Complete Polarimetric and Doppler Measurements with a Single Receiver Radar

DUSAN S. ZRNIĆ

NOAA/Environmental Research Laboratories, National Severe Storms Laboratory, Norman, Oklahoma

(Manuscript received 11 April 1990, in final form 30 August 1990)

ABSTRACT

The concept of the polarimetric scattering matrix applicable to hydrometeors is reviewed to indicate the total number of measurands that is possible from a radar system with two orthogonal linear polarizations. It is shown how to obtain this complete set of polarimetric measurands together with Doppler spectral moments through a single receiver by proper choice of polarization in a transmit–receive sequence pair.

1. Introduction

Polarimetric measurements have been an active area of research in the United States and abroad over the last decade. Radars with circular and linear polarization diversity are in use (Bringi and Hendry 1990). Among the most advanced is the 5-cm wavelength radar operated by the DLR [German Aero Space Research Establishment (Schroth et al. 1988)]; it has two receivers for simultaneous reception of orthogonally polarized electric fields. Furthermore, transmitted polarization can be controlled from pulse to pulse. There are four 10-cm wavelength weather radars in the United States capable of transmitting sequentially linear orthogonal polarizations. Three of these have single receivers and therefore cannot estimate simultaneously the cross-polar component or quantities derived from it. Lack of simultaneous reception does not preclude estimation of all the polarimetric variables. It only requires proper accounting of Doppler shifts and spectral broadening, a chore that is usually desired anyway if one is interested in storm kinematics. A polarimetric variable or measurand is defined in this paper as a nonredundant backscattering quantity that depends on polarization.

The purpose of this paper is to show how all polarimetric measurands can be estimated with a single receiver. I consider vertical and horizontal polarizations because they are available on several 10-cm wavelength radars. For these polarizations I first review the polarimetric covariance matrix in section 2 and use it to identify all possible measurands. Not all of these have equal meteorological significance, but their relative importance is not discussed here as it requires detailed modeling of the backscatter from complex hydrometeors.

Corresponding author address: Dr. Dusan Zrnić, NOAA/National Severe Storms Laboratory, 1313 Halley Circle, Norman, OK 73069.

In section 3 of the paper I show, via examples, the procedure to obtain all polarimetric measurands and the Doppler spectral moments. A practical sequence to achieve this consists of polarization coded transmitted pulses; these are synchronized with the receiver switch to obtain the required polarization-upon-reception.

2. Covariance matrix and polarimetric measurands

The purpose of this section is to identify the number of polarimetric measurands by counting the number of independent terms in the covariance matrix. To make things simpler, propagation effects are neglected at first but will be introduced later. Therefore, a backscattering covariance matrix for echoes from an ensemble of particles within the resolution volume is first examined. I will also list the relationships between the polarimetric measurands and the commonly accepted parameters such as the reflectivity factor, linear depolarization ratio, etc.

Consider a horizontally or vertically polarized electric field component at the radar antenna that is echoed by a point scatterer located at $\mathbf{r}_n$:

$$E_{ij} = P_j^{1/2} \exp(-j2kr_n) s_{ij}(n) \eta^{1/2} I^{1/2}(\mathbf{r}_n) / [2(\pi)^{1/2} r_n^2] \quad (1)$$

where s_{ij} is an element of the backscattering matrix for the nth hydrometeor, k the wavenumber, P_j is the transmitter power that produces linearly polarized incident electric field, η is the free space impedance, and E_{ij} is the received field. In this convention the first index refers to the polarization of the backscattered field and the second to the polarization of the incident electric field on the hydrometeor; only vertical and horizontal polarizations are considered. Here $I(r_n)$ is a weighting function that accounts for antenna beam pattern as well as the transmitted pulse shape. The incident field at the scatterer is given by $P_j^{1/2}\eta^{1/2}/[2(\pi)^{1/2}r_n]$ so that the convention regarding the scattering coefficients in (1) agrees with that of McCormick and Hendry (1975).

