

Response Characteristics of the VACM Compass and Vane Follower

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(Manuscript received 7 March 1991, in final form 22 October 1991)

ABSTRACT

Several simple laboratory experiments have been conducted to study the dynamic behavior of the vector-averaging current meter (VACM) compass and vane follower. They demonstrate that the behavior of the compass and vane follower can be modeled as a damped linear harmonic oscillator for small-amplitude forcing. The combined eddy-current and bearing-friction torque nearly critically damps the free oscillation of the compass and vane follower. This frictional torque is proportional to the angular-velocity difference between the instrument magnet assembly and housing. Dynamic experiments on five compasses indicate a mean (undamped) resonant period of 3–5 s at 41°N. Similar experiments on two vane followers indicate a resonant period of 2–3 s.

For the VACM dynamic compass experiments, frictional torque allowed an angular oscillation of the compass housing to drive an oscillation of the compass magnet, and at resonant forcing, the compass magnet oscillates exactly in phase with its housing. For the VACM vane-follower experiments, angular motion of the vane magnet directly drove the vane follower. For resonant forcing of the vane magnet, the vane-follower oscillation overshoots the forcing slightly and is 90° out of phase with the forcing. The damped linear harmonic oscillator model suggests that a small-amplitude angular forcing of the compass or vane-follower housing (which may occur in the field due to mooring motion) should not cause any error in the vector-averaged headings. However, periodic angular oscillations near the resonant frequencies of the compass or vane follower could cause an error in the magnitude of the vector-averaged velocity. Forcing at frequencies lower than the well-defined resonant frequencies of the instruments should have little effect since directional errors do not exceed the angular resolution of the instruments at periods of ≥ 10 s.

1. Introduction

The vector-averaging current meter (VACM) has been widely used in oceanographic research since its initial introduction in 1973. The VACM utilizes a Savonius rotor and vane magnetically coupled to an internal vane follower to measure the speed and orientation of the flow relative to the instrument case. A magnetic compass senses the orientation of the case relative to magnetic north. This basic instrument is also used as a vector-averaged wind recorder (VAWR) to measure wind speed and direction. While the VACM can provide accurate current observations in steady or slowly varying flow conditions, significant measurement errors can occur in unsteady flow conditions due to the response characteristics of the vane follower and compass, as well as those of the Savonius rotor and

vane itself (see e.g., Saunders 1976, 1980; Beardsley 1987). While it is known that the VACM can experience angular oscillations with periods ranging from a few seconds to several minutes and longer due to mooring motion in unsteady flow, errors induced by these motions are not well understood. This study focuses on the dynamic response of the VACM compass and vane follower to oscillatory rotational forcing as a first step in developing a more comprehensive understanding of the compass and vane-follower behavior and their influence on velocity measurement error.

a. Compass design

The VACM compass is housed in an aluminum cylinder approximately 10.2 cm in diameter and 10.2 cm high. It uses two small bar magnets mounted in parallel to find magnetic north (see Fig. 1). The compass uses a 7-bit optical encoder to measure the orientation of the bar magnets relative to the housing, and the compass is generally considered to be accurate to ± 2 bits or 5.6° (Bryden 1976). Like any mechanical compass, the VACM compass is a damped oscillator. Unlike most compasses that are designed to minimize damping, the VACM compass was designed with near-critical damping to minimize resonant oscillation of the mag-

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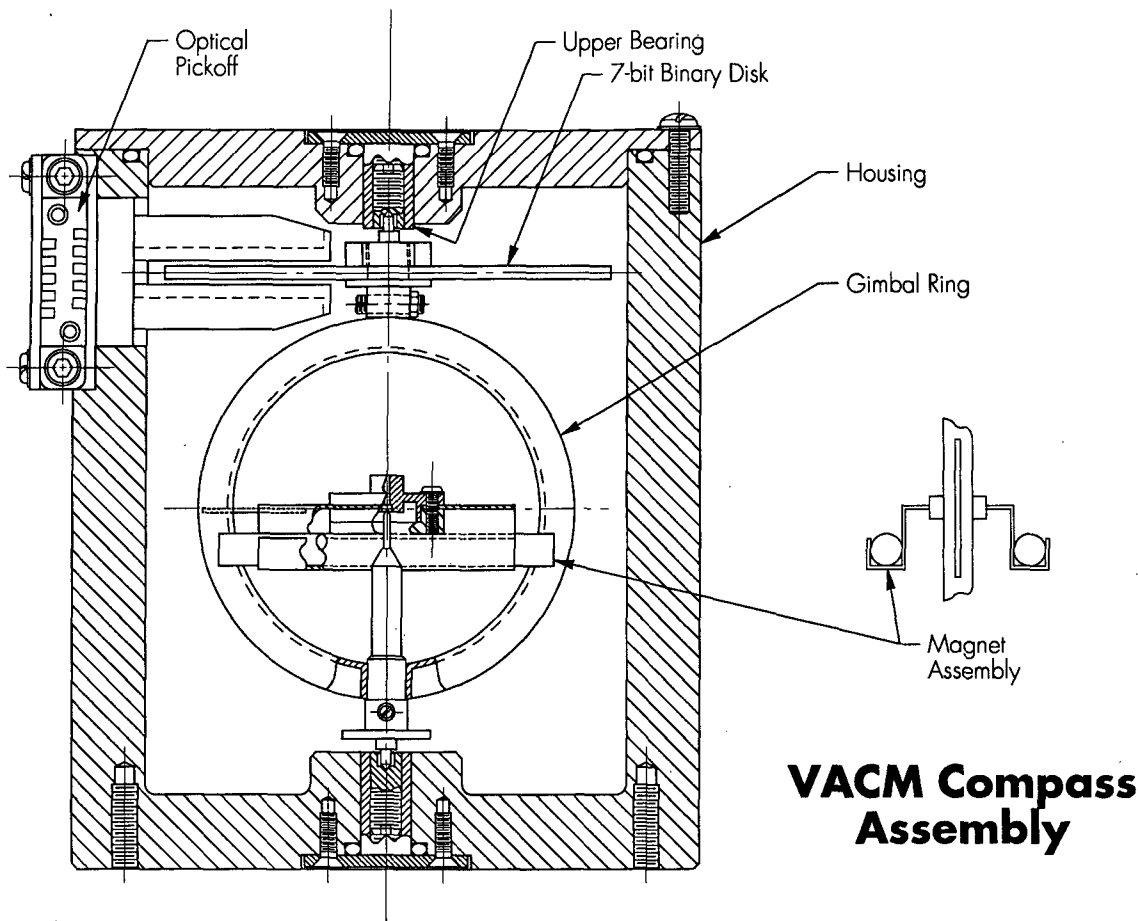


FIG. 1. Cross section of the VACM compass.

