELSEY BRUNNER^a

^a *SRI International, Ann Arbor, Michigan*

(Manuscript received 5 April 2021, in final form 4 August 2021)

ABSTRACT: This paper describes a variational data assimilation algorithm based on the Simulating Waves Nearshore (SWAN) wave spectrum model. The approach allows single-point wave spectrum observations to be used to estimate the wave field for a nearshore region under stationary conditions, assuming a spatially uniform incident wave spectrum at the offshore boundary. The assimilated data are in the form of Fourier directional coefficients, the standard output from operational wave buoys, and are used directly by incorporating the relationship between directional spectrum and the Fourier coefficients into the formulation. The algorithm was tested on data from nearshore buoys deployed off the coast of North Carolina in May 2012, and the estimated wave field is compared to both the input data and independent observation data. The results compare favorably to the independent data with overall RMS errors of 10%–20% for significant wave height, about half a second for mean wave period, and as much as three to four SWAN spectral grid cells for mean direction. Overall, the results show that the algorithm can be effectively used to estimate the offshore boundary spectrum and accurately reproduce wave conditions in the domain.

KEYWORDS: Waves, oceanic; In situ oceanic observations; Inverse methods; Numerical analysis/modeling; Variational analysis; Model comparison

1. Introduction

Mature and accurate computational models exist for many oceanographic applications; however, accurate model inputs, in particular boundary condition data, are not readily available in many cases of practical interest. In this study, we examine nearshore wave estimation using in situ observations of wave directional spectra for a domain that is small enough in cross-shore extent that the conditions can be treated as stationary. As is often the case, the available data are for locations in the interior of the spatial domain and not on the boundary, and we use a variational assimilation approach, along with contemporaneous observations for surface winds and tides, to estimate the offshore boundary condition. The model we use is Simulating Waves Nearshore (SWAN; Booij et al. 1999; Holthuijsen 2009), a wave-spectrum model, which implements conservation of the wave-action spectral density in shallow water. This approach allows us to leverage a few point observations to estimate the wave field for the region of interest, including the effects of variable bathymetry, tides, and winds.

Variational data assimilation approaches are highly developed for atmosphere and ocean forecasts (see, e.g., Bannister 2017), but are not yet widely used in ocean wave models like SWAN, although other simpler methods have been used. Optimal interpolation was employed with satellite-altimeter wave-height data for SWAN-based hindcasts for the Black Sea by Rusu (2015). Thanh et al. (2019) used an ensemble Kalman filter approach to assimilate satellite-altimeter wave heights in

the South China Sea. The performance of three wave models, including SWAN, was investigated by Li and Zhang (2020) in conjunction with an optimal interpolation assimilation scheme to adjust the resulting significant wave heights and wind forcing to be consistent with satellite altimeter observations.

Variational assimilation in the SWAN model was first used by Veeramony et al. (2010) based on the work of Walker (2006), where the adjoint only treated spatial and spectral advection and the source terms were ignored. They used the analytic adjoint of the wave action balance equations, derived using the methods of variational calculus (see, e.g., Bennett 2005), and implemented in a modified version of the SWAN code. Orzech et al. (2013, 2016) extended this approach with a new adjoint, including all the source terms, based on the discrete equations derived using parametric Fortran (Erwig et al. 2007). They examined a number of situations, both stationary and nonstationary, and demonstrated good skill when compared to independent spectrum observations.

Wave spectrum data are obtained from wave buoys or similar instruments that do not produce frequency–direction spectra directly, but instead provide the first five Fourier coefficients of the directional spectrum (Longuet-Higgins et al. 1963). For the variational assimilation studies cited above, buoy data were preprocessed to produce realizations of the frequency–direction spectrum on the SWAN spectral grid. Crosby et al. (2017), in an extension of the work by O'Reilly and Guza (1998), assimilated the Fourier coefficients from the U.S. Army Corps of Engineers Coastal Data Information Program (CDIP) buoys and the global WAVEWATCH III model predictions in a linear wave model that relies on the self-adjoint property of ray projection. Their linear approach is limited to swell waves ($f < 0.09$ Hz) as it is only applicable to wave conditions where dissipation and nonlinear spectral energy transfers are negligible. In the present work, the assimilation is also done using the Fourier coefficients directly by

Denotes content that is immediately available upon publication as open access.

Corresponding author: David Walker, david.walker@sri.com

incorporating a sensor model that relates the SWAN spectrum to the buoy directional coefficients, an approach first used with SWAN for assimilation of synthetic aperture radar data (Walker 2006). This approach eliminates the sometimes ad hoc assumptions inherent in estimating frequency–direction spectra from buoy directional coefficients.

The approach adopted here extends that of Veeramony et al. (2010) and relies upon application of the calculus of variations to the SWAN model equations, now including the source terms. A cost function is defined which quantifies the error in the predicted Fourier coefficients for the location(s) where the observations are available. An adjoint SWAN model equation is derived which has the difference between the predicted and observed spectrum as an input. The solution of the adjoint model yields the adjoint wave-action spectrum. From the adjoint spectrum at the incident-wave (seaward) boundary of the region, the gradient of the cost function with respect to the incident wave spectrum values can be calculated. The cost-function gradient is used together with a conjugate-gradient minimization algorithm to determine the incident-wave spectrum which minimizes the cost function. To ensure the uniqueness of the result, the cost function includes a term which “penalizes” the incident wave spectrum (Bennett and Miller 1991).

Here we wish to estimate the offshore incident wave spectrum, and we expect to have data from, at best, a few wave buoys. For coastal domains approaching mesoscale size (covering say 100 km of coastline) one could expect some variation in the offshore wave field. However, one cannot reasonably expect to be able to estimate a detailed, spatially varying incident spectrum using data from a few points in the interior. In the limit of a single spectrum observation, if we try to estimate a spatially varying offshore spectrum, we will get a narrow-direction spectrum at each boundary location corresponding to the wave rays that pass through the observation location and that boundary point. This would clearly be a nonphysical result. Requiring a spatially uniform offshore spectrum is likely the best one can do in that case, since a single spectrum observation only contains enough information to estimate a single (uniform) incident spectrum. In the present study, we assimilate data from two wave buoys, but choose to keep the incident spectrum uniform to provide redundancy in the data. Methods for relaxing this requirement are being explored at present in separate work for future publications.

As mentioned above, the approach used for this study solves a discrete form of the analytic adjoint, as opposed to the adjoint of the discrete equations (a la Orzech et al. 2013). Comparison of these differing approaches has been investigated quite thoroughly in a number of different application domains. As noted by Griesse and Walther (2004), the discrete adjoint provides the exact derivative of the discrete approximation of the cost function, not an approximation of the exact derivative of the cost function (what the analytic adjoint attempts), so these are different but related approaches. Recent work (Economou et al. 2017; Delay et al. 2017) has concluded that as long as the analytic adjoint is solved accurately and with sufficient resolution, the results from the two approaches are comparable.