In weather radars, echo voltages V_{ij} are processed to retrieve properties of hydrometeors. For a single hydrometeor the voltage V_{ij} is proportional to the scattering coefficient and can be written as

$$v_{ij}(n) = s_{ij}(n)F(\mathbf{r}_n)\exp(-j2kr_n). \qquad (2)$$

The proportionality factor $F(\mathbf{r}_n)$ contains range dependence, attenuation, weighting function of the resolution volume and other system parameters (Doviak and Zrnić 1984). For an ensemble of scattering hydrometeors the composite voltage V_{ij} is a superposition of voltages from each individual scatterer

$$V_{ij} = \sum_n s_{ij}(n)\exp(-j2kr_n)F(\mathbf{r}_n). \qquad (3)$$

The mean value of V_{ij} is zero because contributions by the phase terms in the summation over n cancel each other. Thus, radar meteorologists use various second-order moments, $\langle V_{ij}V^*_{kl}\rangle$, to quantify hydrometeor scatterers. This is different than procedures for observations of the earth surface for which optimum polarizations are sought to maximize differences between echo powers of contrasting surfaces (Kostinski et al. 1988).

Detailed derivation of the relationship between some second-order moments and the scattering coefficients can be found elsewhere (e.g., Jameson 1985). Starting from (2) the expected value of the general term is

$$\begin{aligned}\langle V_{ij}V^*_{kl}\rangle &= \Big\langle \sum_n\sum_m \{s_{ij}(n)s^*_{kl}(m)\\ &\quad\times \exp[-j2k(r_n-r_m)]F(\mathbf{r}_n)F^*(\mathbf{r}_m)\}\Big\rangle\\ &= \sum_n \langle [s_{ij}(n)s^*_{kl}(n)]\,|F(\mathbf{r}_n)|^2\rangle\\ &= \langle s_{ij}s^*_{kl}\rangle \int |F(\mathbf{r}_n)|^2 dV/V_6 \end{aligned} \qquad (4)$$

where V_6 is the resolution volume (Doviak and Zrnić 1984) and brackets denote expectations. In the last equality the summation over n is replaced with the integral over the resolution volume weighting function where it is assumed that scatterer distribution is homogeneous.

For the most general case the second-order moments (4) can be grouped in a four by four covariance matrix, but because of reciprocity the term $V_{ij} = V_{ji}$ so that the covariance matrix reduces to a three by three dimension (Borgeaud et al. 1987). It is evident from (4) that the voltage covariance matrix is a scalar multiple of the scattering covariance matrix defined as

$$\begin{vmatrix} \langle |s_{hh}|^2\rangle & \langle s_{hv}s^*_{hh}\rangle & \langle s_{vv}s^*_{hh}\rangle \\ \langle s_{hh}s^*_{hv}\rangle & \langle |s_{hv}|^2\rangle & \langle s_{vv}s^*_{hv}\rangle \\ \langle s_{hh}s^*_{vv}\rangle & \langle s_{hv}s^*_{vv}\rangle & \langle |s_{vv}|^2\rangle \end{vmatrix}. \qquad (5)$$

The expectations can be expressed in terms of the distribution of the hydrometeor's properties (i.e., equivalent volume diameter, shape, canting angle, etc.). Thus, the general term is

$$\langle s_{ij}s^*_{kl}\rangle = \int N(\mathbf{X})s_{ij}s^*_{kl}d\mathbf{X}. \qquad (6)$$

$N(\mathbf{X})$ is the probability density of the scatterer's properties. These properties are represented by a vector $\mathbf{X}$.

The off-diagonal symmetric terms in the covariance matrix are conjugates of each other; therefore, there are nine real quantities (three real on the main diagonal and the remaining six real from the off-diagonal terms) that a polarimetric radar can measure (Ioannidis and Hammers 1979).

Most terms of the covariance matrix have been used by themselves or in combination with others to infer properties of the scattering hydrometeors. The well-known quantities that are derived from the covariances are

Reflectivity factor at horizontal polarization:

$$Z_h = (4\lambda^4/\pi^4|K_w|^2)\langle |s_{hh}|^2\rangle, \qquad (7)$$

Reflectivity factor at vertical polarization:

$$Z_v = (4\lambda^4/\pi^4|K_w|^2)\langle |s_{vv}|^2\rangle, \qquad (8)$$

Differential reflectivity:

$$Z_{dr} = 10\log(\langle |s_{hh}|^2\rangle/\langle s_{vv}|^2\rangle), \qquad (9)$$

Linear depolarization ratios:

$$\mathrm{LDR}_{hv} = 10\log(\langle |s_{hv}|^2\rangle/\langle |s_{vv}|^2\rangle), \qquad (10a)$$

and

$$\mathrm{LDR}_{vh} = 10\log(\langle |s_{hv}|^2\rangle/\langle |s_{hh}|^2\rangle), \qquad (10b)$$

