Analysis of Rotating Multicylinder Data in Measuring Cloud-Droplet Size and Liquid Water Content

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ABSTRACT

An objective method is presented for the analysis of rotating multicylinder data in measuring the liquid water content and median volume droplet diameter of icing clouds. The method is based on time-dependent numerical modeling of cylinder icing and requires no data on diameters of the iced cylinders. Developing a fully automatic rotating multicylinder instrument is proposed.

1. Introduction

Droplet size and liquid water content have traditionally been measured using various impactor devices (see Stainer and Stow 1976 for review) that require a very laborious analysis in counting individual droplets. Another drawback of the impact methods is their uncertain calibration, which has resulted in systematically erroneous results (Keller 1978; Makkonen and Stallabrass 1984).

Recently, optical devices—such as forward-scattering laser spectrometer probes (FSSPs)—have mostly replaced the older impactors in cloud physics research. However, their use in measuring properties of icing fog and clouds has been limited due to their relatively high cost and the necessary sophisticated electronics they require. These devices also require expert calibration (Dye and Baumgardner 1984; Heymsfield and Baumgardner 1985).

Another possibility is to use icing of rotating cylinders of various diameters for measuring the droplet size and liquid water content. This method was developed in the 1940s (Langmuir and Blodgett 1946) but has been utilized infrequently since then. Systematic rotating multicylinder (RMC) data have been collected only at the Mount Washington Observatory in New Hampshire (Howe 1991). Some liquid water content measurements using a similar principle have been made at the Elk Mountain Observatory in Wyoming (Rogers et al. 1983).

The RMC method has several advantages over optical particle spectrometers. The instrument is inexpensive, robust, and easy to use (Fig. 1). The main problems related to interpretation of data have recently been removed.

Icing wind tunnel experiments by Makkonen and Stallabrass (1987) showed excellent agreement between the theoretical and measured ice weights on rotating cylinders at collision efficiencies larger than 0.07. In these experiments the drop-size distribution and liquid water content (LWC) were determined independently by the FSSP. This provided verification of the Langmuir and Blodgett (1946) theory.

Recently, it was shown both empirically and theoretically (Finstad et al. 1988a) that collision efficiencies calculated by the median volume diameter (MVD) are very close to those calculated using the whole droplet distribution. In other words, we can justify that the drop size derived from the RMC results is, with considerable accuracy, the MVD.

In developing a computerized method of analysis it turned out that improvements over the previous analysis (Langmuir and Blodgett; Howell 1952; Brun et al. 1955; Howe 1960 1991) are possible and, indeed, necessary. These include taking into account time dependence, sublimation, and rejection of data outside the experimentally verified range of the droplet collision efficiency. In addition, more accurate parameterizations of the droplet collision efficiency and impact speed (Finstad et al. 1988b), as well as rime density, (Makkonen and Stallabrass 1984) are utilized in the improved method.

2. Data analysis method

A rotating multicylinder measurement provides ice weights on cylinders of known lengths and diameters. In addition, the exposure duration τ, wind speed V, air temperature t_a, and air pressure P are required. Ice thickness on the cylinders is usually also measured, but it is not used in the analysis method presented here.