In the next section, the SWAN model is described first. Then the cost function is defined and the adjoint SWAN model is derived, along with an expression for the gradient of the cost function with respect to the incident-wave spectrum. After that, the algorithm is applied to real-world data and the results are compared to independent observations.

2. The wave model and observation data

The algorithm used here is based on the open-source SWAN wave spectrum model and makes use of buoy directional spectrum data to estimate boundary data for the model. We first briefly describe the model, and then discuss how the data used relate to the modeled wave directional spectrum. We then derive the inverse model at the core of the algorithm, followed by a discussion of the algorithm implementation.

a. The SWAN wave spectrum model

SWAN is a third-generation wave model for coastal regions (Ris et al. 1999; Booij et al. 1999). The basic equation used is the spectral balance equation for wave action $N(\mathbf{x}, \mathbf{s}, t) = E(\mathbf{x}, \mathbf{s}, t)/\sigma$, where E is the variance density spectrum, σ is the intrinsic wave radian frequency, $\mathbf{x} = (x, y)$ is the spatial location, $\mathbf{s} = (\sigma, \theta)$ is the spectral location, and t is time. The action balance equation is given by

$$\frac{\partial N}{\partial t} + \tilde{\mathbf{V}} \cdot (\tilde{\mathbf{C}}N) = \frac{S}{\sigma}, \quad (1)$$

where $\tilde{\mathbf{V}} = (\partial/\partial x, \partial/\partial y, \partial/\partial \sigma, \partial/\partial \theta)$, and $\tilde{\mathbf{C}} = (C_x, C_y, C_\sigma, C_\theta)$ are the wave-energy propagation velocities in physical and spectral space, consisting of wave group velocities plus ocean currents for physical space and the effects of refraction and wave–current interaction for spectral space. These are described in detail in Booij et al. (1999), as is the source term on the right-hand side of (1). This source term includes models for the effects of wind growth and loss of energy due to whitecapping, bottom friction, and depth-induced breaking, as well as energy transfer within the spectrum due to nonlinear wave-wave interactions (resonant triad and quartet interactions). These source-term models can be written collectively as

$$S = S(E) = A + S_E E, \quad (2)$$

where A , the “linear” wind-growth term, is independent of E , and all other terms are proportional to E , with S_E representing the combined coefficients of all the source terms. Some coefficients for individual source terms in S_E are independent of E , but others are parameterized in terms of integral measures of E , so $S_E E$ is quasi linear in E as a first approximation [see Booij et al. (1999) and the SWAN model documentation for details on the source-term parameterizations].

This set of equations can be solved for the action spectrum for spatial region Ω subject to appropriate initial conditions on Ω and boundary conditions on $\partial\Omega$. For portions of the wave spectrum with propagation velocities that carry energy into Ω , the “incident” wave spectrum $E(\mathbf{x}, \mathbf{s}, t)$ on $\partial\Omega$ must be specified. The spectrum is taken as periodic in θ and the

σ boundaries are located far above and below the energy-containing region of the spectrum, yielding $E = 0$ at these boundaries. In addition to these boundary and initial conditions, complete specification of the mathematical problem requires that the winds and bathymetry, including any tidal offset, be prescribed for Ω . These inputs are required for any wave model, and are not exclusive to our assimilation algorithm, and any errors in these quantities would have the same effect on assimilation results as they would in any application of wave modeling. In the work presented here, we neglect the effects of currents, which are generally small; however, their inclusion is not precluded in this methodology (and we have included them in more recent work, to be described in a future publication).

b. Buoy directional spectrum data

As discussed above, the data produced by most single-point field instruments consist of integral moments of the directional spectrum. In particular, the motion time history from wave buoys can be analyzed using the methods of Earle (1996) [which draws heavily on the analysis of Longuet-Higgins et al. (1963)]. The directional spectrum is approximated as a Fourier series in θ , truncated at the second harmonic, yielding five coefficients that are functions of frequency, given by

$$a_j(\sigma) = \int_{-\pi}^{\pi} E(\sigma, \theta) \cos(j\theta) d\theta, \quad j = 0, 1, 2 \quad (3)$$

and

$$b_j(\sigma) = \int_{-\pi}^{\pi} E(\sigma, \theta) \sin(j\theta) d\theta, \quad j = 1, 2. \quad (4)$$

Here, $a_0(\sigma)$ is proportional to the frequency spectrum, b_0 is zero by definition, and directional information for each frequency band is contained in the a_1 , b_1 , a_2 , and b_2 functions.

Derived wave directional spectrum parameters are calculated from the a_j and b_j coefficients. The significant wave height H_s is computed as 4 times the square root of the total energy (the zeroth moment of a_0). The mean wave period T_m is computed from the first moment (centroid) of the frequency spectrum. The mean wave direction θ_m is computed by calculating $\theta(\sigma) = \tan^{-1}[b_1(\sigma)/a_1(\sigma)]$ and then performing a spectral-density-weighted average over all σ . The peak (or dominant) wave period T_p is computed from the frequency (σ_p) corresponding to the peak in a_0 . The dominant wave direction θ_p is defined as $\theta_p = \tan^{-1}(b_1/a_1)$ evaluated at σ_p .

c. The variational inverse problem formulation

Assimilation of buoy-derived directional spectra is accomplished using a variational inverse model based on the SWAN model equations [Eq. (1)]. We wish to solve a constrained minimization problem where we determine the spatially uniform offshore boundary spectrum that, when used as input to the SWAN model (the constraint), minimizes the squared error between the model estimate of the directional Fourier coefficients and a set of buoy observations. An equivalent unconstrained minimization problem is formulated by defining a cost function J consisting of error terms

augmented with a product of the constraint equations (the SWAN model) and an adjoint wave action spectrum (Lagrange multipliers). The control, the offshore boundary spectrum, is penalized in J to ensure a unique solution (Bennett and Miller 1991). The cost function has the following form:

$$J = J_d + J_c + J_i, \quad (5)$$

where the first component is the model–data error, the second is the constraint (the action balance model), and the third penalizes the model inputs (the controls).