Correlation coefficient at zero lag:

$$\rho_{hv}(0) = \langle s_{vv}s^*_{hh}\rangle/[\langle |s_{hh}|^2\rangle^{1/2}\langle |s_{vv}|^2\rangle^{1/2}]. \qquad (11)$$

This list contains five independent variables, three real from the diagonal terms and one complex off-diagonal term. The other two complex measurands have been less utilized although Jameson (1985) has shown their explicit dependence on the scattering coefficients. Next the three complex terms of the covariance matrix can be expressed in terms of amplitude and phases as

$$\langle s_{vv}s^*_{hh}\rangle = \langle |s_{vv}s^*_{hh}|\exp[j(\delta_{vv}-\delta_{hh})]\rangle \qquad (12)$$

$$\langle s_{hv}s^*_{vv}\rangle = \langle |s_{hv}s^*_{vv}|\exp[j(\delta_{hv}-\delta_{vv})]\rangle \qquad (13)$$

$$\langle s_{hv}s^*_{hh}\rangle = \langle |s_{hv}s^*_{hh}|\exp[j(\delta_{hv}-\delta_{hh})]\rangle \qquad (14)$$

where the phase δ_{ij} of the scattering coefficient s_{ij} corresponds to the lag or lead angle of the backscattered field (polarization i) with respect to the incident field (polarization j) at the location of the scatterer.

Hydrometeors for which the Rayleigh–Gans theory is applicable (usually at S-band frequencies) have very small differences of backscattering phase angles (Jameson and Mueller 1985) so the three terms (12, 13, 14) are nearly real. Thus, the total number of useful measurands would be reduced to six. But at higher frequencies this reduction may not be possible and furthermore the backscatter phase differences may have meteorological significance. Analogously to (11) two more correlation coefficients can be defined as

$$\rho_v = \langle s_{hv}s_{vv}^* \rangle / [\langle |s_{hv}|^2 \rangle^{1/2} \langle |s_{vv}|^2 \rangle^{1/2}] \quad (15)$$

$$\rho_h = \langle s_{hv}s_{hh}^* \rangle / [\langle |s_{hv}|^2 \rangle^{1/2} \langle |s_{hh}|^2 \rangle^{1/2}]. \quad (16)$$

The symbols ρ_h, ρ_v are chosen to designate the correlation coefficients between the copolar (indicated by the subscript) and cross-polar components of the same echo and distinguish these from the correlation between the copolar components ρ_{hv} (where both indices stand for the received polarization).

For hydrometeors, small compared to radar wavelength, the two correlations (15) and (16) are about equal and that would further reduce the number of useful measurands to five. Moreover it remains to be demonstrated that ρ_v and/or ρ_h could add significantly different information to what ρ_{hv} already carries. If there is no added information the number of useful measurands would be four; this is a realistic possibility in view of the observations by Illingworth and Caylor (1989) who claim that ρ_h through the bright band did not contain useful information. Even for non-Rayleigh scatterers with regular shapes, such as spheroids, the correlations (ρ_v and ρ_h) may be similar; but, in general, irregularly shaped scatterers are expected to produce different correlations.

The discussion so far deliberately ignored propagation effects in order to clearly identify the backscattering properties of the hydrometeors in the radar resolution volume. Both attenuation and phase shift along propagation path affect the received signals and correction for these may need to be made (Bebbington et al. 1987). At a 10-cm wavelength differential phase shift along propagation paths can be measured and related to attenuation. Differential attenuation is rather small and is also linearly related to differential phase shifts (Bringi et al. 1990). Therefore I will consider only the differential phase shifts. The shifts can be introduced directly into the off diagonal terms of the covariance matrix. Thus, they also affect the correlation coefficients (11), (15), and (16). Because radar measurements contain both the differential phase shift upon backscatter and the differential phase shift along the propagation path it is instructive to write the correlation coefficient in which the two are separated. Thus the correlation of the copolar signals is given by

$$\rho_{hv}(0) \exp[j(\phi_{hh} - \phi_{vv})] \quad (17)$$

and the correlations of the co- and cross-polar echoes are

$$\rho_v \exp[j(\phi_{vv} - \phi_{hv})] = \rho_v \exp[j(\phi_{vv} - \phi_{hh})/2] \quad (18)$$

$$\rho_h \exp[j(\phi_{hh} - \phi_{hv})] = \rho_h \exp[j(\phi_{hh} - \phi_{vv})/2]. \quad (19)$$

The phase shifts ϕ_{ij} are cumulative for the total round-trip between the radar and the center of the resolution volume. Reciprocity is again invoked by setting $\phi_{hv} = \phi_{vh}$, and the right sides of (18) and (19) are obtained by inserting the equality

$$\phi_{vh} = (\phi_{hh} + \phi_{vv})/2. \quad (20)$$

Note that the differential phase

$$\phi_{dp} = \phi_{hh} - \phi_{vv} \quad (21)$$

can be obtained directly from any one of the equations (17) to (19) if the scattering is Rayleigh because ρ_{hv}, ρ_h, or ρ_v would all be real. Otherwise there would be differential phase shifts upon scattering that can not be separated from ϕ_{dp} without accounting for propagation effects.

