NOTES AND CORRESPONDENCE

The Distribution of C_n^2 as Measured by 50-, 405-, and 915-MHz Wind Profilers

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ABSTRACT

The distribution of backscattered power was computed for three wind profilers in the Colorado network that operated at 50, 405, and 915 MHz. Since the backscattered power is a function of fluctuations in the refractivity index, this power distribution also gives the relative distribution of C_n^2. Similar distributions were found for all three frequencies in the lower troposphere where the atmosphere is often well mixed. But near and above the tropopause the distributions for the three frequencies differed, probably because they responded to different processes in the atmosphere.

1. Introduction

With the increasing use of clear-air radars for wind profiling, it is important to understand how their performance can vary over time and with changes in location.[1] One measure of performance is the backscattered power from the atmosphere. Radar echos are produced in varying strengths as a function of radio frequency (rf) by hydrometeors, particulate matter, and insects in the atmosphere. Radar echo can be caused by fluctuations in the refractive index, although that echo is sometimes undetectable in the background of atmospheric noise. The clear-air backscattered power is proportional to the magnitude of those fluctuations or C_n^2, the structure parameter of refractive index (Gossard and Strauch 1982).

We examined the backscattered power for three different wind profilers of the Colorado network operated by the Wave Propagation Laboratory (WPL) for several years (Strauch et al. 1984). These were a 50-MHz profiler at Fleming, a 405-MHz profiler at Platteville, and a 915-MHz profiler at Denver. Although many more data were analyzed, we present a limited but representative selection here. The power measurements at 50, 415, and 915 MHz were analyzed for January and July 1985 and 1986.

We measured the return power on the wind profiler east beam at several heights. This beam is tilted at 15° from vertical. The 50-MHz wind profiler at Fleming used two beams tilted approximately 15° off vertical to measure the horizontal wind components. The 405- and 915-MHz profilers are more sensitive to hydrometeors than the 50-MHz radar.

Measurements were made 12 times each hour for each height and antenna beam. The 12 measurements were then averaged using a random sample consensus (Fischler and Bolles 1981), requiring a consensus of at least 4 out of 12 radial velocity measurements to fall within a window one-sixteenth of the Nyquist velocity interval. Only the measurements making up the consensus are averaged to produce each hourly report. If a consensus of fewer than four is found on any one beam, then it is assumed that no atmospheric signal is discernible and the measurements are rejected as noise. There is about a one in ten chance that random noise will pass the consensus test on one beam. There were very few times that the 50-MHz profiler failed to produce a measurement below 8 km MSL.

2. Relation of power to C_n^2

The radar equation (Probert-Jones 1962) shows that the received power is proportional to several radar parameters, for example, the transmitted power, the antenna gain, and the rf wavelength. It also shows that the received power is inversely proportional to range squared. These relations are fixed for a given radar, assuming that it does not deteriorate with time (a practical concern). In addition, the received power is proportional to the reflectivity η of the atmospheric target, be it precipitation or clear-air turbulence.

The backscattered signals from the clear air are the result of refractive-index fluctuations at spatial scales of half the radar wavelength, that is, $\lambda/2$. Tatarskii (1961) has shown that, for homogeneous and isotropic

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[1] A demonstration network of 30 UHF (404.37-MHz) wind profilers was installed in the central United States by the National Oceanic and Atmospheric Administration (NOAA) in 1991 to evaluate the utility of continuous wind profiler measurements for routine operations by the National Weather Service (NWS).

turbulence, the refractive-index structure function fluctuations are described by

$$D_n(r) = C_n^2 r^{2/3},$$

where $D_n(r)$ is the refractive-index structure function, r is the spatial scale, and C_n^2 is the refractive-index structure constant. If the $\lambda/2$ lies within the inertial subrange of turbulence then the radar reflectivity is given by (Ottersten 1969)

$$\eta = C_n^2 \lambda^{-1/3}.$$

Since the backscattered power is proportional to the reflectivity and the reflectivity in clear air is proportional to C_n^2, the measurement of power gives a direct measurement of the relative level of reflectivity turbulence in the atmosphere. Absolute levels are measurable only if the power is calibrated. Therefore, the power will be reported as a measure of the relative value for C_n^2.

3. Distribution of C_n^2

Nastrom et al. (1986) found with a 50-MHz radar that C_n^2 had a nearly lognormal distribution, with almost the same variance from one month to the next over one year but with median values for each month changing by 10 dB over the year. If the cumulative distribution of the logarithm of the power (dB) is plotted on a probability chart, then the distribution is linear, the power at the 50th percentile of the distribution is the average power, and the slope is related to the variance (Brooks and Carruthers 1953).