The data term anticipates observations at N discrete spatial locations $\mathbf{x}_i$, and is given by

$$J_d = \frac{1}{2N} \sum_{i=1}^N \sum_{j=0}^2 \int \left[(\hat{a}_j^i - a_j^i)^2 + (\hat{b}_j^i - b_j^i)^2 \right] d\sigma, \quad (6)$$

where $a_j^i(\sigma)$ and $b_j^i(\sigma)$ are the observations at $\mathbf{x}_i$, and $\hat{a}_j^i(\sigma)$ and $\hat{b}_j^i(\sigma)$ are computed from $E(\sigma, \mathbf{x}_i)$, the model spectrum at the observation locations. The imposed constraint is the SWAN action balance model for stationary conditions, here weighted by the adjoint spectrum $A(\mathbf{x}, \mathbf{s})$, yielding

$$J_c = \int_{\Omega} \int_S \sigma^2 A \left[\tilde{\mathbf{v}} \cdot (\tilde{\mathbf{C}}\mathbf{N}) - \frac{S}{\sigma} \right] d\mathbf{s} d\mathbf{x}, \quad (7)$$

where the σ^2 factor is included to keep the equation in terms of energy, rather than wave action. The control term penalizes deviations of the boundary spectrum from zero,

$$J_i = \frac{\phi}{2L} \int_{\partial\Omega} \int_S E^2 d\mathbf{s} d\ell, \quad (8)$$

where the boundary $\partial\Omega$ is taken to be the open, seaward boundary, L is the length of the boundary, and ℓ is the coordinate along the boundary. The factor ϕ assigns a weight to the control term relative to the data term in computing the cost function, and must be finite for a unique solution.

The minimum of J is obtained when δJ , the first variation with respect to (A, E) , vanishes. Setting the first variation of J with respect to A to zero recovers the SWAN model as the constraint; this requires that the computed wave spectrum be a solution of (1). The first variation with respect to E vanishes in the interior of Ω when $A(\mathbf{x}, \mathbf{s})$ is a solution of

$$-\tilde{\mathbf{C}} \cdot \tilde{\mathbf{v}} A = S_E A - \frac{1}{\sigma N} \sum_{i=1}^N \sum_{j=0}^2 [(a_j - \hat{a}_j^i) \cos j\theta + (b_j - \hat{b}_j^i) \sin j\theta] \delta(\mathbf{x} - \mathbf{x}_i), \quad (9)$$

where the weak dependence of S_E on E has been ignored, and the double summation results from (6) combined with definitions (3) and (4), and represents forcing of the adjoint using the model–data errors computed from the observations. (Note that collective source term $S_E E$ is treated as linear in E even though, as discussed above, parts of S_E depend on integral properties of E . While this is an approximation, the effects on the resulting adjoint solution are small compared to the observation error term).

If the adjoint Eq. (9) is satisfied, then δJ is reduced to

$$\delta J = \int_{\partial\Omega} \int_S \left(\sigma A \mathbf{C} \cdot \mathbf{n} + \frac{\phi}{L} E \right) \delta E \, ds \, d\ell, \quad (10)$$

where $\mathbf{C}$ is the spatial wave energy propagation velocity (which, in the absence of currents, is simply the group velocity) and $\mathbf{n}$ is the outward-pointing unit normal on the boundary. For a spatially uniform incident wave spectrum $E_i(\mathbf{s})$ on $\partial\Omega$, this becomes

$$\delta J = \int_S \left(\int_{\partial\Omega} \sigma A \mathbf{C} \cdot \mathbf{n} \, d\ell + \phi E_i \right) \delta E_i \, ds. \quad (11)$$

For a given input spectrum we can reduce J by making δJ negative; this can be accomplished by choosing

$$\delta E_i = -\epsilon \left(\int_{\partial\Omega} \sigma A \mathbf{C} \cdot \mathbf{n} \, d\ell + \phi E_i \right), \quad (12)$$

where ϵ is a positive constant. Here, the term in parentheses is effectively the gradient of J with respect to the input spectrum E_i , and can be calculated from the adjoint spectrum $A(\mathbf{x}, \mathbf{s})$ integrated over the offshore boundary in the manner indicated.

For the cost function (5), if the constraint (the SWAN model) is satisfied, J_c is zero, and J consists only of contributions from J_d and J_i . This yields

$$J = \frac{1}{2N} \sum_{i=1}^N \sum_{j=0}^2 \left[(\hat{a}_j^i - a_j^i)^2 + (\hat{b}_j^i - b_j^i)^2 \right] d\sigma + \frac{\phi}{2} \int_S E_i^2 \, ds \quad (13)$$

as the final working form of the cost function to be minimized.

For the inverse model, Eqs. (1), (9), (12), and (13) are key. The present estimate of the observed spectra is computed using the SWAN model (1), with the present estimate of E_i on the boundary. The adjoint SWAN action balance model (9) is solved with the source term incorporating the difference between the estimated and observed Fourier direction spectrum coefficients (a_j , b_j). The gradient is calculated from the adjoint spectrum A on the boundary, along with the present estimate of the incident wave spectrum E_i using (12). The gradient is used in a descent algorithm to find the incident wave spectrum that minimizes J , calculated using (13). Although the gradient can have either sign, the updated incident spectrum is constrained in practice to be nonnegative.

d. Solution implementation

The algorithm was implemented using a modified version of the SWAN version 40.85 code, and separate Fortran programs to calculate the cost function, the gradient, and the updated boundary spectrum. These programs were executed using a Python script that controlled the iteration process. A separate Python script was used to step from time to time over the period of interest.

The modified version of SWAN can be configured to solve either the forward or adjoint form of the model. When the forward model is solved, the complete five-dimensional action spectrum solution is stored; this solution is then read by the adjoint solver and is used to compute the spectrum-dependent source terms (S_E) in the adjoint equation. This allows all the

FIG. 1. Bathymetry map for the computational domain showing sensor locations for in situ observations of winds, tides, and wave directional spectra.

relevant source terms, consistent with the forward-model solution, to be used in producing the adjoint equation solution. For the forward solution, standard SWAN ASCII spectrum files are written for the observation locations. These are used to compute the a and b values using (3) and (4), respectively, for comparison to the observations.