It has been suggested that the differential propagation constant K_{dp}, which is a range derivative of ϕ_{dp}, can be a good estimator of rainfall (Sachidananda and Zrnić 1986; Sachidananda and Zrnić 1987) and that it can also indicate the rain amount in a mixture of rain with hail (Balakrishnan and Zrnić 1990a). Because K_{dp} is derived from ϕ_{dp} it is not a fundamental quantity and hence will not be discussed further.

3. Processing to retrieve the covariances

Systems that sequentially transmit linear horizontal and vertical fields employ a fast waveguide switch in which the transit time is less than a few microseconds. These are four port devices that may be used to receive simultaneously the copolar and cross-polar echoes and channel them into separate receivers (Fig. 1). There are practical and economical reasons for high-power polarimetric radars to use one receiver because then the absolute calibration of phases is not critical and there is no differential gain through the receiver.

The switch and other pertinent components in Fig. 1 illustrate a polarimetric radar with a single receiver. A brief explanation of its operation follows. If the magnetic field driving the switch is activated for one position (for example H) the transmitted power from port 1 flows to port 2 and is radiated with horizontal polarization. In this switching state the input signal at port 2 flows to a termination at port 3 and the input at port 4 to port 1. Therefore, the cross-polar echo (V) is channeled to the receiver and the co-polar to the termination. If the switch is reversed immediately after transmission, the flow of power is from port 2 to 1 and from port 4 to 3 and the copolar power (H) is channeled to the receiver. Therefore, by proper timing of the switch with respect to the transmitter pulse either copolar or cross-polar echoes can be presented to the receiver. Note that the receiver is not connected to port 3 to avoid an extra set of rotary joints.

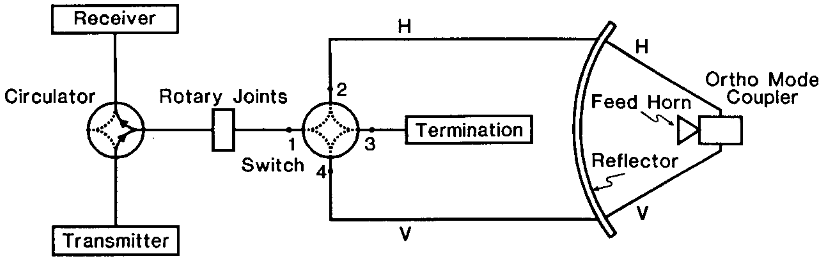

FIG. 1. Radar system with a single receiver and a switch that controls the transmitted polarization and the received polarization.

Mueller (1984) has proposed an algorithm that yields differential phase shift from sequential measurements. Processing to retrieve simultaneously the differential phase shift and Doppler spectral moments from a scheme that transmits alternating horizontal and vertical fields is discussed by Sachidananda and Zrnić (1989) and calculation of the copolar correlation $\rho_{hv}(0)$ is given in Balakrishnan and Zrnić (1990b). The scheme proposed in this paper extends these techniques to include the correlations (ρ_h and ρ_v) and the linear depolarization ratio that heretofore have not been attempted with a single channel. In the course of the discussion the two previous methods are reviewed in order to contrast them to the one that recovers all the covariances.

Consider the following transmit–receive sequence that for reference will be called a pair sequence 1.

Transmit sequence	H	V	H	V	H
Receive sequence	$V_{hh}(4k)$	$V_{vv}(4k+1)$	$V_{vh}(4k+2)$	$V_{vv}(4k+3)$	$V_{hh}(4k+4)$

where $k = 0, 1, 2\cdots$.

