STUDIES OF THE MOTION AND DEVELOPMENT OF LONG WAVES IN THE WESTERLIES

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ABSTRACT

In order to subdue small scale irregular features of the atmospheric circulation and emphasize the large-scale flow patterns, five-day mean charts showing the pressure distribution at 10,000 feet are presented. It is shown qualitatively that the long waves of the westerlies which appear on these charts seem to behave in a manner suggested by a theoretical formula developed by Rossby. Examples are shown wherein complex combinations of troughs and ridges are brought about by different speeds of motion in different latitude belts. Some indications for the development of new troughs are discussed, wherein a rapid readjustment of circulation patterns becomes necessary over most of the hemisphere. Cases of blocking are described where an unexplained disturbance in the atmosphere moves slowly westward, bringing with it increases in mass and diminutions of the speed of the westerlies. In the second portion of the report evidence is presented to show that Rossby's formula for trough displacement is also verified quantitatively, although the displacements are greatly modified by convergence and divergence in the atmosphere. Furthermore, the data reveal a systematic decrease with time in the velocity of troughs, which can be explained by Rossby's theory after taking into account the existence of semipermanent trough or ridge systems in the general circulation due to solenoidal or topographic influences. Finally, the practical use of empirical displacement formulae is discussed.

I. PROBLEMS SUGGESTED BY A SYNOPTIC STUDY OF MEAN 10,000-FOOT CHARTS

By Jerome Namias

Shortly after the initial preparation about four years ago of five-day mean 10,000-foot charts extended over the oceans it was recognized that a basic tool of extended forecasting was at hand. When mean maps of this nature are prepared they bring to light in an effective manner the large-scale features of the general circulation which to a large extent govern the behavior of individual cyclones and anticyclones, and which have a close association with the pattern of weather occurring over the period covered (6). Mean maps at 10,000 feet are for the most part composed of smooth simple isobaric patterns resembling sinusoidal waves, and the complex patterns found on daily maps are rarely encountered. The war in its early stages made the preparation of extended 10,000-foot charts difficult inasmuch as data were lacking in many areas. More recently, however, this unfortunate circumstance has been remedied, and it is now possible to obtain a reasonably accurate picture of the 10,000-foot pressure distribution over approximately two-thirds of the northern hemisphere (e.g., see Fig. 1).

In the current routine practiced by the Extended Forecast Section five-day mean maps of this nature are prepared three times a week, and monthly mean 10,000-foot maps twice a month.

It is the purpose of this paper to indicate some of the practical uses made of these charts in extended weather forecasting, and to raise some problems sug-

gested during the last few years of close observation and study incident to the semiweekly forecast routine.

While the present portion of this paper attempts to present in a descriptive fashion synoptic aspects of long waves, Mr. Clapp's report which follows summarizes the results of some empirical studies.

One of the principal reasons why mean 10,000-foot maps are helpful in long range forecasting is that the troughs and ridges observed on them move with a speed which is only a small fraction of the speed of troughs on daily charts or of the speed of the air particles flowing through the troughs. This is effectively shown in Table 1 where the average zonal speed (in degrees of longitude per day at 40°N) of air particles (zonal index), troughs on the daily maps, and troughs on the five-day mean maps are given for the period January 1941 through June 1943. It is clear that while the zonal speed of both air particles and

TABLE 1
Average Speed at 40°N in Degrees of Longitude per Day
(Period: January 1941-June 1943)

	10,000-Foot index	Daily troughs	Mean troughs
Winter	12.3	11.8	1.2
Spring	9.3	10.4	2.0
Summer	7.0	8.0	1.5
Fall	10.7	9.9	0.8

FIG. 1. Normal 10,000-foot pressure distribution for February.

troughs on the daily maps are very similar, even undergoing similar seasonal variation, the speed of the troughs on the mean maps is less than 1/5 as great. The practical importance of this fact becomes clearer when it is realized that the daily troughs undergo their maximum deepening when entering the area of the mean trough (a fact which may be shown statistically).

There are natural regions where ridges or troughs tend to predominate. These regions are perhaps best illustrated by monthly normal maps of 10,000-foot pressure (7), a sample of one of these for February being given in Figure 1. The explanation of these favorite sites for troughs and ridges is, of course, part of the problem of the general circulation. From a practical standpoint, normal charts give us a working base for making a 10,000-foot prognosis.