3. Results

The inverse model described above was applied for estimation of waves in the area near the New River Inlet on the North Carolina coast during the period of the Office of Naval Research Riverine and Estuarine Transport (RIVET) field experiment in May 2012. The goal was to use nearshore buoy data to estimate the waves in all of Onslow Bay at a number of discrete times by determining the best-fit offshore wave spectrum (boundary condition) for use in the SWAN model, assuming stationary conditions. For the algorithm, the SWAN model Already mentioned version number was run with all the relevant model physics enabled (i.e., wind generation, whitecapping, bottom friction, three-wave and four-wave interactions, and depth-induced breaking) and all input parameters set to their default values. The initial guess for each time was generated using the SWAN model for the observed vector wind with a zero offshore boundary spectrum (referred to as “wind only,” below). The algorithm product is a wave estimate for the region from a stationary SWAN model run for the time of interest that is a best fit to the observation data for that time. The model results can be interrogated at different locations, and comparisons are made to independent observations to assess the overall fidelity of the approach.

a. Problem setup and algorithm performance

Figure 1 shows the bathymetry and the extent of the computational domain. The domain is 295 km alongshore $\times$ 156 km

FIG. 2. In situ observations of environmental conditions during the month of May 2012: (a) wind speed, (b) wind direction (“to” direction measured CCW from east), and (c) tides relative to mean sea level. Wind data are from NDBC 41037 and tide data are the averages of tides from NOS stations 8656483 and 8658163, shown in Fig. 1.

cross-shore, and is discretized at 500 m, yielding 589×311 grid points. The spectral grid has 48 directions yielding 7.5° resolution and 50 frequencies spaced logarithmically from 0.025 to 0.58 Hz. Stationary computations were done between midnight (UTC) on 1 and 30 May 2012, at intervals of 6 h. This 6-h interval is smaller than the time scale for changes in conditions, but larger than the time for 5–10-s waves to transit to the shore. The forward SWAN model is solved with a spatially uniform spectrum at the offshore boundary that ramps to zero at the shore along the lateral boundaries. The bathymetry used is the National Geodetic Data Center Coastal Relief Model at 3-arc-s resolution (about 93 m). The water surface elevation in SWAN was set using the average of National Ocean Service (NOS) tide gauges located at Beaufort, North Carolina (8656483), and Wrightsville Beach, North Carolina (8658163), shown in Fig. 1. Winds were assumed to be uniform over the entire domain and were obtained from National Data Buoy Center (NDBC) meteorological buoy NDBC 41037, near the center of Onslow Bay, also shown in Fig. 1. The time history of the winds and tides for the period of interest are shown in Fig. 2. The wind speeds vary from about 1 m s^{-1} up to about 12 m s^{-1} and the wind direction covers all possible directions. The tides range from about 0.9 m below mean sea level to about 0.7 m above.

Observation data for the inversion algorithm came from two CDIP Datawell Waverider buoys deployed nearshore by the Army Corps of Engineers; these data were accessed via the NDBC web interface where they are known as NDBC 41110 (Masonboro Inlet) and 41109 (New River Inlet), moored in 13 and 17 m of water, respectively. The locations of these buoys are shown in Fig. 1. Independent ground truth data came from two NDBC 3 m discus buoys, 41036 and 41013, deployed further offshore in 30 and 24 m of water, respectively. Also included is a very nearshore acoustic wave and current (AWAC) sensor, NR191702, deployed by the Woods Hole Oceanographic Institute in 4 m of water as part of the RIVET field experiment. The locations of these instruments are shown in Fig. 1.

The algorithm converged for all cases, with details shown in Fig. 3. Figure 3a shows the initial value of the cost function J , from (13), which varied significantly over the time period of interest. In most cases the cost function was improved using the observed wind with zero boundary condition (the wind-only initial guess), and it was further improved using the estimated boundary spectrum. The cost function for each case, normalized by the initial cost function, is shown in Fig. 3b. The median reduction in the cost function is to 17% of the initial value. There are three

FIG. 3. Convergence history for all assimilation cases: (a) initial, wind-only, and final cost function, (b) normalized cost function showing median reduction to 17% of the initial value (red line), and (c) number of iterations required for each case (median is five iterations, indicated by the red line).

cases where the cost function increased slightly (normalized cost functions greater than unity); these were cases where the wind sea essentially matched the observations, with only small contributions from the boundary spectrum (which increased the cost function due to the penalty term in J). Figure 3c shows the number of iterations required for convergence in each case, with a median of five iterations.

b. Comparisons for input data

In this section, we show comparisons between wave parameters derived from the observed and estimated spectra for the buoy data used in the algorithm. We focus here on buoy 41109; buoy 41110 is similar, so it is not shown in detail. Comparisons to independent data are shown in the next section. Here we restrict our presentation to the significant wave height H_s , the mean wave period T_m , and the mean wave direction θ_m , defined above (the comparisons for the peak wave period T_p and direction θ_p are similar in nature to the mean quantities). Mean and RMS errors for all these quantities for both buoys are tabulated, as well.

Figure 4 shows a comparison of estimated wave quantities to observed values for buoy 41109. Figure 4a shows the time history of the estimated H_s , compared to the observed values; also shown is H_s for wind only (zero offshore boundary condition). The agreement between the predicted and observed values is good; however, there appears to be a tendency to overestimate H_s slightly when the wind speed is elevated and the wind-only results are close to the final estimate, e.g., around 19 May. Figure 4b shows a scatterplot comparing the predicted to the observed H_s , where the values cluster around the unit-slope line indicating small variance and minimal bias in the estimates. Figures 4c and 4d, respectively, show time-history and scatterplot comparisons for T_m . Again, the agreement is generally good in the time history, with minimal error exhibited in the scatterplot, although slightly biased toward lower values; however, T_m around 11 and 21 May is somewhat underestimated. Figures 4e and 4f, respectively, show time-history and scatterplot comparisons for θ_m . Again, other than some discrepancies around 11 May the agreement is good.

FIG. 4. Comparison of estimated to observed wave parameters for buoy 41109, used in the estimation algorithm: (a),(b) significant wave height H_s , (c),(d) mean wave period T_m , (e),(f) mean wave direction θ_m . In the time histories open circles show the observed quantities, red dots show the estimates, and gray dots show the wind-only, initial-guess quantities; in the scatterplots the symbols are colored by day.

Quantitative error statistics for H_s , T_m , and θ_m for the buoys 41109 and 41110 are summarized in Table 1, along with corresponding statistics for T_p and θ_p . The mean error in H_s is less than 0.06 m and the mean H_s is around 1 m, while the RMS error is 0.12 m or less. For T_m , the mean and RMS errors are about -0.4 and 0.6 s, respectively, a small fraction of the mean wave period $\bar{T}_m$, calculated at just under 5 s. For θ_m , the mean error is around -2° , about half the directional resolution for the computation, while the RMS errors are

about 15° , or 3 times the SWAN directional resolution. In general, the “peak” statistics exhibit larger RMS errors than the “mean” quantities, as would be expected.

c. Comparisons for independent data

In this section, we show similar comparisons between wave parameters from buoy 41036, buoy 41013, and AWAC NR191702, not used in the algorithm, and those from estimated wave spectra from the corresponding locations. These

TABLE 1. Error statistics for significant wave height (H_s), mean wave period (T_m), and mean wave direction (θ_m) from comparison of the SWAN assimilation results to observation data used for the estimates. Also included are statistics for the peak wave period (T_p) and the peak wave direction (θ_p). The mean error in quantity X is $\bar{\epsilon}_X$, the RMS error is ϵ_X^{rms} . For comparison, the mean and standard deviation of the observations ($\bar{X}$ and σ_X) are included on the second line in parentheses.