The time index k identifies a set of four polarization coded transmit–receive pairs. Thus $V_{hh}(4k)$ is a complex voltage corresponding to the horizontal component of the polarized echo for a horizontally polarized transmitted wave that was emitted at the $4k$th pulse repetition time T_s. Similarly $V_{vh}(4k+2)$ is the vertical component of the polarized echo for the horizontally polarized transmitted field at $(4k+2)T_s$. The period of pair sequence 1 is $4T_s$ which is indicated by the multiplying factor 4; thus the last pair $[H, V_{hh}(4k+4)]$ is at the beginning of the next period.

From the pair sequence 1 it is possible to form variances and covariances by summing products of the sequentially received echoes. Rather than explicitly showing the sums, I will use expectations because then all quantities are exact, the notation is also more compact, and the number of samples does not enter into consideration. Therefore, the powers become

$$P_h = \langle |V_{hh}(4k)|^2 \rangle \quad (22)$$

$$P_v = \langle |V_{vv}(4k+1)|^2 \rangle \quad (23)$$

$$P_{vh} = \langle |V_{vh}(4k+2)|^2 \rangle, \quad (24)$$

and the correlations of various echoes are

$$|\rho(T_s)|\rho_{hv}(0)\exp[j(\omega T_s + \phi_{dp})] = \langle V_{hh}^*(4k)V_{vv}(4k+1)\rangle/(P_hP_v)^{1/2} \quad (25a)$$

$$|\rho(T_s)|\rho_{hv}(0)\exp[j(\omega T_s - \phi_{dp})] = \langle V_{vv}^*(4k+3)V_{hh}(4k+4)\rangle/(P_hP_v)^{1/2} \quad (25b)$$

$$|\rho(2T_s)| = |\langle V_{vv}^*(4k+1)V_{vv}(4k+3)\rangle|/P_v \quad (26)$$

$$|\rho(T_s)|\rho_v \exp[j(\omega T_s - \phi_{dp}/2)] = \langle V_{vv}^*(4k+1)V_{vh}(4k+2)\rangle/(P_vP_{vh})^{1/2} \quad (27a)$$

$$|\rho(T_s)|\rho_v \exp[j(\omega T_s + \phi_{dp}/2)] = \langle V_{vh}^*(4k+2)V_{vv}(4k+3)\rangle/(P_vP_{vh})^{1/2} \quad (27b)$$

$$|\rho(2T_s)|\rho_h \exp[j(2\omega T_s + \phi_{dp}/2)] = \langle V_{hh}^*(4k)V_{vh}(4k+2)\rangle/(P_hP_{vh})^{1/2} \quad (28a)$$

$$|\rho(2T_s)|\rho_h \exp[j(2\omega T_s - \phi_{dp}/2)] = \langle V_{vh}^*(4k+2)V_{hh}(4k+4)\rangle/(P_hP_{vh})^{1/2}. \quad (28b)$$

In these equations $\rho(T_s)$ is the correlation coefficient at lag T_s between two echoes with identical polarizations; its argument ωT_s is caused by the mean Doppler shift ω. Relative motion of scatterers causes Doppler spectral broadening or reduction in the correlation. Note that reciprocity implies $P_{hv} = P_{vh}$, which is used several times in the equations. On the right side of these equations are the quantities measurable by radar and on the left are the desired parameters, which can be related to hydrometeors in the resolution volume.

The most important aspect of Eqs. (25), (27) and (28) is the fact that various correlation coefficients between components of either polarization at one sample time and components at another sample time can be expressed as a product of coefficients due to Doppler spread and due to polarization effects. Sachidananda and Zrnić (1985) capitalized on that fact in their analysis of the variance of differential reflectivity estimates. Because of its fundamental importance to the present exposition some further discussion follows.

Two spectral broadening mechanisms are pertinent here. One is the relative motion of scatterers in the radar resolution volume and the other is the change of shape and/or wobbling of the hydrometeors (Zrnić and Doviak 1989). Because both are contained in the Doppler correlation coefficient, the only other decorrelating effect due to physical reasons that remains to be accounted for is caused by the change in polarization. This partition and independence is justified by the fact that the relative motions and positions of scatterers are independent of their sizes and shape. Independence of positions and velocities holds also for the mean motion at low elevation angles. But at high elevation angles the fall speeds of hydrometeors may cause mean Doppler shifts in the co- and cross-polar channels to be different (Metcalf 1986) and that would require modifications of Eqs. (25) to (27).

I have assumed that the correlations ρ_{hv} and ρ_v, ρ_h are real. This is not a restriction because, if they were complex, then their arguments would be combined with the differential phases. At present there is no method to separate these two in a measurement at a single range location regardless if one or two receivers are used.