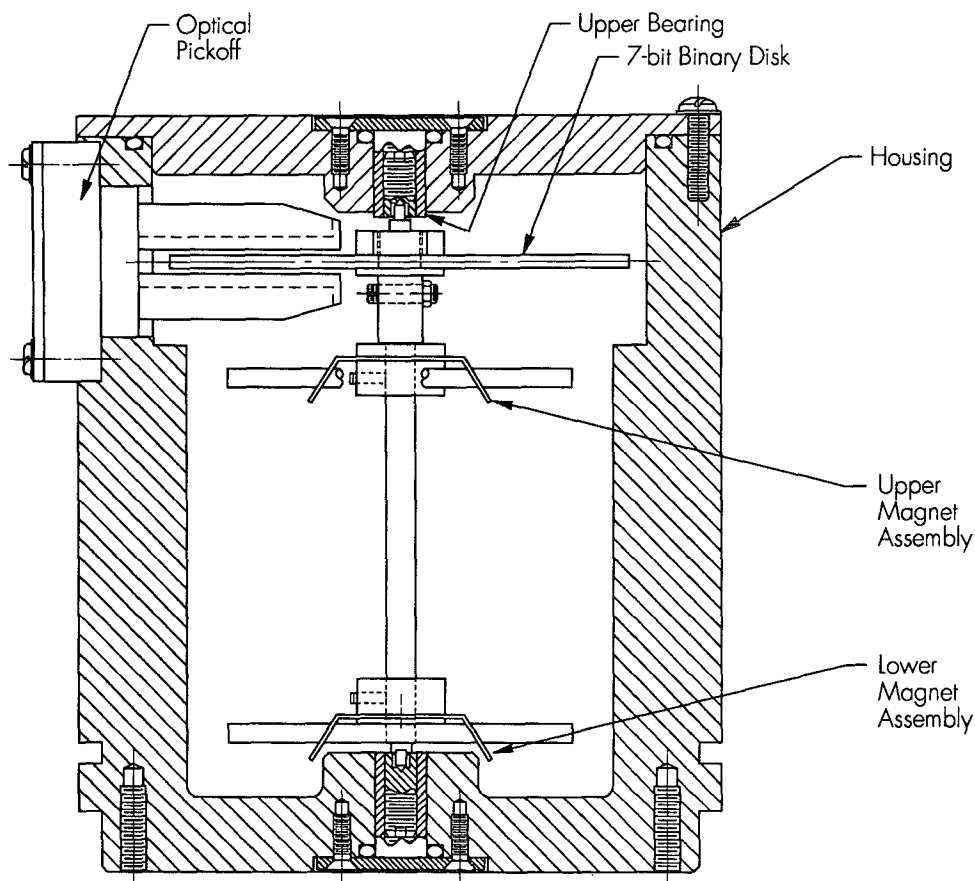
nets that could occur if the compass housing were moving about on a mooring. The compass housing is made of aluminum so that electrical eddy currents induced by the bar magnets as they swing within the housing damp the motion of the magnets. The VACM compass was originally designed by J. McCullough and R. Koehler in the early 1970s to be critically damped. This was done by adjusting the dimensions of the aluminum housing, since eddy damping is the main source of damping (provided good bearings are used). Additional damping is due to bearing friction.

b. Vane-follower design

The VACM vane follower is very similar in design to the VACM compass (see Fig. 2). The main difference between the instruments is the magnet assemblies. The compass has one gimballed magnet assembly mounted in the center of the housing. The vane follower has two magnet assemblies mounted near the top and bottom of the vane-follower housing. These are not gimballed, and thus, are constrained to rotate

as a unit about the same axis as the vane. The two vane-follower magnet assemblies have opposite orientations so that the net torque exerted on the vane follower by the earth's magnetic field is zero.

The VACM vane follower is forced by a pair of bar magnets encased in a nylon holder on top of the vane. These magnets are approximately 7.0 cm from the lower vane-follower magnet assembly and 12.2 cm from the upper. At these distances the vane magnets can be approximated as dipole sources whose strength is proportional to r^{-3} , where r is the distance from the source. The resultant magnetic torque on the top and bottom vane-follower magnet assemblies, though oppositely directed, is very different in magnitude because of the r dependence, and the vane-follower response is effectively that of a single assembly forced by the vane magnets. The effective angular forcing of the vane follower is 3–10 times the forcing of the compass by the earth's magnetic field, depending on the strength of the vane magnets and the magnitude of the earth's magnetic field. For clarity, the two forcing magnets encased in the nylon holder will be referred to as the vane magnet, while the magnets in the vane-follower housing



VACM Vane Follower Assembly

FIG. 2. Cross section of the VACM vane follower.

will be referred to as the vane-follower magnet assembly.

c. Theoretical instrument behavior

The simplest model of the dynamical response of the compass or vane follower to angular forcing is the damped harmonic oscillator. The equation for the free angular motion of either instrument is then

$$a\ddot{\theta}_I + b\dot{\theta}_I = -c \sin\theta_I, \quad (1)$$

where θ_I is the angle between magnetic north of the forcing (earth's magnetic north for the compass or the vane magnet north for the vane follower) and the orientation of the instrument magnets, a and b are parameters related to the instrument moment of inertia and friction divided by the magnetic moment of the instrument, and c is the strength of the horizontal component of the earth's local magnetic field in the case of the compass and the effective vane-magnet strength in the case of the vane follower.

Equation (1) was used to estimate a and b from static tests performed on several compasses. In these tests the compass housing was held fixed and the compass response as it returned to magnetic north from several initial displacements was examined. Data from these trials were used to estimate the coefficients a and b using a least-squares fit. Static-test values of a and b exhibited considerable variability due to several sources of error. Further information and preliminary results from these tests can be found in Patch et al. (1990). These initial tests led to the development of two dynamic tests for the compass and vane follower that allowed more accurate determination of a and b .