Station	$\bar{\epsilon}_{H_s}$ ($\bar{H}_s$)	$\epsilon_{H_s}^{\text{rms}}$ (σ_{H_s})	$\bar{\epsilon}_{T_m}$ ($\bar{T}_m$)	$\epsilon_{T_m}^{\text{rms}}$ (σ_{T_m})	$\bar{\epsilon}_{\theta_m}$ ($\bar{\theta}_m$)	$\epsilon_{\theta_m}^{\text{rms}}$ (σ_{θ_m})	$\bar{\epsilon}_{T_p}$ ($\bar{T}_p$)	$\epsilon_{T_p}^{\text{rms}}$ (σ_{T_p})	$\bar{\epsilon}_{\theta_p}$ ($\bar{\theta}_p$)	$\epsilon_{\theta_p}^{\text{rms}}$ (σ_{θ_p})
41109	0.06 m (0.92 m)	0.12 m (0.28 m)	-0.38 s (4.7 s)	0.64 s (0.73 s)	-2.0° (133°)	14° (34°)	-0.32 s (6.8 s)	1.3 s (1.7 s)	-0.5° (132°)	15° (25°)
41110	0.03 m (1.02 m)	0.11 m (0.33 m)	-0.44 s (4.9 s)	0.59 s (0.68 s)	-2.5° (145°)	15° (36°)	-0.66 s (7.2 s)	1.7 s (1.6 s)	-2.7° (148°)	20° (24°)

FIG. 5. Comparison of estimated to observed wave parameters for buoy 41036, not used in the estimation algorithm: (a),(b) significant wave height H_s , (c),(d) mean wave period T_m , (e),(f) mean wave direction θ_m . In the time histories open circles show the observed quantities, red dots show the estimates, and gray dots show the wind-only, initial-guess quantities; in the scatterplots the symbols are colored by day.

comparisons show the general performance of the algorithm. We again restrict our presentation to the significant wave height H_s , the mean wave period T_m , and the mean wave direction θ_m .

Figure 5 shows comparisons for buoy 41036 located in 30 m of water; error statistics are shown in Table 2. As is expected the errors in general are larger for these independent data. Figures 5a and 5b show the estimated H_s , compared to the observed values. Here the wave height is generally 20%–30% larger than at the nearshore locations described above, extending up to 3 m late in the month, and the agreement between the predicted and observed values is good; however, it is clear from Fig. 5b that H_s is consistently slightly overestimated. Figures 5c and 5d show T_m , and again the agreement is good, but the mean period is biased low by about half a second. Again, T_m around 11 and 21 May is somewhat underestimated, but there are comparable errors at other times. Figures 5e and 5f show comparisons for θ_m , and the errors are larger than for the cases above, with a bias to lower values (i.e., the mean propagation direction rotated clockwise from the true direction) during the first half of the

observation period. There are some substantial variations in mean direction around 11 May, which corresponds to some of the largest errors. The “peak” statistics, shown in Table 2, have bias errors comparable to those in the mean quantities, but larger RMS errors, as above.

Detailed comparisons for buoy 41013, located to the southwest of the other sensors in 23.5 m of water, are similar to those for buoy 41036. However, as seen in Table 2 for 41013, there is bias in the wave direction to lower angles, i.e., the wave propagation direction is to the right of (clockwise from) the true mean direction, likely related to the location of this buoy well away from the center of the domain.

To evaluate algorithm performance closer to the shore, we look at comparisons to NR191702, an AWAC sensor deployed by Woods Hole Oceanographic Institute in about 9 m depth, offshore of the inlet ebb shoal; these are shown in Fig. 6. Figures 6a and 6b show the estimated and observed H_s which is substantially lower than at the offshore locations. There is no significant bias in the estimates, and agreement is good, other than low estimates at the

TABLE 2. Error statistics for significant wave height (H_s), mean wave period (T_m), and mean wave direction (θ_m) from comparison of the SWAN assimilation results to independent observation data. Also included are statistics for the peak wave period (T_p) and the peak wave direction (θ_p). The mean error in quantity X is $\bar{\epsilon}_X$, the RMS error is ϵ_X^{rms} . For comparison, the mean and standard deviation of the observations ($\bar{X}$ and σ_X) are included on the second line in parentheses.

Station	$\bar{\epsilon}_{H_s}$ ($\bar{H}_s$)	$\epsilon_{H_s}^{\text{rms}}$ (σ_{H_s})	$\bar{\epsilon}_{T_m}$ ($\bar{T}_m$)	$\epsilon_{T_m}^{\text{rms}}$ (σ_{T_m})	$\bar{\epsilon}_{\theta_m}$ ($\bar{\theta}_m$)	$\epsilon_{\theta_m}^{\text{rms}}$ (σ_{θ_m})	$\bar{\epsilon}_{T_p}$ ($\bar{T}_p$)	$\epsilon_{T_p}^{\text{rms}}$ (σ_{T_p})	$\bar{\epsilon}_{\theta_p}$ ($\bar{\theta}_p$)	$\epsilon_{\theta_p}^{\text{rms}}$ (σ_{θ_p})
41036	0.10 m (1.25 m)	0.21 m (0.45 m)	−0.52 s (5.4 s)	0.76 s (0.83 s)	−6.2° (144°)	27° (42°)	−0.36 s (7.1 s)	1.6 s (1.7 s)	−2.7° (46°)	36° (143°)
41013	0.03 m (1.32 m)	0.27 m (0.54 m)	−0.60 s (5.5 s)	0.79 s (0.74 s)	−9.8° (152°)	29° (46°)	−0.64 s (7.43 s)	2.0 s (1.7 s)	−12.7° (152°)	49° (46°)
NR191702	−0.01 m (0.81 m)	0.15 m (0.23 m)	−0.36 s (4.8 s)	0.77 s (0.86 s)	2.7° (120°)	11° (23°)	0.29 s (6.6 s)	1.7 s (1.7 s)	5.1° (119°)	18° (23°)

beginning of the observation period (through 5 May). Figures 6c and 6d show estimates for T_m exhibiting similar good agreement; however, as above, T_m around 10/11 May is again underestimated. Results for θ_m , shown in Figs. 6e and 6f, show a smaller range of wave directions compared to deeper water, but otherwise generally good agreement.

(Note that missing observation data during the last day of the period are indicated as zeros in the comparisons).

d. Selected spectrum comparisons

To illuminate some of the features seen in the time histories of wave characteristics shown above, we can examine the

FIG. 6. Comparison of estimated to observed wave parameters for sensor NR191702, not used in the estimation algorithm: (a),(b) significant wave height H_s , (c),(d) mean wave period T_m , (e),(f) mean wave direction θ_m . In the time histories open circles show the observed quantities, red dots show the estimates, and gray dots show the wind-only, initial-guess quantities; in the scatterplots the symbols are colored by day.