A few words about the number of measurands, their relations to the terms of the covariance matrix, and the procedure to calculate these are in order. Equations (25a) and (25b) are needed to compute ω and ϕ_{dp} by taking products and conjugate products. There are practical advantages to compute first ϕ_{dp}, correct it for possible ambiguities (ϕ_{dp} is unambiguous over 180° and usually increases with range and this continuity can be used to resolve the ambiguity in ϕ_{dp}), and then calculate ω as done by Sachidananda and Zrnić (1989). The average of the magnitudes (25a) and (25b) is just the product $|\rho(T_s)|\,|\rho_{hv}(0)|$ from which $|\rho(T_s)|$ must be separated to obtain $|\rho_{hv}(0)|$; this is possible for weather echoes because their correlation coefficients have Gaussian shape and hence satisfy

$$|\rho(T_s)| = |\rho(2T_s)|^{1/4}. \tag{29}$$

Thus, the sole purpose of (26) is to use it to obtain the Doppler correlation at lag T_s given by (29).

One equation in each pair (27a,b) and (28a,b) is redundant but can be used to improve the estimates. Assuming that the Doppler parameters are calculated, as suggested in the previous paragraph, the total number of independent equations is six; three real, (22), (23) and (24), and three complex, (25a), (27a) and (28a). Obviously the powers given by the real equations are proportional to the diagonal terms of the covariance matrix. The complex equations produce the off-diagonal terms.

The outlined procedure is suitable for real time calculation because all data are from at most five consecutive echoes and each cumulative sum spans at most two pulse repetition times (PRTs). For brevity I will refer to it as a sequential method.

There is another way to obtain the magnitudes of correlation coefficients but it requires storage of the power sequences. Figure 2a illustrates the method a variant of which has been used successfully by Caylor and Illingworth (1989) to compute $|\rho_{hv}(0)|$; we have tested it and found no significant difference between it and the sequential method. But in these experiments the transmit–receive H and V sequences were alternating so that only one sample needed to be interpolated between any two consecutive samples (as in the P_v sequence of Fig. 2a). In Fig. 2a the P_h and P_{vh} sequences require interpolation across three T_s intervals and this could introduce unacceptable errors. The importance of interpolation schemes stems from the fact that they are suitable for incoherent radar systems. Correlation coefficients obtained from the interpolated power sequences equal to the square of the desired coefficients in the case of weather echoes. For example $|\rho_h|$ can be computed from

$$|\rho_h|^2 = \mathrm{cor}(P_h, P_{vh}) \tag{30}$$

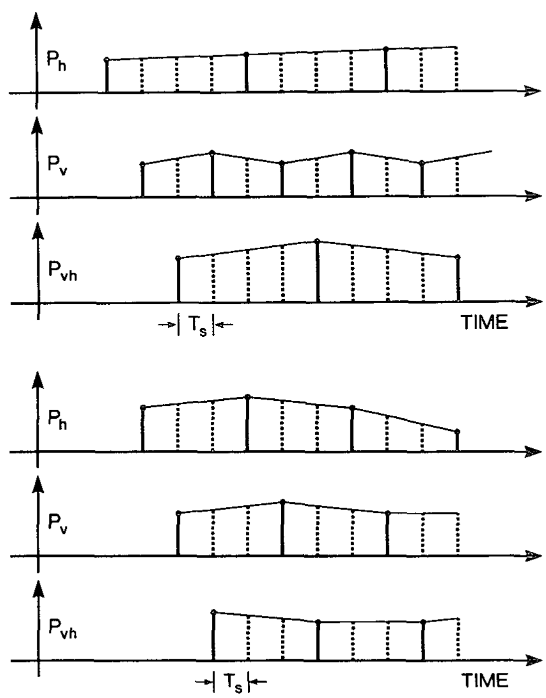

FIG. 2. (a) Power samples available at the receiver for the pair sequence 1. Dashed samples are not available and must be obtained by interpolation. (b) As in (a) except that it is for the sequence pair 2.

where cor signifies the correlation coefficient between the interpolated sequences.

The sequence pair 1 is not the shortest nor the only one of the same length that can yield all the terms of the polarimetric covariance matrix. For example, an identical result can be obtained with a complementary sequence pair in which all H polarizations are changed to V and vice versa. Also reversal of the time axis (i.e., a sequence pair run backwards) produces a satisfactory variant.