In dynamic testing of the compass, the compass housing is rotated with (a) a constant angular velocity (slued) and (b) a sinusoidal angular velocity. For both tests the relevant equation is

$$a(\ddot{\theta}_H + \ddot{\theta}_I) + b\dot{\theta}_I = -c \sin(\theta_H + \theta_I), \quad (2)$$

where θ_H is the orientation of the housing relative to magnetic north and θ_I is the angle measured by the

instrument. The total angle between magnetic north and the compass magnet becomes $\theta_H + \theta_I$, and the angular acceleration of the compass in inertial space is $a(\ddot{\theta}_H + \ddot{\theta}_I)$. Since friction is proportional to the angular velocity of the compass relative to the housing and not the total angular velocity, it is given by $b\dot{\theta}_I$ alone. The sluing test eliminates the acceleration term so that b can be found independently of a . The oscillating test then allows a to be determined.

In dynamic testing of the vane follower, the vane magnet is rotated at a constant and sinusoidal angular velocity while the vane follower housing is kept stationary. Because the housing is kept stationary no inertial term arises from housing motion (unlike the compass tests), and the model equation becomes

$$a\ddot{\theta}_I + b\dot{\theta}_I = -c \sin(\theta_H + \theta_I). \quad (3)$$

In all analyses, a , b , and c will be taken as constant, and in oscillating tests, $\sin(\theta_H + \theta_I)$ will be linearized as $(\theta_H + \theta_I)$, assuming small-amplitude periodic forcing. Experimental results confirm the applicability of the damped linear oscillator model. Also, because of design features such as the relatively thin cylinder wall and wide gap between the magnet assemblies and the aluminum wall of the housing, the friction due to eddy damping should be linear (R. Samelson, personal communication 1989), and no statistically significant experimental evidence for a nonlinear term (like $\dot{\theta}|\dot{\theta}|$) in (1), (2), or (3) at high forcing frequencies was found.

2. Experimental procedures and results

a. Dynamic compass tests

A sequence of two dynamic tests were conducted on each compass to determine the two coefficients a and b in the model equation (2). A steady-sluing test was first conducted to determine the friction coefficient b , independent of inertial effects. The compass was mounted on a motor-driven turntable that spun the compass at a constant angular velocity ($\ddot{\theta} \equiv 0$). The resulting equation of motion was

$$b\dot{\theta}_I = -c \sin(\theta_H + \theta_I) \quad (4)$$

so that compass position lagged table position by a constant that was equal to the $\arcsin(-b\dot{\theta}_I/c)$. Thus, as the angular velocity decreased, so did the phase lag. The minimum angle that could be measured (described below) determined the smallest phase lag that could be measured. This limited the slowest angular velocity that could be used to calculate b . For the compass, this minimum angular velocity was approximately 0.05 rad s^{-1} . The minimum angular velocity used to compute b varied for different compasses but was always greater than 0.05 rad s^{-1} . Over this range estimates of b remained fairly consistent (see Fig. 3). Angular velocity was constant during each trial but varied between trials. The horizontal component of the earth's mag-

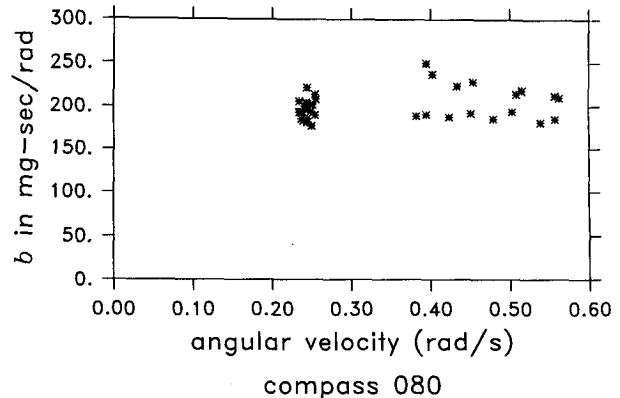


FIG. 3. Frictional coefficient b (mG s rad^{-1}) versus angular velocity (rad s^{-1}) for compass 080. All recorded trials were conducted at angular velocities greater than 0.05 rad s^{-1} . Below this angular velocity the phase lag is not well determined due to the resolution limit of the compass.

netic field c was measured with a Hewlett-Packard (HP) magnetometer and checked for spatial uniformity. An Onset (model 4) Tattletale attached to a laptop computer recorded both compass and table position every 0.02 s . Ten seconds worth of data were collected per trial. An estimate for b was calculated by linear least squares for each trial. The major sources of error in these tests were errors in measuring the earth's magnetic field and the resolution of angular position measurements. The HP magnetometer used in all tests has a manufacturer's stated accuracy of $\pm 5\%$. Great care was taken to measure θ_I relative to θ_H accurately and to ensure that the compass did not move during a trial. With the procedure outlined above, errors caused by the digitization of the compass readings (one compass bit is approximately 0.05 rad) set the resolution limit on angular measurement. Before a compass was tested, ten measurements of the initial stationary positions of compass and table were taken. These trials were averaged and used to calculate the offset between the table and compass zero. Sluing trials were then conducted. These were interspersed with some stationary readings. After sluing trials were completed, ten more sets of stationary readings were taken. The latter stationary readings were used to make sure the compass did not slip during the course of the experiment. After stationary and sluing trials were completed, the magnetic field was once again measured to check uniformity in time. Fits and confidence intervals for b are listed in Table 1.

The second dynamic test was used to estimate a . The compass housing was sinusoidally forced to simplify the inertial term $a(\ddot{\theta}_H + \ddot{\theta}_I)$ in the equation of motion (2). The compass was placed on a rotating turntable that could be driven with a sinusoidal angular velocity by a variable-speed motor and scotch yoke assembly. The amplitude and frequency of oscillation

TABLE 1. Coefficients a and b estimated for five VACM compasses and two VACM vane followers based on least-squares fits to (4) and (10) for the compasses and (4) and (13) for the vane followers. The 95% confidence limits are also shown. The undamped resonant period of the compass or vane follower due to oscillation of the VACM housing is given by (16). The damped resonant period of the vane follower due to vane magnet forcing is given by (18). The degree of damping $d = \pi b (ac - b^2/4)^{-1/2}$ is also listed.

	a (mG s ² rad ⁻¹)	b (mG s rad ⁻¹)	c (mG)	Undamped T_r (s)	Damped T_r (s)	d
Compass number						
067	116 ± 18	204 ± 11	190	4.9	—	5.9
080	126 ± 9	200 ± 12	235	4.4	—	4.5
109	115 ± 8	234 ± 14	235	4.4	—	6.4
615	153 ± 28	171 ± 10	243	2.8	—	3.1
695	112 ± 11	219 ± 15	238	4.3	—	5.7
Vane follower number						
265	129 ± 3	421 ± 36	1150	2.1	3.3	4.1
318	127 ± 4	360 ± 36	1150	2.1	2.8	3.4

varied between trials but were constant during each trial. Amplitudes of oscillation ranged from 12° to 20°; periods ranged from 1 to 12 s. A potentiometer attached to the turntable recorded its position. Once the system reached steady state, both the table and compass position were read every 0.02 s using the Tattletale and laptop computer.