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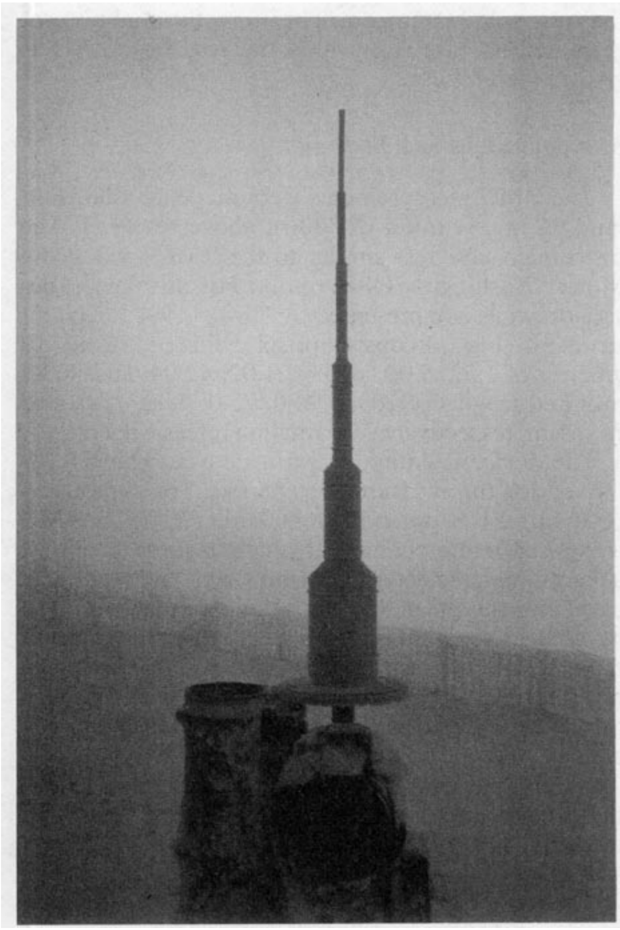

FIG. 1. Rotating multicylinder in operation.

The foundation of the data analysis is the fact that when MVD and LWC are known accurate time-dependent modeling of the ice weight on a small rotating cylinder at given V, t_a, and P is possible (Makkonen 1984; Makkonen and Stallabrass 1987). The reverse of this is then used; that is, by trial and error such a combination of MVD and LWC is found that the time-dependent numerical icing model predicts the measured ice weight for the period that the RMC was exposed to icing.

In principle, only two cylinders of sufficiently different diameters are required, since there are two unknowns (MVD and LWC) in the system. The accuracy of the method is, however, improved by including several cylinders and using a numerical optimation method. This is done as follows.

The modeled ice weights at given small-guessed MVD and LWC are tested against the measured ice weights by the linear regression analysis using the method of weighted least squares. In this method, the absolute error at each point is scaled by the measured value; that is, the square sum of relative errors is minimized. The linear correlation coefficient with respect to the regression line through the origin is then calculated. Next, the same procedure is repeated for an MVD that is 0.1 μm larger than the "previous guess." This goes on until the correlation coefficient is at maximum; that is, until it starts to decrease. The program then corrects the LWC by calculating the value that gives the 1:1 line at the MVD with the highest correlation. The calculation starts again using these "optimum" values of LWC and MVD as "first guesses" for the next phase. This continues until the optimized LWC and MVD are sufficiently stable. The procedure is schematically shown in Fig. 2.

The time-dependent numerical icing model used is that by Makkonen (1984) with the following modifications. The collision efficiency and droplet impact-speed parameterizations are from Finstad et al. (1988b). They are in even better agreement with the experimental data in Makkonen and Stallabrass (1987) than the original data by Langmuir and Blodgett (1946).