The shortest possible sequence pair from which one can reconstruct the covariance matrix must have a period of at least three PRTs. Two PRTs are needed to transmit and receive a pair of likewise polarizations and one PRT is needed for cross-polar transmission–reception. A candidate for such a shortest sequence pair, which will be called sequence pair 2, is

Transmit sequence	H	V	H	H
Receive sequence	$V_{hh}(3k)$	$V_{vv}(3k+1)$	$V_{vh}(3k+2)$	$V_{hh}(3k+3)$.

Naturally a time-reversed sequence pair, a complement of this sequence pair, and the time-reversed complement would also serve the purpose. The indices and the notation here are the same as in the sequence pair 1; thus, the period is three and the last pair [H, $V_{hh}(3k+3)$] is at the beginning of the next period.

To save space and because powers can be written by inspection I will write only the cross-covariances for the sequence pair 2.

$$|\rho(T_s)|\rho_{hv}(0)\exp[j(\omega T_s+\phi_{dp})] = \langle V_{hh}^*(3k)V_{vv}(3k+1)\rangle/(P_hP_v)^{1/2} \quad (31)$$

$$|\rho(T_s)|\rho_v\exp[j(\omega T_s-\phi_{dp}/2)] = \langle V_{vv}^*(3k+1)V_{vh}(3k+2)\rangle/(P_vP_{vh})^{1/2} \quad (32)$$

$$|\rho(T_s)|\rho_h\exp[j(\omega T_s-\phi_{dp}/2)] = \langle V_{vh}^*(3k+2)V_{hh}(3k+3)\rangle/(P_hP_{vh})^{1/2} \quad (33)$$

$$|\rho(3T_s)| = |\langle V_{hh}^*(3k)V_{hh}(3k+3)\rangle|/P_h. \quad (34)$$

From one of the two pairs [(31), (32)] or [(31), (33)] one can solve for ω and ϕ_{dp} and then the Doppler correlation coefficient $|\rho(T_s)|$ can be eliminated because it equals the sixth root of (34) if spectra have Gaussian shape. But at lag three the correlation may be more corrupted by noise than at lag two because its value is smaller. This is not the only disadvantage of this scheme. Note that either Eq. (32) or (33) that must be used to estimate the Doppler shift contains the cross-polar component, which is typically two orders of magnitude smaller than the copolar component. Thus, the estimate of ω would have a larger variance than the one from the pair [(25a), (25b)] and may not be acceptable. Interpolation to obtain power correlation coefficients (that do not depend on the Gaussian assumption for the spectral shape) is better balanced in this example because there are two samples that need to be interpolated within $3T_s$ for any one of the powers (Fig. 2b).

4. Summary

The motivation behind this paper is to indicate how a radar with a single receiver and a switchable polarization can obtain all polarimetric measurands that are normally expected from two receivers. This first required identification of all the measurands. For that purpose, a three by three polarimetric covariance matrix proved very useful. The matrix contains nine independent real quantities, all of which can be calculated providing a special sequence pair of transmitted–received polarizations is used. Two such sequence pairs are given; one of them has the shortest possible period of three PRTs.

Fundamental to the proposed scheme is a trade-off between parallel processing (that requires two receivers) and sequential processing with one receiver. This trade-off is possible because weather echoes are statistically stationary and highly correlated. Thus, the value of the correlations at zero lag as well as the mean Doppler shift can be estimated even though the measurements are not made simultaneously. Discussion of the estimation accuracy is beyond the scope of the present paper, yet it is clear that longer dwell times are needed because of sequential processing. It is relatively straightforward to eliminate the Doppler parameters from the equations for polarimetric measurands. Application of the method to noncoherent polarimetric radars is also possible and as expected no phase terms can be retrieved.

In general, differential phase shifts that may be estimated are composites of propagation and scattering phase shifts. At a 10-cm wavelength, phase shifts due to scattering are negligible (except in large hail or large wet snowflakes) and, therefore, the dominant phase term in the equations is from differential propagation.

In summary, it is demonstrated that simple to compute procedures requiring storage of conjugate products can yield all the polarimetric and Doppler parameters simultaneously if sequence-specific switching of the polarization-on-reception is employed. Thus, it is possible to derive well-known parameters such as the reflectivity factors, differential reflectivity, linear depolarization ratio, differential propagation constant, and the correlation coefficient between copolar received echoes. Two lesser known correlations between orthogonal echoes are also available, but their interpretation in the context of the ensemble of scattering hydrometeors needs to be explored.