Rossby (8) showed that the spacing of the waves in the free atmosphere, as exemplified by 10,000-foot maps, is a function of the strength of the zonal circulation. For the steady state

$$L_s = 2\pi \sqrt{\frac{\bar{U}}{\beta}}, \quad (1)$$

where L_s is the stationary wave length, $\bar{U}$ is the zonal speed of the westerlies (the zonal index), and β is the

rate of change of the Coriolis parameter northward. His idealized sketch for the wave pattern with low index is shown in Figure 2, where the initial perturbation in the westerlies is formed off the Asiatic coast by strong heating of cold monsoon air from Asia as it

FIG. 2. Idealized flow pattern aloft for low index (after Rossby).

moves over the Japanese current (producing cyclonic vorticity). Following the formation of this fixed trough (fixed through geography) other troughs result downstream at distances determined chiefly by the over-all zonal speed. For high zonal speeds (large $\bar{U}$) the troughs should be farther apart. Examples of observed five-day mean maps for extremes of indices are shown in Figure 3 and 4 together with the corresponding sea level maps.

Unfortunately, a steady state is rarely approximated in the atmosphere, and for this reason troughs are in a continual state of readjustment—attempting to approach the ideal conditions indicated in formula (1) but being disturbed by fluctuations in $\bar{U}$ and by complex geographical and thermal influences. These disturbances of equilibrium lead to motions of the troughs, the trough speed C being given by Rossby as:

$$C = \bar{U} - \frac{\beta L^2}{4\pi^2}. \quad (2)$$

Formula (1) is a special case of the above expression when $C = 0$.

The extent to which this expression applies to observed data will be treated in some detail by Mr. Clapp. It will suffice here to say that of the parameters involved, $\bar{U}$ and L , L has been found especially significant in determining trough motions on mean maps at 10,000 feet.

One obvious restriction on the use of expression (2) for trough speed concerns cases where troughs form or disappear, for in these cases the wave lengths undergo rapid changes. Obviously, if a new trough

develops between two extended troughs, the effective wave length is appreciably shortened and a rapid readjustment of the systems must occur in an attempt to approach equilibrium. In other words, the L term in the above expression changes abruptly as the new trough develops. Similarly, if a mean trough fills and disappears, the original wave length upon which the computation was based will be too short, the previously indicated eastward motion will be too rapid, and often the trough may even retrograde (C becomes negative).

One of the ways in which a new trough may form occurs when, after a period of high index, troughs are spaced far apart and the 10,000-foot index drops sharply. In this case, a sudden readjustment of wave lengths is necessary so that the effective spacing between troughs must decrease. Sometimes this readjustment takes place by means of relative motion, but it frequently happens that a new trough forms somewhere between the two extended troughs. The formation of such new troughs is most likely to occur in regions where troughs are favored in the normal state, or in regions where topographic factors favor trough development. Such a region is the area just east of the Continental Divide where dynamic factors operating on a westerly current tend to create troughs in the lee of the mountain ranges. It is therefore important to watch for new troughs whenever the wave length is especially large and particularly when a flat ridge between extended troughs is located in an area which is a favorite site for troughs (as given by the normal 10,000-foot maps).

FIG. 3. Observed five-day mean isobars at sea level and at 10,000 feet for a period of high index (zonal sea-level index 5.1 mps; 10,000-foot index 17.7 mps).

FIG. 4. Observed five-day mean isobars at sea level and at 10,000 feet for a period of low index (zonal sea-level index -2.3 mps, 10,000-foot index 7.4 mps).