The density of rime is calculated by Eq. (1) (Makkonen and Stallabrass 1984),

$$\rho = 0.378 + 0.425\,(\log R) - 0.0823\,(\log R)^2, \quad (1)$$

where R is the Macklin's parameter,

$$R = \frac{V_0(\mathrm{MVD})}{2t_i}. \quad (2)$$

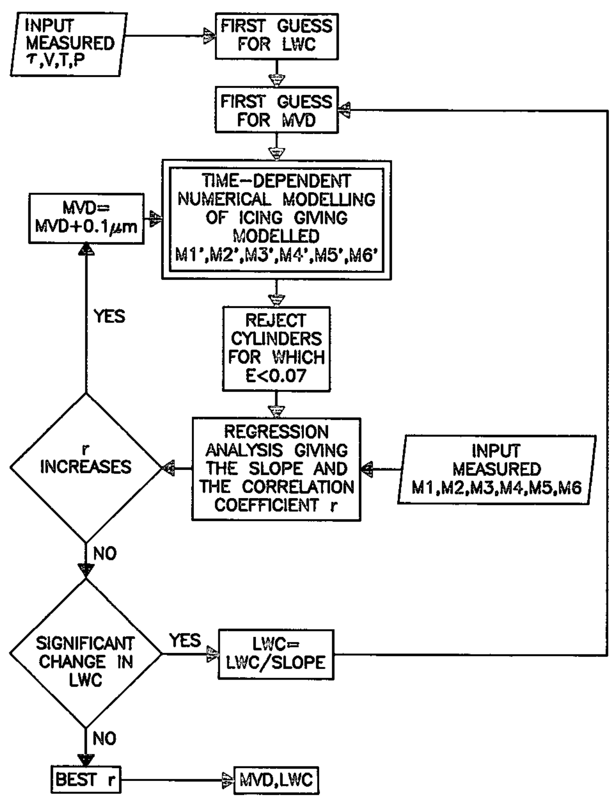

FIG. 2. Block diagram of the numerical RMC data analysis method.

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Here V_0 is the impact speed of the droplets (m s^{-1}) at the stagnation line calculated using the MVD (μm), and t_i is the modeled mean ice surface temperature (°C).

Equation (1) is based on laboratory measurements on rimed cylinders in which the droplet size was measured by a carefully calibrated FSSP. Due to the errors involved in impactor methods used earlier, the density values differ from those put forward by Macklin (1962).

The need to include sublimation in the model became evident when first analyzing the RMC data. The resulting modeled ice weights, when plotted against the measured ice weights, were not on a straight line. This is to be expected at low collision efficiencies, due to limitations of the theory (Finstad et al. 1988b), but not at collision efficiencies in the theoretically feasible and experimentally verified range. In addition, the measured ice thicknesses were consistently higher than predicted even when the ice weights were predicted correctly.

Both of the aforementioned discrepancies indicate that there is additional growth of low-density hoarfrost at least on the largest cylinders. Consequently, a subroutine to include the mass growth rate due to sublimation was incorporated into the model.

Sublimation is modeled using the modeled mean transfer coefficient and the ice surface temperature and water vapor pressure equations by Huffman and Norman (1988). It is assumed that the cloud is saturated with respect to a water surface; that is, the relative humidity is 100% and no ice particles are present in the cloud. Curvature effects on the equilibrium water vapor pressure over the ice surface are not considered because no rime or hoarfrost feathers were observed on the rotating cylinders. Rotation and simultaneous drop impingement apparently destroy the dendritic microstructure typical to hoarfrost (Schneider 1978) and soft rime (Gates et al. 1988) on fixed objects.

The value of 0.06 g cm^{-3} for hoarfrost density was chosen based on the smallest densities measured on rotating cylinders (Jones 1988). The overall accretion density ρ for each time step is then calculated as

$$\rho = \frac{M_R + M_H}{M_R/\rho_R + M_H/\rho_H}, \quad (3)$$

where M_R and M_H are the mass increments and ρ_R and ρ_H the densities of rime and hoarfrost, respectively.

The analysis program includes simulation of the ice surface temperature (Makkonen 1984) and rejects data for which $t_i \geqslant 0$°C. This avoids exceeding the Ludlam limit (Ludlam 1951; Fraser et al. 1953); that is, not attempting an analysis when the growth had been wet and water was being shed from the cylinders.

The RMC method is limited to cloud droplets only. For drops in the precipitation range, the collision efficiency becomes insensitive to changes in the MVD and makes the RMC analysis less accurate. Big drops may also splash at impact. Therefore, the analysis program rejects cases for which the resulting MVD is greater than 50 μm.

3. Applications and discussion

The RMC measurements were made at Ylläs, Finland, at an elevation of 700 m above sea level. The instrument design is similar to the RMC used at the Mount Washington Observatory. For this device, design drawings are presented by Howe (1991). The instrument (Fig. 1) consists of six cylinders having diameters of 1.52, 5.00, 10.15, 21.02, 42.04, and 80.80 mm and lengths of 103.5, 90.0, 75.0, 75.0, 75.0, and 75.0 mm, respectively. The rotation rate is 4 rev min^{-1}.