FIG. 7. Comparison of estimated to observed primary directional spectrum coefficients (a_0 , a_1 , b_1 vs f) for all buoys for 1800 UTC 8 May 2012. Data from (a) NDBC 41110 and (b) NDBC41109 were assimilated. (d) NDBC 41013, (e) WHOI NR191702, and (f) NDBC 41036 are independent validation data. (c) Significant wave height and dominant wave direction, with the wind magnitude and direction indicated; assimilated buoy locations are red circles and validation buoy locations are white circles.

underlying wave spectra. Here, we will first show results that are typical of the good agreement generally seen between the predicted and observed wave statistics, shown above, specifically for a case where the waves are small (1800 UTC 8 May 2012), and when they are larger (0000 UTC 27 May 2012). We will then show spectra from select times when agreement is poor—a time where the mean period T_m was underestimated by more than a second for nearshore buoys (0600 UTC 11 May 2012), a time where significant wave height H_s showed errors of 10%–20% for most buoys (1200 UTC 19 May 2012), and a time where mean period T_m is underestimated by 2 s offshore and smaller amounts close to shore (0000 UTC 21 May 2012).

Spectrum comparisons and wind and wave conditions from 1800 UTC 8 May 2012 are shown in Fig. 7. At this time wind is low with $U_w = 2.6 \text{ m s}^{-1}$ northward and waves are also small with $H_s < 1.1 \text{ m}$ over the entire study region. Agreement between estimated and observed directional spectrum coefficients is quite good for the nearshore buoys (41110, 41109, and NR191702). Agreement is not as good for the offshore buoys (41013 and 41036) as seen in Figs. 7d and 7f. While the significant wave height is well predicted, offshore the computed a_0 shows energy shifted from below 0.2 Hz to above, and the direction of the low-frequency waves is predicted to be cross-shore rather than to the north, consistent with the wind direction, as in the data. For this case the low-frequency

waves have turned cross-shore and their energy has been dissipated as they approach the nearshore buoy locations used for assimilation. The assimilation algorithm does not undo these effects for this case, and the offshore wave estimate resembles the nearshore waves more than is appropriate, even though the wave statistics are accurate.

Consistently good agreement between predicted and observed spectra is found at 0000 UTC 27 May 2012, as shown in Fig. 8. Wind and waves are much higher at this time, with $U_w = 10.5 \text{ m s}^{-1}$ southwestward and offshore waves reaching as high as 3 m. Overall agreement is very good, but there are small discrepancies in the spectra which mainly reflect differences in the details of the local peaks and oscillations in the observed spectra. Both 41109 and 41013 overshoot the peaks in a_0 and a_1 around 0.12 Hz by 10%–20% (Figs. 8b,d, respectively); 41036 underestimates secondary peaks by about half in a_0 and a_1 around 0.16 Hz (Fig. 8f), while NR191702 overestimates similar secondary peaks (Fig. 8e).

Wave conditions at 0600 UTC 11 May 2012 are similar to those for 1800 UTC 8 May 2012 (Fig. 7, above), although wind speed has doubled to 5.8 m s^{-1} and the wind direction is opposite (southward). For this time, unlike the earlier case, the agreement between the predicted and observed θ_m is consistently poor and T_m is underestimated for nearshore buoys. The SWAN spectra at the offshore locations overestimate the

FIG. 8. Comparison of estimated to observed primary directional spectrum coefficients (a_0, a_1, b_1 vs f) for all buoys for 0000 UTC 27 May 2012. Data from (a) NDBC 41110 and (b) NDBC41109 were assimilated. (d) NDBC 41013, (e) WHOI NR191702, and (f) NDBC 41036 are independent validation data. (c) Significant wave height and dominant wave direction, with the wind magnitude and direction indicated; assimilated buoy locations are red circles and validation buoy locations are white circles.

spectrum below 0.2 Hz by as much as 50%. The spectra have a broad bump in the tail ($f > 0.2$ Hz) of the a_0 and a_1 coefficients that is roughly consistent with the observations offshore, but is not in evidence in the nearshore observations (Fig. 9). This is evidence of short-fetch wave growth, due to the offshore wind direction; however, in the SWAN results, there is constant wind over the whole domain (resulting in wind waves near shore) while lack of wind waves in the observations likely indicates that the region near the shore is sheltered for winds to the south. We note that estimates of T_m and θ_m can be poor while H_s is good, as seen here (and in Fig. 11, below); that can be attributed to cancellation of errors in a_0 that lead to equivalent integrated energy.

For 1200 UTC 19 May 2012, shown in Fig. 10, we have an intermediate case for wind and wave conditions, with offshore waves reaching about 2 m and wind speed $U_w = 12.0 \text{ m s}^{-1}$ toward the west-southwest. There is a north-south pattern of estimated H_s response to the wind direction, with H_s overestimated by up to 20% at the northern, nearshore buoys and underestimated offshore and southward (by 30% at 41013). This pattern is consistent with a wind gradient or nonuniform wind field that is not captured by the assumption of uniform wind over the domain as the buoys with the best agreement are closest to the wind buoy. Levels of a_0, a_1 , and b_1 are all overestimated for nearshore buoys 41109 and NR191702 (Figs. 10b,e, respectively). The a_0 and a_1

coefficients are slightly underestimated for buoys 41110 and 41036 (Figs. 10a,f, respectively), leading to 5%–10% errors in wave height. Coefficients a_0 and a_1 are significantly underestimated for the southernmost offshore buoy 41013 (Fig. 10d) resulting in the large wave height errors observed.

Last, T_m is underestimated by up to 1.5 s at 0000 UTC 21 May 2012, shown in Fig. 11. The wave height H_s is low (under 1.5 m) over the entire study region and wind speed is $U_w = 6.5 \text{ m s}^{-1}$ westward. As at 0600 UTC 11 May 2011 (Fig. 9), the predicted spectra have a broad bump in the tail that is more distinct than in the observations. Estimated spectra for nearshore buoys show good agreement with the observations below 0.2 Hz but for higher frequencies (wind waves) the spectrum is overestimated by a factor of 2–3, leading to a higher mean frequency, and lower T_m . For the offshore buoys, spectra are underestimated by a factor of 2–4 (frequency offset the missing energy to yield a correct H_s for 41036 (Fig. 11f) but not for 41013 (Fig. 11d). Unlike the other times examined, this one is dominated by the wind sea, with very little swell energy, and the observed winds were steadily decreasing over the previous two days. Under these circumstances, errors in the wind wave portion of the spectrum appear to be enhanced, but the origin of the errors is unclear. Possible contributors are the uniform wind field, or the stationary

FIG. 9. Comparison of estimated to observed primary directional spectrum coefficients (a_0, a_1, b_1 vs f) for all buoys for 0600 UTC 11 May 2012. Data from (a) NDBC 41110 and (b) NDBC 41109 were assimilated. (d) NDBC 41013, (e) WHOI NR191702, and (f) NDBC 41036 are independent validation data. (c) Significant wave height and dominant wave direction, with the wind magnitude and direction indicated; assimilated buoy locations are red circles and validation buoy locations are white circles.

nature of our calculation during a period where the wind speed is steadily dropping.