Another manner in which new troughs may form occurs when troughs extend their influence to higher or lower latitudes. In the circumpolar whirl there are frequently troughs within certain latitude bands which do not penetrate other latitudes. Thus we speak of low latitude and high latitude troughs, the latter being troughs which are too weak to extend southward into lower latitudes. These systems of troughs often move quite independently of one another, that is, there may be more rapid motions taking place in northern than in southern latitudes, or vice versa, owing to the different index and wave-length conditions prevailing in the different latitude bands. Since they change their intensity (owing to numerous factors), troughs which have been operative only at high latitudes may, at times, extend their influence into low latitudes and in this manner profoundly influence the wave patterns in the new environment. An example of the different rate of motion of troughs in different latitude bands and the consequent development of a new trough is shown in Figure 5. This figure shows the twice-weekly mean 10,000-foot maps for the period April 9–27, 1941. The positions of the troughs are defined by lines connecting the points of lowest latitude of the isobars, and the trough lines are constructed solidly for the current position, broken for the previous half week's position, and dotted for the preceding whole week's position. Following the history of the trough over the western United States, we see that the northern portion moves regularly forward, while the lower portion makes very little eastward progress. The upper trough finally breaks

off, leaving the low-latitude trough to extend itself to higher latitudes later on. In fact, the acceleration of the northern portion of the trough during the last week of this period appears to be due to the shortened wave spacing resulting from northward development of the low-latitude trough. The conditions in part responsible for this interesting shearing action are brought to light in the behavior of the indices during this period. While the zonal index and the 10,000-foot index fluctuated only slightly about the normal, the index of the subtropical easterlies had been rising rapidly to values much above normal, retarding the motion of the lower-latitude trough.

The reverse case, when troughs at northern latitudes retrograde while forward motions proceed in low latitudes is illustrated in Figure 6 which covers the three-week period from October 18 to November 9, 1941. Here retrogression takes place in the trough originally off the Atlantic coast (note the large wave length between northern-latitude troughs just preceding retrogression), while the trough originally over the southwestern United States moves eastward until the troughs coalesce into one deep trough extending from Hudson Bay southward to Yucatan. This resulted in a cold wave extending far south. The indices associated with this differential motion revealed that the easterly index had been rapidly falling and reached values considerably below normal favoring forward motions of troughs, while the zonal index fell moderately and the 10,000-foot index remained about normal following a sharp drop in the preceding periods.

One of the most frequent ways in which a new

FIG. 5. Series of five-day mean 10,000-foot charts for the period April 9-27, 1941, illustrating how two troughs are formed from one because trough speed is greater in northern than in southern portions.

trough is formed and produces tremendous effects on the entire atmospheric circulation over the Northern Hemisphere occurs in winter when cold arctic air from Alaska is forced to flow into the Gulf of Alaska. This flow at upper levels is often engendered by retrogression of a strong ridge from the Gulf of Alaska westward through the Aleutians or by the development of a strong ridge in the Bering Sea. An outstanding case of this nature is illustrated in Figure 7 in which are shown the four five-day mean

10,000-foot charts drawn in weekly intervals during the period March 3-28, 1943. From the period March 3-7 to March 10-14, retrogression of the long waves has taken place. The strong ridge originally present in the Gulf of Alaska moves westward to the Aleutians while the principal trough moves westward from the Lakes region and Ohio Valley to the Northern Plains. The continuity of retrogression of the trough is shown by the half-week positions indicated by the dotted and broken trough lines. On the basis of continuity

FIG. 6. Series of five-day mean 10,000-foot charts for the period October 18 to November 9, 1941, illustrating how two troughs combine into one as a result of retrogression in northern latitudes and forward motion in lower latitudes.

alone, it might be naively assumed that retrogression of all these features will continue. However, a sudden reversal of motion takes place in the principal trough during the next week and by the following week it is off the Atlantic coast. At first glance, it appears that this peculiar behavior is a haphazard and unforecastable phenomenon. However, a more careful analysis of this situation reveals that the principal ridge of high pressure originally in the Gulf of Alaska does continue to retrograde throughout the four-week

period, but as the ridge retrogrades, it brings into the Gulf of Alaska a northwest flow of extremely cold arctic air. This arctic air reached temperatures as low as -38°C at the 10,000-foot level over Alaska. As soon as this northwest current of frigid air strikes the relatively warm waters of the Gulf of Alaska, tremendous and sudden convection takes place. Some idea of the intensity of this convection may be obtained by noting that with temperatures as low as -38°C at the 10,000-foot level (3050 meters) and

FIG. 7. Series of five-day mean 10,000-foot charts for the period March 3-28, 1943, illustrating the formation of a new important trough in the Gulf of Alaska which reverses the direction of motion of troughs farther east.