The device is equipped with a cover, which is removed for the measurement period. The typical exposure time is between 30 and 60 min. While the RMC is exposed, a microcomputer system records data from a thermometer and a deiced wind sensor and eventually calculates the mean values of the exposure period to be used in the analysis. After exposure, the cover is replaced and the instrument is removed from the support and the motor. The cylinders are then removed one at a time and weighed indoors using an electric balance with a resolution of 1 mg.

As an example of the RMC results and data analysis, the model output and measured values for the first measurement at Ylläs on 26 November 1987 are presented in Table 1.

Table 1 shows that the weighted linear correlation coefficient for the best-fit line is very high. The mean correlation coefficient of the 27 cases analyzed is .9983. The difference between the modeled and measured ice mass on the smallest cylinder (number 1) is less than 1%. This is comparable to the accuracy of the LWC value, since the error caused by inaccuracy of E is smallest for this cylinder. These results indicate that the accuracy of the RMC method (when using the numerical analysis presented here) is better than that of, say, the FSSP.

Another interesting feature of the results in Table 1 is the good agreement between the modeled and measured ice densities. This is significant because the correct diameter at each time step is necessary for accurate icing modeling. There is a discrepancy in the ice density only for the largest cylinder (number 6). There are several possible reasons for this discrepancy. First, the accuracy of a density measurement is by far the worst for the largest cylinder. Second, Table 1 shows that the fraction of rime mass is underestimated by the model for cylinder 6, since the theory is not valid for E this small. Finally, the approximation of the fixed hoarfrost density becomes more critical when the hoarfrost fraction increases. The data by Schneider (1978) indicate that hoarfrost density may, in fact, increase with increasing growth rate. There are, however, no data for combined rime and hoarfrost growth.

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TABLE 1. Example of conditions, measurement results, and model output for an RMC run. The largest cylinder (number 6) is rejected in the analysis because it is outside the verified range of the collision-efficiency theory.

	Exposure time 36 min 34 s		Wind speed 20.6 m s^{-1}		Air temperature −9.6°C	
Cylinder number	Measured ice weight (g)	Modeled ice weight (g)	Modeled hoarfrost (g)	Modeled ice temperature (°C)	Modeled ice density (g cm^{-3})	Measured ice density (g cm^{-3})
1	3.209	3.235	−0.027	−8.50	0.72	0.72
2	3.834	3.741	−0.015	−8.58	0.70	0.66
3	3.755	3.812	0.014	−8.69	0.65	0.62
4	4.077	4.187	0.107	−8.88	0.49	0.48
5	3.464	3.401	0.327	−9.07	0.28	0.27
6	1.970	1.379	0.723	−9.20	0.08	0.22
		Output: MVD	LWC	r		
		11.4 μm	0.237 g m^{-3}	.9996		

It is worth noting in Table 1 that sublimation decreases the ice mass for the two smallest cylinders and increases the mass for the larger ones. This, too, is typical of the analysis results. The origin of this phenomenon is demonstrated in Fig. 3 and can be explained as follows. For the big cylinders, the rime-icing intensity is small due to the small collision efficiency. Consequently, the latent heat flux released increases the ice surface temperature only slightly (Table 1). Thus, the saturation water vapor pressure on the ice surface is lower than the saturation water vapor pressure in the air (with respect to water surface). This results in condensation on the surface. In contrast, the ice surface temperature is higher (Table 1) on smaller cylinders due to a higher rime-icing intensity. This increases the saturation water vapor pressure on the ice surface to a value higher than that in air, resulting in evaporation. A general result of these mechanisms is that the lower the rime growth rate the higher the hoarfrost growth rate.