In these oscillating tests, the compass magnet assembly would remain stationary and always point north if the compass was undamped. Within the reference frame of the compass this behavior would be measured as an oscillation exactly mirroring the sinusoidal motion of the turntable. Damping has two effects on the oscillation measured relative to the moving compass housing: it reduces the magnitude of the oscillation of the compass magnet assembly, and it introduces a phase shift between the oscillations of the housing and the compass magnet assembly. At very low frequencies, damping is small, and the compass orientation θ_I nearly mirrors the forcing θ_H (i.e., $\theta_I + \theta_H$ approaches zero), and the forcing and response are nearly the same amplitude and 180° out of phase. At very high frequencies, inertia dominates the motion of the compass magnet, and θ_I also approaches $-\theta_H$. Near the resonant period, the amplitude of θ_I approaches zero, and the phase leads the housing forcing by 90° just below the resonant period and lags the housing forcing by 90° just above the resonant period. At the resonant period, the amplitude of oscillation relative to the housing is zero, and the phase is not determined. Viewed from outside the housing, the compass magnet assembly moves along with the forced housing at resonance.

Least-squares fits to sinusoids were computed for both the compass and table data. The fit gave the amplitudes and phases of the compass and table data. The ratio of compass to table amplitudes of oscillation and

the phase shift between them were then calculated. This information was used to compute a . Because the amplitudes of oscillation were small, the forcing term in the equation of motion (2) was linearized to simplify data analysis,

$$a(\ddot{\theta}_H + \ddot{\theta}_I) + b\dot{\theta}_I = -c(\theta_H + \theta_I). \quad (5)$$

The steady-state sinusoidal forcing and response data were represented as complex exponentials. In complex notation, $\theta_H = \theta_{H0} \exp(i\omega t)$ and $\theta_I = \theta_{I0} \exp[i(\omega t - \phi)]$. These expressions were substituted into the linearized equation of motion (5) to give the following complex equation,

$$-a\omega^2(1 + \alpha e^{-i\phi}) + i\omega b\alpha e^{-i\phi} + c(1 + \alpha e^{-i\phi}) = 0, \quad (6)$$

where $\alpha = \theta_{I0}/\theta_{H0}$, the ratio of compass to table oscillation amplitude. The real part of this equation was solved for a ,

$$a = \frac{c}{\omega^2} + \frac{\alpha b \sin \phi}{\omega(1 + \alpha \cos \phi)}. \quad (7)$$

This expression contains both amplitude α and phase ϕ information. Though both types of information were obtained experimentally, the phase information was poorly resolved near resonance. To overcome this problem the imaginary part of (6) was also solved for a to give,

$$a = \frac{c}{\omega^2} - \frac{b \cos \phi}{\omega \sin \phi}. \quad (8)$$

Equating (7) and (8) gave a consistency relationship between α and ϕ ,

$$\alpha = -\cos \phi, \quad (9)$$

which was used with (7) to give an expression for a purely in terms of α ,

$$a\omega^2 = c - \frac{b}{\omega \tan[\arccos(-\alpha)]}. \quad (10)$$

The values of b , c , ω , and α for 20–30 trials were used to write (10) as a matrix equation with a as an unknown value. A least-squares fit for a was then calculated. Results for five compasses are listed in Table 1, and a typical example of the data and linearized solution are presented in Fig. 4 for compass 080. In general, there was good agreement, however, several sources of error arose in estimating a . One possible source of error is variability of the local measured magnetic field. In an effort to control this, the magnetic field was measured immediately before and after each experiment and checked for spatial uniformity. Another source of error resulted from the fact that for frequencies near resonant frequency, the data yielded inaccurate phase information. For this reason, amplitude information was used to estimate a and the results

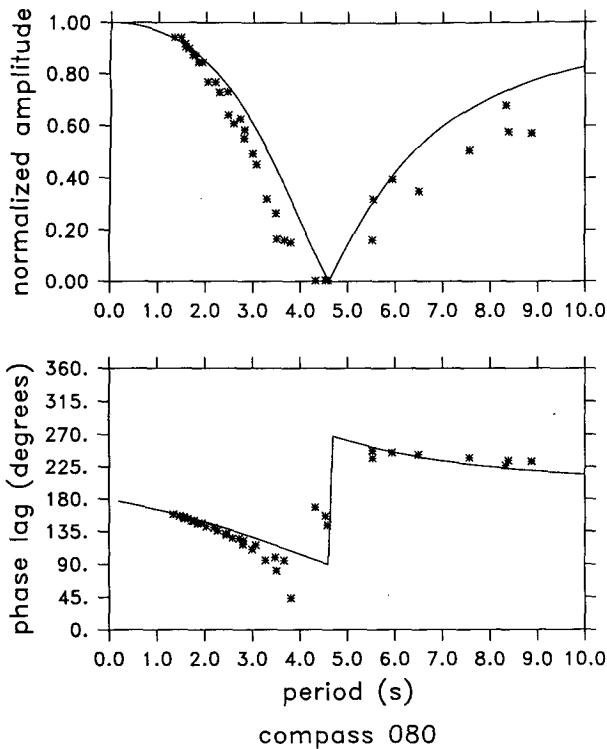


FIG. 4. Plot of (a) observed compass oscillation amplitude divided by the housing oscillation amplitude (upper panel) and (b) observed phase lag of compass behind housing (lower panel) versus forcing period for compass 080. The theoretical values of normalized amplitude and phase predicted with a solution of the linearized equation of motion (5), using the values of a , b , and c listed in Table 1, are shown by the solid line. The drop in amplitude to zero near the resonant period of 4.0 s indicates that the compass is oscillating exactly with the housing.

checked against phase information. Figure 5 shows values of a as calculated from all trials used to find a single estimate of a . In general, values of a are lower near the resonance period when the amplitude of oscillation relative to the housing is lowest. This may indicate that the friction coefficient b is not constant, since the inertial coefficient a should remain constant over all forcing periods, but the friction may exhibit stick-slip behavior for small oscillations.

b. Dynamic vane-follower tests

Similar dynamic tests were performed on two vane followers holding the housing steady and rotating the vane magnets. In sluing and oscillating experiments with the VACM compass, the local magnetic field could be measured and checked for uniformity in space and time. To determine the strength of the forcing by the vane magnets a different procedure was needed. The vane magnets were aligned with the earth's field, and the total field was measured as a function of vertical distance. The local spatially uniform field was subtracted off, and the remainder was taken to be the field due to the vane magnets. These values were fitted to

the field caused by two dipoles located at the positions of the vane magnets. Measured values and the fitted field are shown in Fig. 6. Tests with several sets of vane magnets showed they varied in strength but were always significantly stronger than the earth's magnetic field at distances within the vane-follower assembly. Based on the fitted field and the positions of the magnet assemblies within the vane follower, the effective field strength for the vane-magnet set used in subsequent tests was estimated as $1.15 \text{ G} \pm 0.05 \text{ G}$. Any errors in this estimated value will be systematically carried through later data analysis.