temperatures around 0°C over the sea surface, the lapse rate, were it not for convective overturning, would be appreciably steeper than the dry adiabat! This violent convection and resulting convergence soon set up strong cyclonic vorticity in the northwest arctic stream, so that a new trough quickly develops in the Gulf of Alaska, eventually leading to a closed cyclone during the fourth week. It is this strong new trough that changes the wave length so radically that the direction of motion of the trough in the central United States is reversed. It will also be noted that retrogression of a trough from southwest Europe takes place during the second and third weeks, but this too reverses its direction of motion and moves eastward during the fourth week after pronounced eastward motion of the trough farther west has occurred.

At sea level, the above development is associated with the rapid formation of an intense cyclone in the Gulf of Alaska and the split Aleutian Low (Fig. 8). Once the arctic air empties into the Gulf of Alaska, cyclogenesis occurs with amazing rapidity. In the above case, a fall of pressure of over 20 mb took place within 24 hours in the Gulf of Alaska.

This remarkable chain of events appears to hinge

largely upon the meteorological phenomena taking place in the North Pacific. It is such developments and long-period sequences of events which hold out the hope for general weather forecasts for periods as long as a month in advance, particularly since the phenomenon shows up on monthly mean maps.

The speed at which readjustment of wave length occurs as a new trough is formed is often rapid even at locations far removed from the site of the new trough. Cases have been observed where a trough originally located near the Azores (as in Fig. 8) is forced eastward to western Europe in about four days following the development of the new trough in the Gulf of Alaska. Clearly, it cannot be simply the trajectories of the particles creating the new trough that move downstream and bring about the new positions of the troughs farther east, for the zonal speed of the air particles seldom averages more than 15° of longitude per day at latitude 40°N . Rather it appears that a chain of events is established in which the pressure distribution downstream is to some degree affected as soon as the new cyclonic vorticity is created in the Gulf of Alaska.

Obviously, any attack on long range forecasting

FIG. 8. Series of five-day mean sea-level charts for the period March 3-28, 1943, corresponding to the 10,000-foot charts shown in Figure 7.

which utilizes only the data from one small section of the northern hemisphere cannot hope to explain, let alone forecast, some effects similar to the one shown in Figure 7.

In order to forecast the weather it is necessary not only to forecast the movements of mean troughs and ridges but also their changes in intensity. Indeed this problem at the present time appears to be more complex than that of forecasting the trough motions. Here we shall discuss only one special phenomenon related to the problem of deepening and filling, namely the "blocking" mechanism. This is defined as a retardation of the zonal circulation which appears first over western Europe and the eastern Atlantic and subsequently progresses farther westward, affecting particularly North America. Statistical tests have lent support to the reality of the phenomenon (13).

Several waves of blocking were observed during the winter of 1943-44. Two of the most pronounced cases are shown graphically in Figure 9 where the speed of the westerlies at 10,000 feet is shown as a function of time in various sections embracing 40° of longitude and progressing from western Europe (0° to 40° W) westward to the central Pacific. The sharp minimum

in the uppermost curve which first appears over western Europe at the end of December can be followed in the subsequent curves, but delayed (as indicated by the broken line through points marked I). Thus the decline in zonal circulation reaches a minimum point over eastern North America and the western Atlantic about a week and a half *after* it was reached in western Europe. The minimum is not recorded in the west-central Pacific until about two weeks later. Another well-defined wave of blocking is shown later on by the broken line running along positions marked II.

As the decline in circulation proceeds westward it is accompanied by an increase in mass of air. This is shown graphically in Figure 10 where the mass of air above 10,000 feet in the same 40° longitude zones as used in Figure 9 (but from latitudes 30° N to 70° N) is plotted as a function of time. The mass of air was computed by graphical integration from pressure profiles constructed for the various zones and periods. Comparing the curves of Figures 9 and 10, it is clear that the minima of zonal flow at 10,000 feet are associated with maxima of mass above this level. The correlation of five-day mean values making up

the curves in Figures 9 and 10, that is, between the zonal flow in the various sections and the simultaneous mass of air above 10,000 feet are