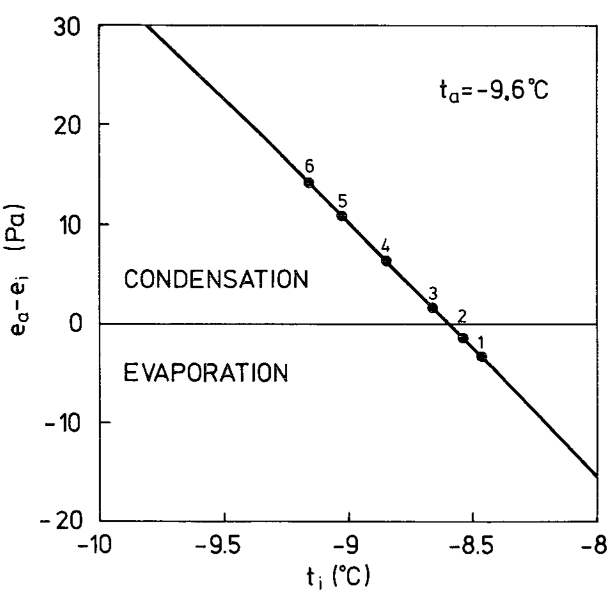

FIG. 3. Difference between the water vapor pressure e_a in a saturated cloud consisting of liquid droplets and at a temperature of −9.6°C and the water vapor pressure e_i over ice at a temperature t_i. During riming the ice temperature rises above the air temperature due to latent heat of fusion released at the surface. The modeled ice temperatures on the cylinders in the example in Table 1 are marked as dots showing evaporation on the small cylinders (numbers 1 and 2) and condensation on the large cylinders (numbers 3–6).

The belief that sublimation affects the ice weights was supported by trials on the model version with and without sublimation. The linear correlation coefficient between the modeled and measured ice weights increased when sublimation was included. This increase in r was insignificant in each separate case but occurred systematically in all cases. Moreover, with sublimation included, the agreement between the modeled and measured ice densities became much better, particularly on cylinders 4 and 5. An example to compare with Table 1 is that ice densities on these cylinders are 0.62 and 0.50 g cm^{-3}, respectively, when modeled without sublimation. In such a case, the resulting MVD is 11.8 μm, LWC is 0.230 g m^{-3}, and r is .9993.

There are some rare cases in our data where the calculated correlation coefficient r increases with increasing MVD but then suddenly decreases when the program accepts data from a large cylinder that were rejected at the previous MVD. When MVD further increases, the correlation coefficient starts to increase again and eventually reaches another maximum. In other words, there may be two maximum correlation coefficients and two sets of optimum MVD and LWC values. There is no objective means to determine which of these is the real set other than to compare the correlation coefficients or their statistical significance. This is a doubtful method because there are typically only four to six data points and the number of points is different in the two sets. The situations with dual output

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are related to wind speeds lower than 10 m s^{-1}. The present numerical method rejects these cases.

The numerical RMC data analysis method presented in this paper was compared with the manual analysis procedure used by the Mount Washington Observatory. Data collected at Mount Washington in 1986–1987 was used in the comparison. The results of the comparison in Figs. 4 and 5 show that the two methods give similar results. The numerical method gives a 6% higher LWC and a 5% larger droplet size in the mean. The linear correlation coefficient is also quite high (r = .967) for the MVD, in spite of the resolution of the manually analyzed MVD data being only 1 μm while being 0.1 μm for the numerically analyzed data.

It should be noted, however, that the results of the manual method are actually worse than indicated by Figs. 4 and 5 because the erroneous values, rejected by the numerical method, are not shown in the comparison. The raw data consisted of 224 measurements. Out of these a result from the numerical analysis method was obtained for 207 cases. Eleven measurements were rejected where the model showed wet growth; three where data for the smallest cylinder was missing; and another three where the modeled MVD was larger than 50 μm.