4. Discussion and conclusions

In this paper we have described the development and testing of a variational data assimilation algorithm for the SWAN nearshore wave-spectrum model. The method requires the calculation of a cost function to quantify the error between observed wave directional spectrum coefficients and those derived from wave spectra predicted by the SWAN model. An adjoint SWAN model equation is then solved. The gradient of the cost function with respect to the incident wave spectrum is calculated from the adjoint spectrum solution. A conjugate-gradient algorithm is used to find the incident spectrum that minimizes the cost function. This improved incident spectrum should result in improved agreement between observed and estimated directional wave spectra as well as wave parameters.

The algorithm is tested using real-world data from nearshore buoys in the area near the New River Inlet during the period of the RIVET field experiment in May 2012. The wave field was treated as stationary, consistent with the cross-shore extent of the domain, and the time rate of change in offshore conditions. The wave spectrum at the offshore boundary was considered constant along the boundary, and the winds,

obtained from a buoy in the domain, were considered spatially uniform across the domain. Tides were accounted for using tide-gauge data. Wave spectrum data from two of the nearshore buoys are assimilated while data from three additional sensors are withheld as independent ground truth. Estimated and observed significant wave height H_s , mean wave period T_m , and mean wave direction θ_m agree well over the observation period for all of the buoys. Mean errors $\bar{\epsilon}_X$ are generally less than 10% while RMS errors ϵ_X^{rms} fall between 10% and 20% when compared to the means for all wave parameters (Tables 1 and 2). The RMS errors for mean and peak wave direction are about twice the direction resolution in the SWAN spectral grid. Mean and RMS errors are higher for peak parameters than mean parameters as there is greater uncertainty in determining the frequency corresponding to the peak than in performing a spectral-density-weighted average over all frequencies.

For the independent sensors, errors are generally higher for the offshore buoys (Table 2) than those used for assimilation (Table 1) while errors for nearshore AWAC NR191702 are comparable to the assimilated buoys. Having said that, however, the mean H_s and T_m values are also higher for the offshore buoys so the errors are essentially equivalent represented as a percentage of the mean. For the offshore observations, the RMS errors in wave direction increase to 3–4 times the SWAN directional resolution. This

FIG. 10. Comparison of estimated to observed primary directional spectrum coefficients (a_0 , a_1 , b_1 vs f) for all buoys for 1200 UTC 19 May 2012. Data from (a) NDBC 41110 and (b) NDBC 41109 were assimilated. (d) NDBC 41013, (e) WHOI NR191702, and (f) NDBC 41036 are independent validation data. (c) Significant wave height and dominant wave direction, with the wind magnitude and direction indicated; assimilated buoy locations are red circles and validation buoy locations are white circles.

may be due to inherent limitations in directional resolution in buoy observations: the direction is defined as the “peak” of what is effectively a broad maximum.

It is perhaps surprising how well the estimates agree with the independent observations, particularly given how different the water depths are from the assimilated buoys’ depths (41036 and 41013 are about twice as deep, while NR191702 is about 3–4 times as shallow) as well as the offshore buoys’ distance from the assimilated data with similar wave and environmental conditions, i.e., NR191702. Perhaps agreement is so good at all of the buoy locations as the domain is relatively small and the bathymetry is uncomplicated. Additional testing in a larger domain with more complex bathymetry and more buoys will help determine the limits of the assimilation algorithm’s performance.

We took a more in depth look at the spectra from some of the dates with larger, consistent errors among the buoys. These dates include 11 May, which underestimated T_m and had θ_m errors, 19 May, which had H_s errors, and 21 May, which underestimated T_m . All of these dates have different wind speeds, wind directions, and tidal phases so there does not appear to be a bias caused by any of these properties.

Additionally, peak wind speeds of 15 m s^{-1} from the same approximate direction are found on both 19 and 27 May (Fig. 2), but H_s agreement is good on 27 May (Fig. 8) but not on 19 May (Fig. 10), all of which suggests that wind response alone cannot explain these differences.

We also found that T_m is consistently biased low by about 0.5 s, although it is notably underestimated on both 11 and 21 May. The directional wave spectra from these 2 days (Figs. 9 and 11) are also notably different than the other spectra that we inspected. At all five buoy locations, the a_0 and a_1 coefficients have a broad bump in the tail above $f = 0.2 \text{ Hz}$ that also serves as a secondary peak in the spectra. In some cases the magnitude of this “secondary” peak is on the same order of magnitude or larger than the primary (lower frequency) peak. This is in stark contrast to the higher-frequency tail of the other spectra which simply decays toward zero. It is unclear why this broad bump is present in the estimates as it is not present in the assimilated observations, although it does exist to a smaller extent in some of the independent observations. Likely this is due to excessive wind growth in SWAN, unrelated to the boundary conditions being estimated, but it is clear that T_m is sensitive to accuracy in the high-frequency tail as an excess of high-frequency energy results in lower T_m .

In conclusion, we have developed a variational data assimilation algorithm for the SWAN nearshore wave-spectrum

FIG. 11. Comparison of estimated to observed primary directional spectrum coefficients (a_0 , a_1 , b_1 vs f) for all buoys for 0000 UTC 21 May 2012. Data from (a) NDBC 41110 and (b) NDBC 41109 were assimilated. (d) NDBC 41013, (e) WHOI NR191702, and (f) NDBC 41036 are independent validation data. (c) Significant wave height and dominant wave direction, with the wind magnitude and direction indicated; assimilated buoy locations are red circles and validation buoy locations are white circles.

model. Comparison of significant wave height, mean wave direction, and mean period calculated from observed and estimated wave spectra show good agreement, validating the assimilation algorithm. When predicted spectra are compared to independent ground truth buoys the RMS errors are 10%–20% for significant wave height, about half a second for mean wave period, and up to three to four SWAN spectral grid cells for mean direction. Additional testing with real-world data is ongoing for a larger, more complicated study region. Further development of the algorithm is also ongoing to make improvements, add additional features, and take advantage of recent improvements to wind growth modeling implemented in SWAN version 41.20 (SWAN Team 2019), based on the work of Rogers et al. (2012), and for application of this algorithm to larger domains under nonstationary conditions.