Steady sluing experiments were first conducted on the vane follower to determine the friction coefficient b . The experimental setup and procedure were similar to that in the compass sluing tests, except that the vane magnet rather than housing was slued. This set $\dot{\theta}_f$ to zero when the system reached steady state, and b could again be determined from (4) independent of inertial effects. Angular velocity was again constant during each trial and varied between trials. The effective vane-magnet forcing c was determined as the aforementioned value. Data were recorded as in the dynamic compass tests, and identical procedures were followed to estimate b for each trial. Errors in determining the magnetic forcing and resolution limits on the vane-follower angle measurement remained the major limitations on accuracy.

As in the compass tests, the 0.05-rad resolution limit of the vane follower limited the lowest angular velocity that could be used to calculate b . Below this resolution limit the phase lag and, hence, b were not well determined. This behavior is shown in Fig. 7 for one vane follower. Because of this, final values of the friction coefficient used in periodic forcing runs were calculated averaging only the data for which the vane magnet rotated faster than 0.2 rad s^{-1} . Over this range, estimates of the friction coefficient remained relatively constant. A higher minimum angular velocity is required for the vane follower than for the compass because the vane follower's forcing is much stronger. For

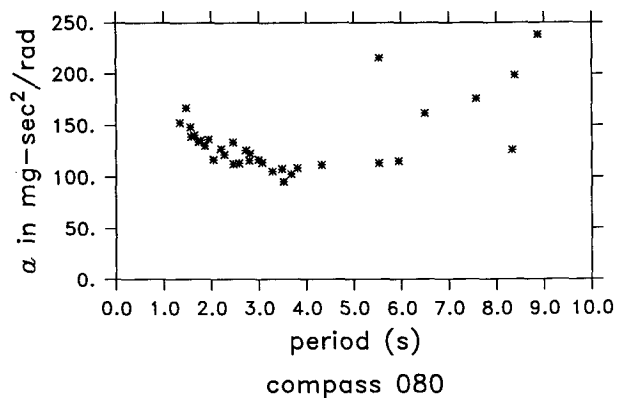


FIG. 5. Calculated a ($\text{mG s}^2 \text{ rad}^{-1}$) versus forcing period (s) for compass 080.

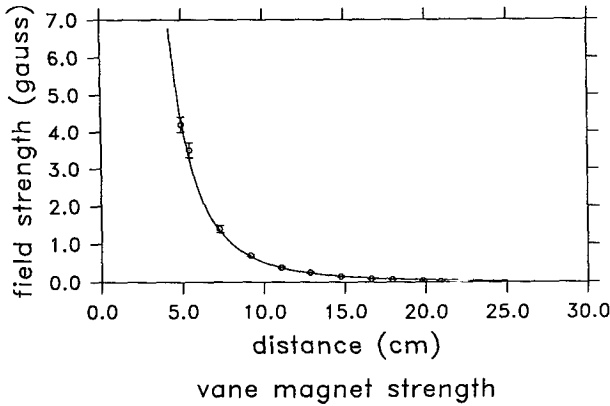


FIG. 6. Measured and modeled magnetic field c (mG) for the vane magnet used in all dynamic vane-follower tests as a function of distance from the source. Bars indicate uncertainty in measurements. Solid line indicates curve fit.

a given b and angular velocity this decreases phase lag. Values of b for the two vane followers tested are given in Table 1. Both vane followers have higher friction coefficients than all compasses tested. One explanation for this is that each vane follower contains two magnet assemblies (the compass contains only one). The two magnet assemblies may cause greater eddy-current damping and/or bearing friction in the vane follower.

Having determined b , sinusoidal forcing experiments were next conducted on the vane follower to determine a . The experimental setup and procedures were again similar to those employed for the compass periodic forcing test with several minor differences. The amplitude of the angular oscillation of the vane magnet was held constant at 20° , and the period of the forcing was varied from 1 to 9 s rather than from 1 to 12 s. At each chosen forcing period three trials were conducted. All three sets of trials were consistent and were used in finding a single estimate of a .

Forcing the vane magnet rather than the housing produces a different response than forcing the housing and holding the magnet still. At low frequencies, the balance is between the forcing and friction. The damping is small, and the vane-follower response lags the vane forcing by a small amount, just as in the forced housing case examined with the compass. At high frequencies, inertia dominates and the vane follower remains nearly stationary. The phase lag of the response approaches 180° for frequencies much higher than the resonance frequency. At the resonant frequency, the vane follower tends to overshoot the forcing by a small amount. This response is 90° out of phase with the forcing.

The data was again fitted to sinusoids, and the amplitudes, phases, and frequencies of the forcing and response were calculated. This information was then used to solve for the a coefficient in the linearized equation of motion for the vane follower

$$a\ddot{\theta}_I + b\dot{\theta}_I = -c(\theta_H + \theta_I), \tag{11}$$

where θ_I is the angular orientation of the vane-follower magnet assembly relative to the housing and θ_H is the orientation of the housing relative to the vane magnets.