The cumulative distributions were also found to be lognormal or nearly lognormal at the lower heights for all the 50- and 405-MHz wind profilers, with a departure at 915 MHz. Figures 1, 2, and 3 show the power distributions during January 1985 for 50, 405, and 915 MHz, respectively. The distributions plotted on a probability scale show that the cumulative-distribution

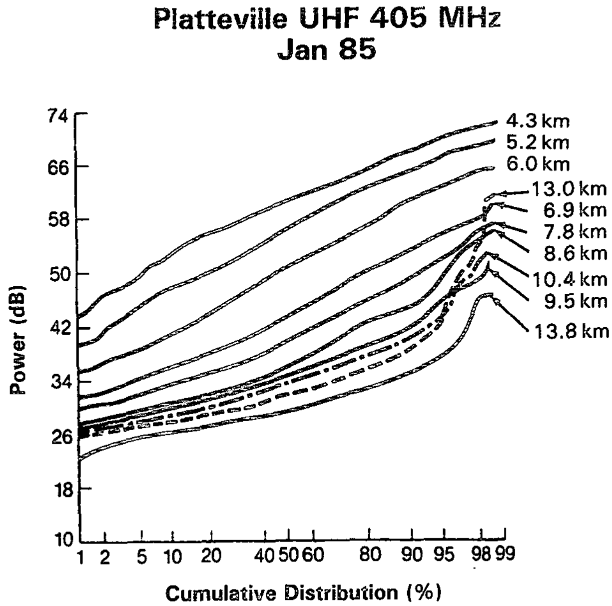

FIG. 2. Power distribution for the 405-MHz wind profiler at Platteville, Colorado. The distribution is given during January 1985 at a number of heights throughout the troposphere up to and above the tropopause.

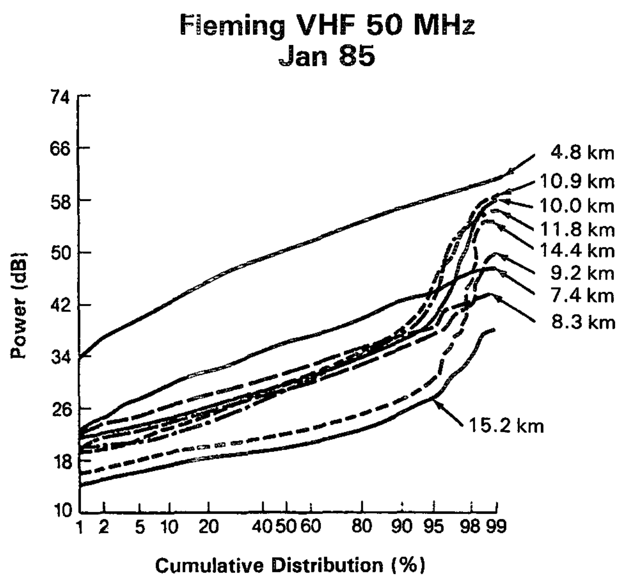

FIG. 1. Power distribution for the 50-MHz wind profiler at Fleming, Colorado. The distribution is given during January 1985 at a number of heights throughout the troposphere up to and above the tropopause.

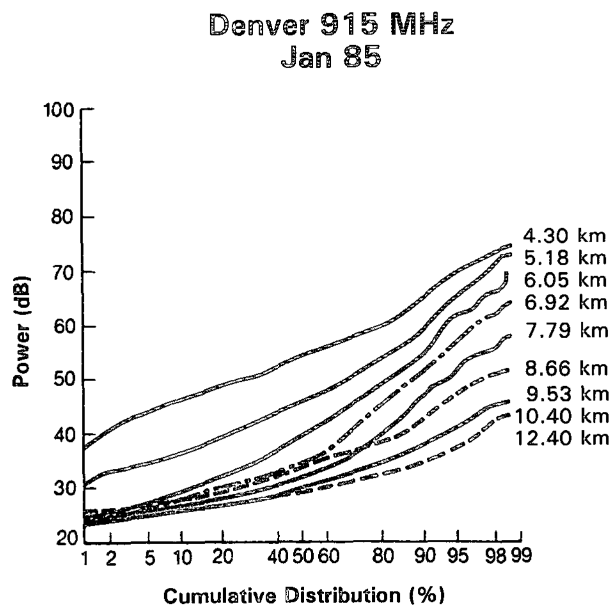

FIG. 3. Power distribution for the 915-MHz wind profiler at Denver, Colorado. The distribution is given during January 1985 at a number of heights throughout the troposphere up to and above the tropopause. The 10.4- and 12.4-km measurements are so close that they cannot be separated on this plot.