Acknowledgments. This work was supported by the Office of Naval Research Littoral Geoscience and Optics program under Contract N0014-10-C-0507. Access to the AWAC NR191702 wave data used for comparison near the shore, provided by S. Elgar of the Woods Hole Oceanographic Institution, is greatly appreciated. An early version of the Python scripts used for this study was developed by M. Masarik.

Data availability statement. All wave buoy data, tide data, wind data, and bathymetry used in this study are publicly

available from the U.S. National Oceanic and Atmospheric Administration. The specific datasets and sources are as described in the text.

REFERENCES

- Bannister, R. N., 2017: A review of operational methods of variational and ensemble-variational data assimilation. *Quart. J. Roy. Meteor. Soc.*, **143**, 607–633, <https://doi.org/10.1002/qj.2982>.
- Bennett, A. F., 2005: *Inverse Modeling of the Ocean and Atmosphere*. Cambridge University Press, 260 pp.
- , and R. N. Miller, 1991: Weighting initial conditions in variational assimilation schemes. *Mon. Wea. Rev.*, **119**, 1098–1102, [https://doi.org/10.1175/1520-0493\(1991\)119<1098:WICIVA>2.0.CO;2](https://doi.org/10.1175/1520-0493(1991)119<1098:WICIVA>2.0.CO;2).
- Booij, N., R. C. Ris, and L. H. Holthuijsen, 1999: A third-generation wave model for coastal regions: 1. Model description and validation. *J. Geophys. Res.*, **104**, 7649–7666, <https://doi.org/10.1029/98JC02622>.
- Crosby, S. C., B. D. Cornuelle, W. C. O'Reilly, and R. T. Guza, 2017: Assimilating global wave model predictions and deep-water wave observations in nearshore swell predictions. *J. Atmos. Oceanic Technol.*, **34**, 1823–1836, <https://doi.org/10.1175/JTECH-D-17-0003.1>.
- Delay, F., H. Badri, M. Fahs, and P. Ackere, 2017: A comparison of discrete versus continuous adjoint states to invert groundwater flow in heterogeneous dual porosity systems. *Adv. Water Resour.*, **110**, 1–18, <https://doi.org/10.1016/j.advwatres.2017.09.022>.

- Earle, M. D., 1996: Nondirectional and directional wave data analysis procedures. National Data Buoy Center Tech. Doc. 96-01, 43 pp., <https://www.ndbc.noaa.gov/wavemeas.pdf>.
- Economou, T. D., J. J. Alonso, T. A. Albring, and N. R. Gauger, 2017: Adjoint formulation investigations of benchmark aerodynamic design cases in SU2. *35th Applied Aerodynamics Conf.*, Denver, CO, American Institute of Aeronautics and Astronautics, <https://doi.org/10.2514/6.2017-4363>.
- Erwig, M., Z. Fu, and B. Pflaum, 2007: Parametric Fortran: Program generation in scientific computing. *J. Software Maint. Evol.*, **19**, 155–182, <https://doi.org/10.1002/smr.346>.
- Griesse, R., and A. Walther, 2004: Evaluating gradients in optimal control: Continuous adjoints versus automatic differentiation. *J. Optim. Theory Appl.*, **122**, 63–86, <https://doi.org/10.1023/B:JOTA.0000041731.71309.f1>.
- Holthuijsen, L. H., 2009: *Waves in Oceanic and Coastal Waters*. Cambridge University Press, 387 pp.
- Li, J., and S. Zhang, 2020: Mitigation of model bias influences on wave data assimilation with multiple assimilation systems using Wavewatch III v5.16 and SWAN v41.20. *Geosci. Model Dev.*, **13**, 1035–1054, <https://doi.org/10.5194/gmd-13-1035-2020>.
- Longuet-Higgins, M., D. Cartwright, and N. Smith, 1963: Observations of the directional spectrum of sea waves using the motions of a floating buoy. *Proc. Ocean Wave Spectra Conf.*, Easton, MD, Prentice-Hall, 111–136.
- O'Reilly, W. C., and R. T. Guza, 1998: Assimilating coastal wave observations in regional swell predictions. Part I: Inverse methods. *J. Phys. Oceanogr.*, **28**, 679–691, [https://doi.org/10.1175/1520-0485\(1998\)028<0679:ACWOIR>2.0.CO;2](https://doi.org/10.1175/1520-0485(1998)028<0679:ACWOIR>2.0.CO;2).
- Orzech, M. D., J. Veeramony, and H. Ngodock, 2013: A variational assimilation system for nearshore wave modeling. *J. Atmos. Oceanic Technol.*, **30**, 953–970, <https://doi.org/10.1175/JTECH-D-12-00097.1>.
- , —, —, S. Flampouris, and I. Souopgui, 2016: Recent updates to SWANFAR, a 5DVAR data assimilation system for SWAN. Naval Research Laboratory Memo. Rep. NRL/MR/7320-16-9705, 43 pp.
- Ris, R. C., L. H. Holthuijsen, and N. Booij, 1999: A third-generation wave model for coastal regions: 2. Verification. *J. Geophys. Res.*, **104**, 7667–7681, <https://doi.org/10.1029/1998JC900123>.
- Rogers, W. E., A. V. Babanin, and D. W. Wang, 2012: Observation-consistent input and whitecapping dissipation in a model for wind-generated surface waves: Description and simple calculations. *J. Atmos. Oceanic Technol.*, **29**, 1329–1346, <https://doi.org/10.1175/JTECH-D-11-00092.1>.
- Rusu, L., 2015: Assessment of the wave energy in the Black Sea based on a 15-year hindcast with data assimilation. *Energies*, **8**, 10 370–10 388, <https://doi.org/10.3390/en80910370>.
- SWAN Team, 2019: SWAN: Scientific and technical documentation. Delft University of Technology Tech. Rep., 140 pp.
- Thanh, N., N. Huan, and T. Tien, 2019: Application of data assimilation method for wave height in eastern Vietnam Sea by the ensemble Kalman filter. *Vietnam J. Mar. Sci. Technol.*, **18**, 358–367, <https://doi.org/10.15625/1859-3097/18/4/12474>.
- Veeramony, J., D. Walker, and L. Hsu, 2010: A variational data assimilation system for nearshore applications of SWAN. *Ocean Modell.*, **35**, 206–214, <https://doi.org/10.1016/j.ocemod.2010.07.008>.
- Walker, D. T., 2006: Assimilation of SAR imagery in a near-shore spectral wave model. General Dynamics Advanced Information Systems Final Rep. 200236, 39 pp.