At steady state, the solution to this equation with sinusoidal forcing can again be represented by complex exponentials, $\theta_H = \theta_{H0} \exp(i\omega t)$ and $\theta_I = \theta_{I0} \exp[i(\omega t - \phi)]$. Substituting these into (11) yields

$$-a\omega^2\alpha + i\omega b\alpha + c(e^{i\phi} + \alpha) = 0, \tag{12}$$

where $\alpha = \theta_{I0}/\theta_{H0}$, the ratio of vane follower to vane-magnet oscillation amplitude. Separating the real and imaginary parts of (12), solving them for $c \cos\phi$ and $c \sin\phi$, respectively, and dividing the imaginary part by the real part gives an equation that can be written as

$$a\omega^2 = c + \frac{b}{\tan\phi}. \tag{13}$$

All parameters in this equation, except for a , have been determined. The frequency of oscillation ω , the relative amplitudes α , and the phase lag ϕ have been determined experimentally. As in the case of the compass, the trials were combined to write (13) as a matrix equation. A least-squares fit was then found for a using both phase and amplitude information separately. Phase information was used directly in (13). Fits to amplitudes were also done by solving the imaginary part of (12) for ϕ ,

$$\phi = \arcsin\left(\frac{-b\omega\alpha}{c}\right), \tag{14}$$

and substituting this expression for the measured ϕ in (13).

Both of these methods gave consistent results. However, care has to be taken when using amplitude information. Near resonance frequency, the phase lag is

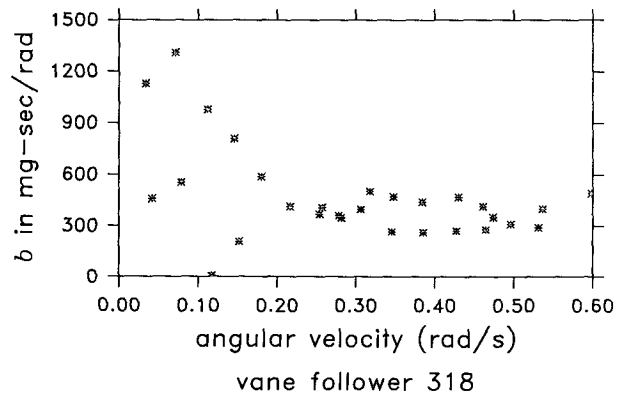


FIG. 7. Frictional coefficient b (mG s rad^{-1}) versus angular velocity (rad s^{-1}) for vane follower 318. Only trials with an angular rotation rate greater than 0.2 rad s^{-1} were used to estimate b for the vane followers. Below this rotation rate the phase lag is less than the resolution limit of the vane follower (0.05 rad), and b is not well determined.

near 90°, and α is slightly greater than one. Therefore, small errors in b or c could cause $b\omega\alpha/c$ to be greater than one, and ϕ could not be calculated from (14). Data points where this occurred were not used, but estimates of a from amplitude information near the resonance frequency may still be inaccurate. Also, at low frequencies, these same small errors cause estimates of a to be inaccurate, since two large values are being subtracted to find a in (13). As a result, phase information gave the most consistent estimates of a (see Fig. 8). In contrast, compass test phases were variable near resonance, therefore, the relation $\phi = \arccos(-\alpha)$ was derived to use amplitude information instead. This relationship was always used, since α was never greater than one and the compass phase information was unreliable near the resonance frequency.

Best-fit values for a are given in Table 1. From a , b , and c analytic solutions to the linear equations are calculated and presented in Fig. 9, with the observed response as a function of period for vane follower 318. Values of a determined using both phase and amplitude information are plotted. The magnitudes of a for both vane followers tested are quite similar to each other and to the compass inertial coefficients. Predicted and observed amplitude indicates a resonant period of around 2–3 s, roughly half that of the compasses tested. There tended to be better agreement between the analytic model and observations for the vane followers than for the compasses, at least at high frequencies. This may be due to stronger magnetic forcing on the vane followers and magnetic, rather than frictionally driven, oscillation of the vane magnet assembly. At low frequencies agreement is not as good between theoretical and observed amplitudes. This may be due to an inadequacy in determining an average value of b , or the greater number of high-frequency trials may bias the least-squares fit to a single a (see Fig. 9).

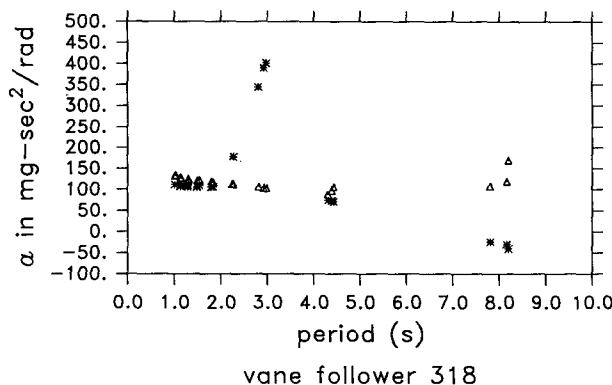


FIG. 8. Calculated a ($\text{mG s}^2 \text{rad}^{-1}$) versus forcing period (s) for vane follower 318. Asterisks denote values obtained from amplitude information, and triangles denote values obtained from phase information. Phase determined values were consistent for all forcing trials and were used to estimate a . Amplitude determined values were inaccurate for trials near the resonance period and at long periods.

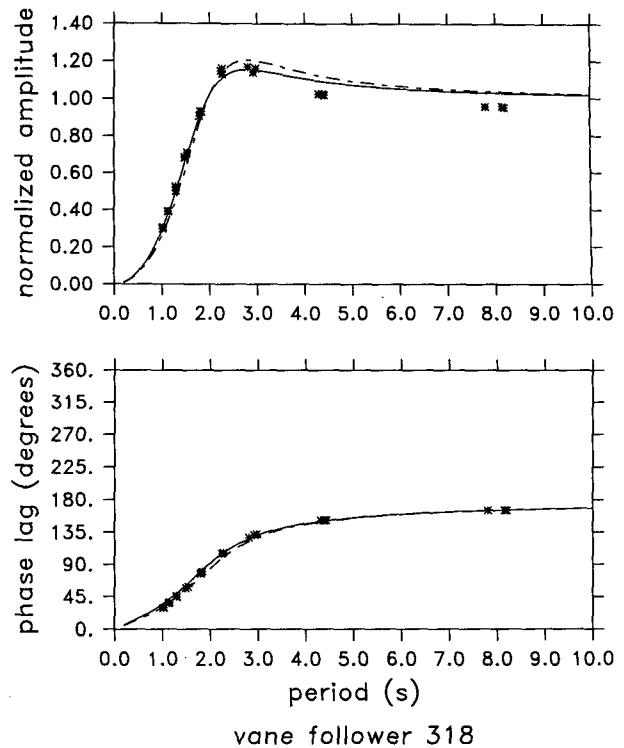


FIG. 9. Plot of (a) oscillation amplitude divided by the vane-magnet oscillation amplitude (upper panel) and (b) phase lag of follower behind vane magnet (lower panel) versus forcing period for vane follower 318. The solid line indicates theoretical values of normalized amplitude and phase predicted with a solution of (11) to amplitude information ($a = 112 \text{ mG s}^2 \text{rad}^{-1}$). The dashed line indicates a fit to phase information ($a = 126 \text{ mG s}^2 \text{rad}^{-1}$). Both curves use $b = 360 \text{ mG s rad}^{-1}$ obtained from the sluing tests (Table 1) and a vane-magnet field intensity of $c = 1150 \text{ mG}$. The peak in amplitude near 2.5 s indicates resonance. Asterisks indicate observations.