functions at all three frequencies are linear and have nearly the same slope at all heights below the 60th percentile showing that the backscattered power distribution is lognormal in this range. However, above 8 km, the 50- and 405-MHz distributions depart from this linear change abruptly between the 80th and 90th percentile. In this percentile range, the cumulative-distribution function can almost be represented by another linear function with a much steeper slope. This function extrapolated to the 50th percentile would have a much lower average power. The departure could be due to occasional stronger stratification in the upper-level winter atmosphere where vertical motion due to gravity waves may infrequently generate larger values of C_n^2. Stable and strong stratification is less likely to persist for long periods in the well-mixed lower troposphere; therefore, the distribution of C_n^2 is a single lognormal distribution. The 915-MHz distributions, on the other hand, deviate from lognormal with the largest deviations at 6.9 and 7.79 km, showing that the distribution is platykurtic; that is, there is a larger number of values in the center of the distribution than would be expected from the first or last part of the distribution function.

The cumulative distributions over one month were used because that period is long enough to remove diurnal and other short-term variations in C_n^2 but short enough to preserve seasonal and yearly changes (Frisch et al. 1990a; Frisch et al. 1990b). In order to see if there were seasonal and yearly changes, we compared a winter month and a summer month during two consecutive years. Figure 4 shows the 50-MHz power distributions during January and July of 1985 and 1986 at approximately 8 km. The slopes are not too dissimilar, but the power levels are higher during both months in the second year. This increased power must be due to interannual trends in C_n^2 because it is believed that

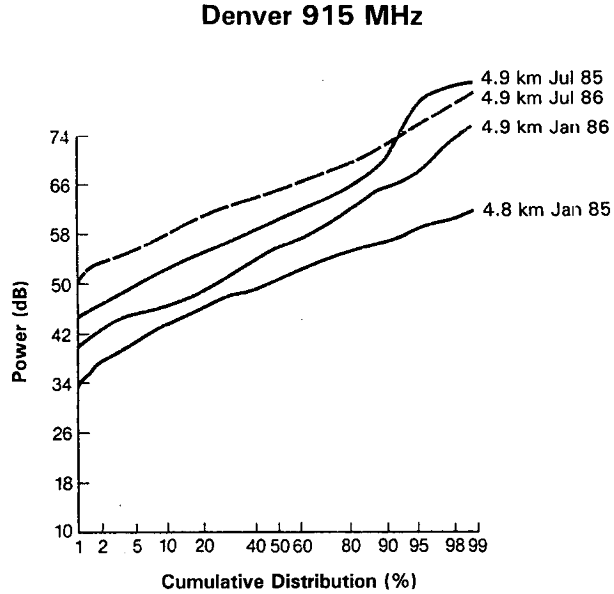

FIG. 5. Power distribution for the 915-MHz wind profiler at Denver, Colorado. The distribution is given during January and July in 1985 and 1986 at a height of approximately 5 km.

none of the radar parameters changed to produce greater sensitivity.

Figure 5 shows the 915-MHz power distributions at about 5 km during the same four months as in Fig. 4. Again, the slopes are similar and the power in 1986 is greater than that in 1985, supporting the upward trend in C_n^2. However, July 1985 had a small percentage of cases with the highest power. It is thought that they were due to unusually strong convective storm activity. This conjecture is supported by Fig. 6, which shows the 915-MHz power distributions near 12 km for the

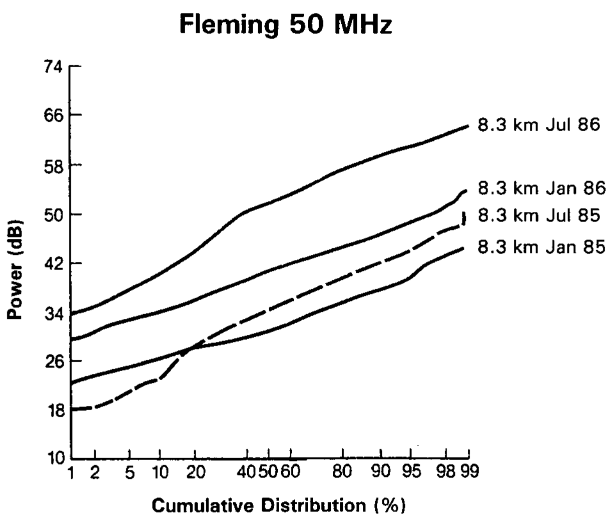

FIG. 4. Power distribution for the 50-MHz wind profiler at Fleming, Colorado. The distribution is given during January and July in 1985 and 1986 at a height of approximately 8 km.