Acknowledgments. I appreciate the patience and encouragements of Drs. R. J. Doviak and P. Mahapatra who listened to my frustrations at the time that I was trying to put on paper what seemed to be such a simple idea. Through their review of the manuscript Drs. R. J. Doviak and M. Chandra enhanced the clarity of presentation, and Dr. Doswell provided suggestion that strengthened the text.

REFERENCES

Balakrishnan, N., and D. S. Zrnić, 1990a: Estimation of rain and hail rates in mixed-phase precipitation. *J. Atmos. Sci.*, **47**, 565–583.

——, and ——, 1990b: Use of polarization to characterize precipitation and discriminate large hail. *J. Atmos. Sci.*, **47**, 1525–1540.

Bebbington, D. H. O., R. McGuinness and A. R. Holt, 1987: Correction of propagation effects in S-Band circular polarisation-diversity radars. *IEE Proceedings*, **134**, 431–437.

Borgeaud, M., R. T. Shin and J. A. Kong, 1987: Theoretical models for polarimetric radar clutter. *J. Electromag. Waves Appl.*, **1**, 73–89.

Bringi, V. N., V. Chandrasekar, N. Balakrishnan and D. S. Zrnić, 1990: An examination of radar propagation effects in rainfall at microwave frequencies. *J. Atmos. Oceanic Technol.*, **7**, 829–840.

——, and A. Hendry, 1990: Technology of polarization diversity radars for meteorology. *Radar in Meteorology*, D. Atla, Ed., Amer. Meteor. Soc., 153–190.

Caylor, I. J., and A. J. Illingworth, 1989: Identification of the bright band and hydrometeors using co-polar and dual polarization radar. Preprints, *24th Conf. on Radar Meteorology*, Tallahassee, Amer. Meteor. Soc., 9–12.

Doviak, R. J., and D. S. Zrnić, 1984: *Doppler Radar and Weather Observations*, Academic Press, pp. 458.

Illingworth, A. J., and I. J. Caylor, 1989: Cross polar observation of the bright band. Preprints, *24th Conf. on Radar Meteorology*, Tallahassee, Amer. Meteor. Soc., 323–327.

Ioannidis, G. A., and D. E. Hammers, 1979: Optimum antenna polarizations for target discrimination in clutter. *IEEE Trans. Antenn. Propag.*, **AP-27**, 357–363.

Jameson, A. R., 1985: Microphysical interpretation of multiparameter radar measurements in rain. Part III: Interpretation and measurement of propagation differential phase shift between orthogonal linear polarizations. *J. Atmos. Sci.*, **42**, 607–614.

——, and E. A. Mueller, 1985: Estimation of propagation-differential phase shift from sequential orthogonal linear polarization radar measurements. *J. Atmos. Oceanic Technol.*, **2**, 133–137.

Kostinski, A. B., B. D. James and W. M. Boerner, 1988: Polarimetric matched filter for coherent imaging. *Can. J. Phys.*, **66**, 871–877.

McCormick, C. G., and A. Hendry, 1975: Principles for the radar determination of the polarization properties of precipitation. *Radio Sci.*, **10**, 421–434.

Metcalf, J. I., 1986: Differential Doppler velocity observations in melting precipitation. Preprints, *23rd Conf. on Radar Meteorology*, Snowmass, Amer. Meteor. Soc., JP288–JP291.

Mueller, G. A., 1984: Calculation procedure for differential propagation phase shift. Preprints, *22nd Conf. on Radar Meteorology*, Zurich, Amer. Meteor. Soc., 397–399.

Sachidananda, M., and D. S. Zrnić, 1985: Z_{DR} measurement considerations for a fast scan capability radar. *Radio Sci.*, **20**, 907–922.

——, and ——, 1986: Differential propagation phase shift and rainfall rate estimation. *Radio Sci.*, **21**, 235–247.

——, and ——, 1987: Rain rate estimated from differential polarization measurements. *J. Atmos. Oceanic Technol.*, **4**, 588–598.

——, and ——, 1989: Efficient processing of alternately polarized radar signals. *J. Atmos. Oceanic Technol.*, **6**, 173–181.

Schroth, A. C., M. S. Chandra and P. F. Meischner, 1988: A C-band coherent polarimetric radar for propagation and cloud physics research. *J. Atmos. Oceanic Technol.*, **5**, 803–821.

Zrnić, D. S., and R. J. Doviak, 1989: Effect of drop oscillations on spectral moments and differential reflectivity measurements. *J. Atmos. Oceanic Technol.*, **6**, 532–536.