3. Discussion

In order to estimate a and b for the compass and vane follower from dynamic tests, the linearized equations of motion (5) and (11) were solved for a and b . The damped linear harmonic oscillator model can be examined, and several statements can be made about the behavior of the compass and vane follower with these estimated parameters.

The resonant frequency or period can be derived for both instruments. In the case of the compass, the relative amplitude response to sinusoidal forcing of the compass housing can be found from (5) to be

$$\alpha = \frac{a\omega^2 - c}{[(c - a\omega^2)^2 + (b\omega)^2]^{1/2}} \quad (15)$$

At resonance there is a minimum in the response, $\alpha = 0$. This occurs when

$$\omega = \omega_R = \left(\frac{c}{a}\right)^{1/2} \quad \text{or} \quad T_R = 2\pi\left(\frac{a}{c}\right)^{1/2} \quad (16)$$

These are the same frequency and period as the undamped oscillator. The resonant period is calculated and presented in Table 1. Note that the resonant frequency is dependent on the forcing strength c . Because the compass is gimballed, the forcing strength is given by the horizontal component of the earth's magnetic field. At 41°N, where these experiments were performed, c is about 200 mG. However, c can be the order of 400 mG near the equator. This decreases the resonance period by about one-third, relative to the values determined here.

For the dynamic vane-follower experiments, the relative amplitude response to sinusoidal forcing by the vane magnet is given from (11) by

$$\alpha = \frac{c}{[(c - a\omega^2)^2 + (b\omega)^2]^{1/2}} \quad (17)$$

At resonance, there is a maximum in α that occurs when

$$\omega = \omega_R = \left(\frac{c}{a} - \frac{b^2}{2a^2}\right)^{1/2}$$

$$\text{or } T_R = 2\pi\left(\frac{c}{a} - \frac{b^2}{2a^2}\right)^{-1/2}, \quad (18)$$

and the resonant frequency is decreased (resonant period is increased) relative to the undamped oscillator. Resonant periods for the two vane followers tested are given in Table 1. The resonant period of the vane follower does not depend on geographic location but rather on the strength of the vane magnet forcing it. It is important to note that the expression for the vane-follower resonant frequency was derived for the case of the sinusoidally forced vane magnet. If the vane magnet were held fixed and the vane-follower housing oscillated (as in the case of the compass), then the vane-follower response would be governed by (5), and the resonant frequency would be given by (16).

The compass and vane follower were designed to be critically damped. Knowing a , b , and c , a linearized version of (1) can be used to examine this. The solution to the linearized version of (1) is

$$\theta_I(t) = \exp\left(\frac{-bt}{2a}\right) \cos\left[\left(\frac{c}{a} - \frac{b^2}{4a^2}\right)^{1/2} t\right]. \quad (19)$$

If the instruments were critically damped, then the undamped resonant frequency $(c/a)^{1/2}$ would equal the decay time of the instrument $b/2a$. If the decay-time scale is larger, then the system is overdamped, and if the decay time is smaller, it is underdamped. All experimental results showed that the instruments tested were very nearly critically damped. Most results indicated slight underdamping. To gauge the size of the underdamping, the solution (19), at one time may be divided by the solution at one period later to give the

relative oscillation amplitude from one period to the next,

$$\exp[-\pi b(ac - b^2/4)^{-1/2}] = \exp(-d), \quad (20)$$

where d is the degree of damping. Values of d are calculated and presented in Table 1. The magnitudes of d predict that any oscillatory free response is very nearly damped out in one period.

Sources and magnitudes of experimental error have been described in the previous sections. In addition, several tests were conducted to determine the validity of the linear models for the compass and vane-follower behavior. Besides checking the identities [i.e., (9) and (14)] resulting from the linear equations for the compass and vane follower, a numerical model of the compass oscillating tests was used to help justify linearization of the forcing term in the equation of motion (2). Agreement between the model and the linear solution was better than 1% even for the largest amplitudes of oscillation used (20°). Agreement was always worst for the compass at resonant frequency because at that frequency amplitude goes to zero: at very high and very low frequencies, $\theta_H \equiv \theta_I$, and the linear and nonlinear models agree nearly perfectly. Both linear models agree well with the data (Figs. 4 and 8).

The implications of these laboratory experiments on VACM velocity measurements made in the field are as follows. Consider a moored VACM oscillating about its vertical axis due to wave and current induced mooring motion. Near its resonance frequency the compass tends to oscillate in phase with its housing and fails to report the full angular motion of the VACM to its on-board vector computer, which assumes that the VACM instrument orientation is given instantaneously by the vane-follower heading relative to the compass heading. Thus, even if the speed (Savonius rotor) and relative direction (vane and vane-follower) sensors were perfect, near resonance forcing of the compass will introduce an error in the amplitude of the vector-averaged velocity. This error arises because a steady flow will not be recorded as such but instead as an oscillatory flow with angular excursions of amplitude $\theta_H + \theta_I$ about the flow direction. It is less than or equal to θ_H at all frequencies (θ_I is out of phase with and tends to cancel θ_H). An upper-bound estimate for the reduction in measured velocity can then be made by considering angular excursions of the form $\theta_{H0} \cos(\omega t)$. The vector-averaging program records components of the excursion parallel and perpendicular to the actual steady flow. Perpendicular components average out to zero in time (so that the measured direction is correct), but the average of parallel components is