4. Concluding remarks

The technique presented in this paper provides an easy method to analyze RMC data. The data can be inserted directly into a microcomputer, and the processing takes only a few minutes. This analysis method also improves the accuracy of the RMC measurements,

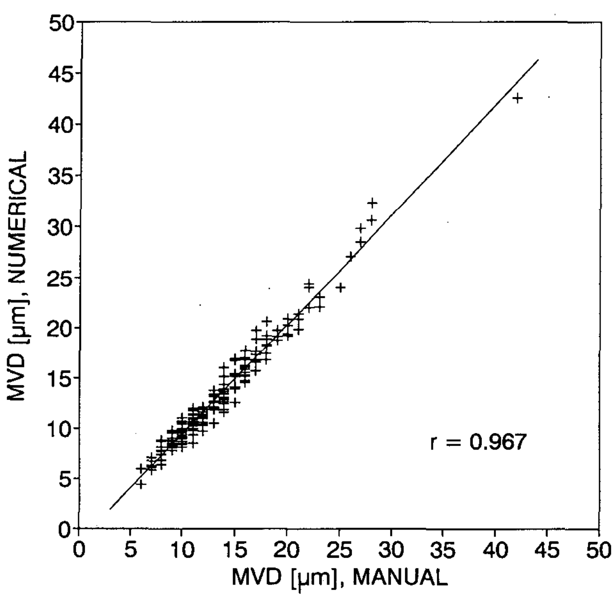

FIG. 4. Median volume droplet diameter (MVD) calculated by the improved numerical method versus MVD obtained using the manual method. Data from Mount Washington (1986–1987).

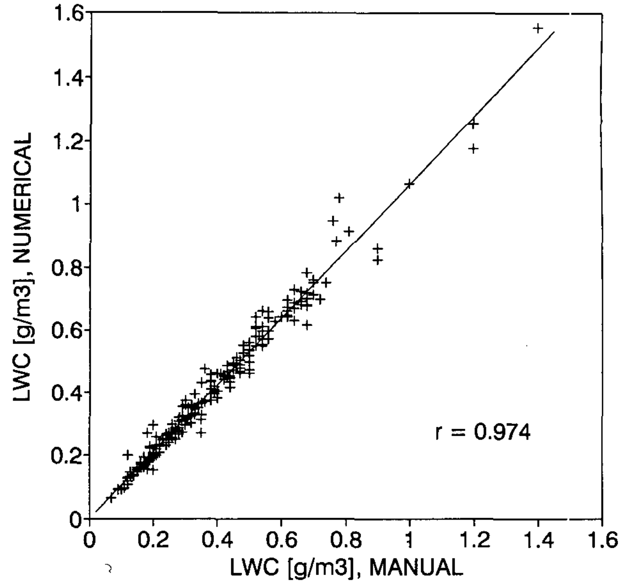

FIG. 5. Liquid water content (LWC) calculated by the improved numerical method versus LWC obtained using the manual method. Data from Mount Washington (1986–1987).

although a detailed error analysis is very difficult to make due to the complexity of the theory. Based on the application runs and the comparisons with the FSSP in Makkonen and Stallabrass (1987), the error in MVD and LWC is estimated to be less than 2% when the measurement is made carefully at wind speeds higher than 10 m s^{-1}.

The RMC method, as such, can be used in measuring the properties of cold clouds only, since it is based on icing. However, the new analysis method makes RMC measurements so much more attractive that modifying the RMC instrument for warm-cloud measurements should be considered. There are two options: either make the cylinders from a water-absorbing material or artificially cool the cylinders. The necessary subroutines to allow for evaporation and condensation in such applications are already included in the analysis program.

Automation of RMC measurement is also within reach since the improved method requires no ice diameter data. It is possible to design an RMC instrument that automatically measures the weight of ice on the cylinders. The device would include a shelter that covers the cylinders during weighing and a heater that removes the ice after the weights have been measured.