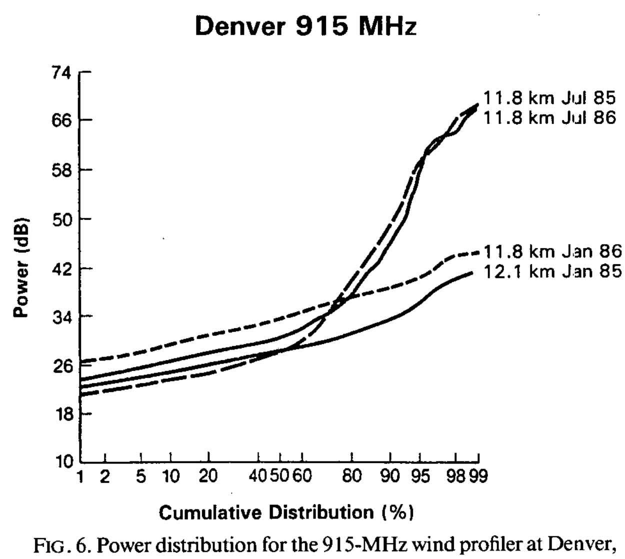

FIG. 6. Power distribution for the 915-MHz wind profiler at Denver, Colorado. The distribution is given during January and July in 1985 and 1986 at a height of approximately 12 km.

same winter and summer months in 1985 and 1986. The summer months show a bimodal distribution where the 90th percentile of the distribution function can be associated with very infrequent and very large values of C_n^2 and possibly with hydrometeors near the tropopause due to convective storms. On the other hand, the 50-MHz distributions around 12 km show the same pattern for both winter and summer months (Fig. 7). If the highest received powers are due to occasional gravity wave motion during winter and summer, then this demonstrates how the different radar frequencies respond to different atmospheric phenomena.

4. Discussion and conclusions

There appears to be a universal lognormal distribution of C_n^2 measured by the 50- and 405-MHz profilers in the lower troposphere and pronounced seasonal changes in this distribution near and above the tropopause when measured at 915 MHz (33 cm, Fig. 6) but not at 50 MHz (600 cm, Fig. 7). The 405-MHz profiler was not operated in July. We can estimate confidence limits of the distributions from their slopes and the number of measurements (Brooks and Carruthers 1953). The slopes of all the distributions below the 80th percentile are nearly the same. Since the slope is related to the standard deviation on this type of plot, the standard deviation can be estimated and confidence limits can be placed on the distributions below the 80th percentiles. The standard deviation is approximately 4.6–6.3 dB, and the number of samples is 720 for each distribution. Using the 6.3 dB for the standard deviation, the upper limit for the confidence interval can be estimated as ±0.9 dB at the 1st and 99th percentiles, ±0.5 at the 5th and 95th percentiles, and ±0.3 at the 50th percentile.

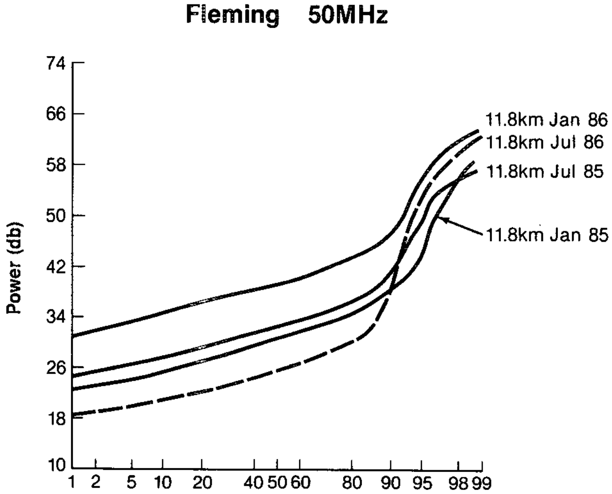

FIG. 7. Power distribution for the 50-MHz wind profiler at Fleming, Colorado. The distribution is given during January and July in 1985 and 1986 at a height of approximately 12 km.