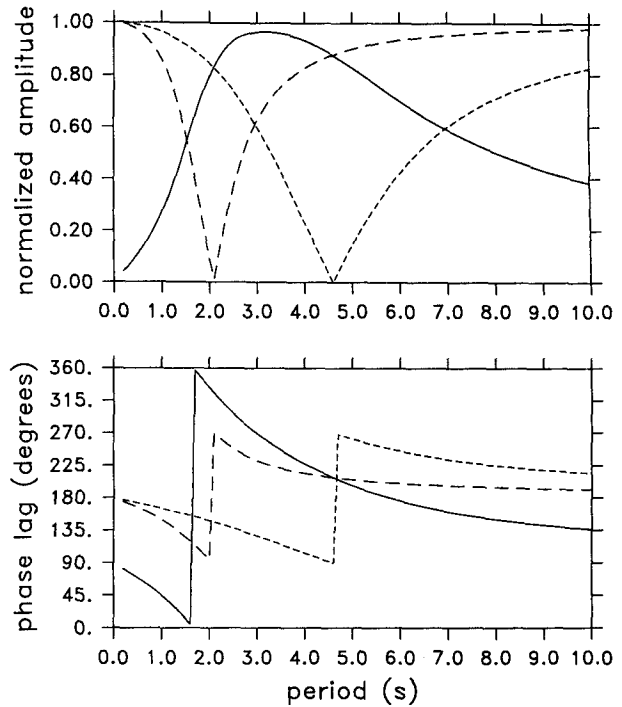
$$\frac{\overline{\cos[\theta_{H0} \cos(\omega t)]}}{\cos[\theta_{H0} \cos(\omega t)]} \approx 1 - \frac{[\overline{\theta_{H0}^2 \cos^2(\omega t)}]}{2}$$

$$\approx 1 - \frac{\theta_{H0}^2}{4}, \quad (21)$$

where the overbar represents a time average, and the cosine has been approximated using the first two terms of its infinite series. Thus, a crude upper-bound estimate of the fractional reduction in vector-averaged velocity is $0.25\theta_{H0}^2$, where θ_{H0} is the amplitude of the housing oscillation. Fortunately, for θ_{H0} less than 25° , the reduction is less than 5%. In reality, the vane follower is not perfect but will be driven simultaneously by the same angular oscillation of the VACM that drives the compass. In the case of steady unidirectional flow with angular housing motion, the compass and vane follower obey the same governing equation (2) so that their behavior is similar, although they have different resonance frequencies due to their different values of the coefficients a , b , and c . If the compass and vane follower had exactly the same resonance frequency and damping, then $(\theta_H + \theta_I)$ would vanish at all frequencies, and the vector-averaged velocity would be correct and unaffected by oscillatory mooring motion. However, because the compass and vane follower have well-separated resonance frequencies, their response to identical housing motion differs (see Fig. 10), especially between the two resonance frequencies. Since the resonant frequency of the compass response depends on the horizontal component of the earth's magnetic field, it is not practical to modify it or the vane follower to match their resonant frequencies. Fortunately, the combined response to oscillatory forcing is less than the amplitude of the forcing; therefore, the actual error in the vector-averaged velocity caused by near-resonant forcing of the compass and vane follower should be less than the aforementioned crude upper-bound estimate.

4. Conclusions

Several simple laboratory experiments have been conducted to study the dynamic behavior of the VACM compass. These experiments demonstrate that the combined eddy-current and bearing-friction torque on the compass magnet is proportional to the angular-velocity difference between the compass magnet and housing. This combined frictional torque tends to critically damp the free oscillation of the compass (as originally intended by the compass's designers), although some underdamping may occur. Experiments on five compasses (with good bearings) indicate a resonant period of 3–5 s at 41°N . At lower latitudes where the horizontal component of the earth's magnetic field is greater, this resonant period will be decreased. The combined frictional torque allows an angular oscillation of the compass housing to drive an oscillation of the compass magnet, and at resonant forcing, the compass magnet oscillates exactly in phase and with equal amplitude with the housing. For small amplitude forcing, the behavior of the compass can be modeled with a linear constant-coefficient equation of motion, which



combined response to housing oscillation

FIG. 10. Vane follower 318 (long dashed line), compass 080 (short dashed line), and combined (solid line) responses to sinusoidal VACM housing motion. Since the combined response to housing motion (rather than vane-magnet motion) is being considered, (5) is used to model the vane-follower response. The compass response is modeled using a magnetic field strength of 235 mG, and the compass oscillation is subtracted from that of the vane follower to give the combined response. For high- and low-frequency forcing vane follower and compass responses tend to cancel one another. Near the instruments' resonant frequencies, however, an error in measured velocity magnitude may occur (see text). Near the earth's magnetic equator the magnetic field strength will increase for the compass and, hence, decrease its resonant period, but resonant periods of the compass and vane follower remain mismatched.

suggests that an asymmetric angular forcing of the compass housing (which might occur in the field due to mooring motion) should not cause any error in the mean compass heading.

Several simple laboratory experiments have also been conducted to study the dynamic behavior of the VACM vane follower. In these experiments, the vane magnet was rotated at either a constant or sinusoidal rate, and the vane-follower housing was held fixed. This forcing simulates high-frequency angular forcing on the vane rather than forcing of the entire mooring as studied for the compass. In reality both types of forcing could affect the vane-follower response, though, presumably, the vane is more readily forced than the entire mooring. A linear model describes the response of the vane follower to small amplitude sinusoidal forcing quite well. Results from this model show that the vane

follower has a resonant period of 2–3 s and is slightly underdamped.

Unfortunately, relatively little is known about the variability in θ_f induced by high-frequency mooring motion. Preliminary compass data obtained at 4 Hz on a surface mooring by Santala (1991) suggest that periodic horizontal translation of the compass may excite an oscillation of the compass due to the difference in position of the moment of inertia and center of mass of the compass magnet assembly. Such oscillations should exhibit a peak at the resonant period in the compass response to broadband translational forcing. When new information on the compass response to translation is obtained it should be possible to construct a theoretical model of the compass and vane-follower response to rotational and translational mooring motion. Such a model, together with a more accurate knowledge of mooring motion near the resonant frequencies of these sensors, could then be used to assess the influence of more realistic instrument motion on the VACM and VAWR measurement accuracy.

Acknowledgments. Laboratory experiments on the VACM compass were conducted during the summers of 1989 and 1990 by Sarah Patch, with help from Gary Haywood, Jerry Dean, Joe Poirier, and Sandy Williams. VACM vane-follower experiments were con-

ducted during the fall of 1990 by Ed Dever, with help from Jerry Dean. Roger Samelson examined several theoretical models of eddy damping and concluded that eddy damping in the VACM compass should be linear, a prediction these tests confirmed within their accuracy. Andy Solow helped with technical aspects of the data processing. Their help is greatly appreciated. Financial support was provided by a National Science Foundation (NSF) Grant OCE 88-16937 with an NSF REU supplement.

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