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REFERENCES

Brun, R. J., W. Lewis, P. J. Perkins, and J. S. Serafini, 1955: Impingement of cloud droplets on a cylinder and procedure for measuring liquid-water content and droplet sizes in supercooled clouds by rotating multicylinder method. Lewis Flight Propulsion Laboratory, Cleveland, Ohio, Report No. 1215, 43 pp.

Dye, J. E., and D. Baumgardner, 1984: Evaluation of the forward scattering spectrometer probe. Part I: Electronic and optical studies. *J. Atmos. Oceanic Technol.,* **1,** 329–344.

Finstad, K. J., E. P. Lozowski, and L. Makkonen, 1988a: On the median volume diameter approximation for droplet collision efficiency. *J. Atmos. Sci.,* **45,** 4008–4012.

——, ——, and E. M. Gates, 1988b: A computational investigation of water droplet trajectories. *J. Atmos. Oceanic Technol.,* **5,** 160–170.

Fraser, D., C. K. Rush, and D. Baxter, 1953: Thermodynamic limitations of ice accretion instruments. *Bull. Amer. Meteor. Soc.,* **34,** 146–154.

Gates, E. M., A. Liu, and E. P. Lozowski, 1988: A stochastic model of atmospheric rime icing. *J. Glaciol.,* **34,** 26–30.

Heymsfield, A. J., and D. Baumgardner, 1985: Summary of a workshop on processing 2-D probe data. *Bull. Amer. Meteor. Soc.,* **66,** 437–440.

Howe, J. B., 1960: Handbook for the rotating multicylinder method. Aeronautical Icing Research Laboratories, Tech. Note No. 568, 39 pp.

——, 1991: Rotating multicylinder method for the measurement of cloud liquid-water content and droplet size. U.S. Army Cold Regions Research & Engineering Laboratory, CRREL Report 91-2, 21 pp.

Howell, W. E., 1952: Comparison of three multicylinder icing meters and critique of the multicylinder method. NACA, Tech. Note No. 2708.

Huffman, G. J., and G. A. Norman, Jr., 1988: The supercooled warm rain process and the specification of freezing precipitation. *Mon. Wea. Rev.,* **116,** 2172–2182.

Jones, K. F., 1986, The density of natural ice accretions. *Fourth Int. Workshop on Atmospheric Icing of Structures,* Paris, Electricite de France, 114–118.

Keller, R. G., 1978: Measurement and control of simulated environmental icing conditions in an outdoor, free jet, engine ground test facility. NATO, AGARD Conf. Proc., No. 236, 7.1–7.13.

Langmuir, I., and K. Blodgett, 1946: A mathematical investigation of water droplet trajectories. *Collected Works of Irving Langmuir,* Vol. 10, Pergamon Press, 348–393.

Ludlam, F. H., 1951: The heat economy of rimed cylinders. *Quart. J. Roy. Meteor. Soc.,* **77,** 663–666.

Macklin, W. C., 1962: The density and structure of ice formed by accretion. *Quart. J. Roy. Meteor. Soc.,* **88,** 30–50.

Makkonen, L., 1984: Modeling of ice accretion on wires. *J. Climate Appl. Meteor.,* **23,** 929–939.

——, and J. R. Stallabrass, 1984: Ice accretion on cylinders and wires. National Research Council of Canada, Division of Mechanical Engineering, Tech. Rep. TR-LT-005, 50 pp.

——, and ——, 1987: Experiments on the cloud droplet collision efficiency of cylinders. *J. Climate Appl. Meteor.,* **26,** 1406–1411.

Rogers, D. C., D. Baumgardner, and G. Vali, 1983: Determination of supercooled liquid water content by measuring rime rate. *J. Climate Appl. Meteor.,* **22,** 153–162.

Schneider, H. W., 1978: Equation of the growth rate of frost forming on cooled surfaces. *Int. J. Heat Mass Transfer,* **21,** 1019–1024.

Stainer, R. D., and C. D. Stow, 1976: Direct methods for the measurement of small water drops. *New Zealand J. Sci.,* **19,** 135–143.

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