The 50- and 405-MHz profilers experienced significant departures from a lognormal backscattered-power distribution relative to the extrapolated distribution function above the 80th percentile at the upper measurement heights. This departure indicates a few higher than expected values in the backscattered power. The Fleming 50-MHz profiler experienced this departure in both winter and summer. The 915-MHz wind profiler backscattered-power cumulative distribution at the higher elevations shows a strong change in the distribution at the 60th percentile in the summertime relative to the wintertime. This departure is puzzling. We can speculate that above the tropopause the gravity waves become unstable at times and generate more turbulence than the normal "background" level of turbulence. This increase in turbulence mixes with the stable temperature gradient generating more refractive-index fluctuations. However, in the wintertime, these increases in the background turbulence are not enough to generate an innerscale of turbulence at the wavelength matched to the Bragg condition for scattering at 915 MHz. In the summertime, there can be additional turbulence generated by convection, and it may be enough to generate a small enough innerscale of turbulence that the Bragg condition is satisfied for the 915-MHz profiler.

Frisch et al. (1990a) showed how monthly averages in received power can be useful in understanding the long-term performance of wind profilers. In addition, the power distributions presented here along with the radar parameters can be used to predict seasonal performance of wind profilers at different frequencies and at different radar sensitivities (Frisch et al. 1986). They could also be helpful in predicting the performance of profilers at different locations. For example, Fig. 8 shows the power distribution for the Platteville 405-MHz wind profiler from April through June 1985. If this wind profiler can detect an atmospheric echo only above 60 dB then the point where the cumulative distribution crosses the 60-dB level gives the percentage of time the profiler will make an observation. In this example, the 60-dB level corresponds to the 38th percentile on the distribution, meaning that the wind profiler will have no detectable signal 38% of the time. If the radar sensitivity is increased by 5 dB, then the distribution shifts upward by that amount. Therefore, the 55-dB level is used on this distribution, giving only 17% failure to detect a signal. On the other hand, if the sensitivity is reduced by 15 dB then the 75-dB level indicates that 97% of the time no signal is detected. Similar analysis can be done at other profiler frequencies. The Denver 915-MHz data indicate that there could be large differences in the performance between winter and summer at the upper heights. For example, in Fig. 6, if a profiler were designed to detect a signal at 42 dB then it would measure winds less than 2% of the time at 12 km in January and about 20% of the time in July.

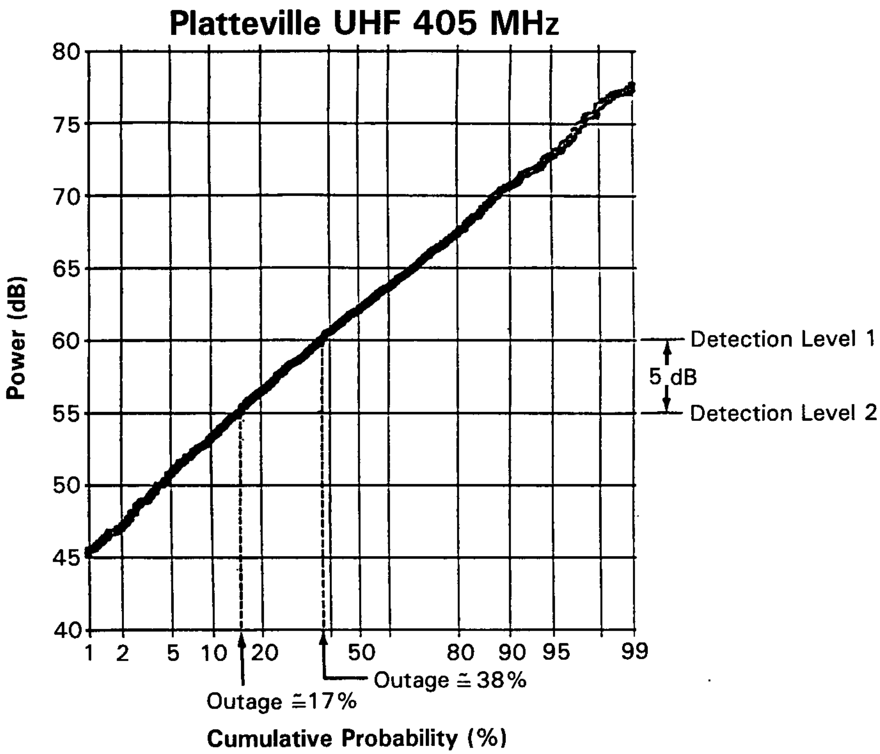

FIG. 8. Power distribution for the 405-MHz wind profiler at Platteville, Colorado. The distribution is given for April through June of 1985 at a height of approximately 6 km.

The meteorological causes for the different distributions of power are of interest, and some speculations concerning these have been presented. Regardless of the cause, the existence of such a wide range of distributions as a function of season, year, and radar frequency is significant for predicting wind profiler performance.

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