TABLE 2

Section	Correlation coefficient (r)	Number of values (n)
40°E - 0°	-0.52	40
0° - 40°W	-0.65	40
40°W - 80°W	-0.64	40
80°W - 120°W	-0.51	40
120°W - 160°W	-0.79	40
160°W - 160°E	-0.67	40

Clearly, any physical explanation of blocking must take into consideration this relationship. The increase in mass can hardly be explained as a simple mass transport of air from the west since the currents aloft, at least up to some distance beyond the tropopause, are prevailing from the west and the phenomenon proceeds from east to west. Rather, there appears to be some form of convergence operating progressively westward, the physical mechanism of which is highly complex, to say the least. It is almost unnecessary to remark that a better understanding of this phenomenon is one of the principal problems of long range weather forecasting.

The increase of mass aloft is usually associated with the development of high-level ridges at progressively westward positions. This is illustrated in Figure 11 which shows the five-day mean 10,000-foot maps covering the first period of blocking treated above. The arrows show the intensification of the ridges (change in latitudinal position of an isobar) from the preceding week. Note the progressively westward intensification of ridges.

FIG. 9. Five-day mean values of the zonal speed (35°N-55°N) at 10,000 feet for progressively westward sections of 40° longitude from December 22, 1943 to March 22, 1944.

FIG. 10. Average pressure at 10,000 feet (mass of air above 10,000 feet) in progressively westward sections of 40° longitude (30°N to 70°N) for five-day periods from December 22, 1943, to March 22, 1944.

Summary.

1. When treated in the form of means over five-day intervals, 10,000-foot pressure charts reveal comparatively simple wave patterns of a sinusoidal nature to which Rossby's theoretical analysis may be applied, at least qualitatively.

2. Troughs on these mean maps may often be subdivided into high-latitude and low-latitude troughs. The motions in different latitude belts may differ considerably but appear to be governed chiefly by the wave length and zonal flow in the particular latitude band in which they are embedded.

3. The different motions lead to a form of shearing action which may culminate in fracturing or combining of troughs.

4. New troughs are generally formed when two basic troughs are far apart and the zonal index falls. They are formed chiefly in areas naturally favorable to troughs (as shown in the normal state).

5. The formation of a new trough profoundly affects not only the circulation in its immediate vicinity, but also the circulation in other distant parts of the hemisphere. These changes in circulation are carried downstream with speeds many times the zonal speeds of the individual air particles.

6. "Blocking" operates in the form of a progressively westward decline in the speed of the zonal circulation at 10,000 feet. It is also associated with an increase in mass of air above 10,000 feet which occurs simultaneously with the decreased zonal flow.

FIG. 11. Five-day mean 10,000-foot pressure charts for the period December 18, 1943 to January 19, 1944 when a pronounced wave of blocking was in operation.

II. EMPIRICAL STUDIES OF TROUGH MOTION

By Philip F. Clapp

1. Introduction. In the preceding discussion by Namias the appearance and synoptic behavior of mean circulation patterns at 10,000 feet were described. The apparent qualitative verification of some of Rossby's theories concerning the motion of troughs and ridges was also illustrated. It is the purpose of this section to present the results of a more quantitative statistical test of these theories.

It may be pointed out that the forecasting procedure of the Extended Forecast Section requires the prognostication of the west-east motion of troughs and ridges for periods a half week (3 or 4 days) and a full week in advance of the latest available mean map. Therefore, the immediate problem to be solved is to find the best means of determining these "half-week" and "full-week" displacements.

2. Statistical studies of the Rossby formula assuming constant velocity.

A. Formula for trough displacement. Rossby's formula for the motion of long waves in the westerlies is:

$$C = \bar{U} - \frac{\beta L^2}{4\pi^2}, \quad (2)$$

where C is the instantaneous west-to-east speed of the wave, $\bar{U}$ is the average speed of the westerlies, β is the rate of change of the Coriolis parameter with latitude, and L is the wave length.

This formula has been used in practice for a number of years, but it has been only recently that enough data have been available to check the formula by statistical means. However, a preliminary check by J. Namias (unpublished) revealed essentially the same results as the more extensive work to be described below. The present study is based on approximately 3 years of 5-day mean 10,000-foot charts.

Before we can use the above formula to obtain an estimate of trough displacement, it is necessary to make certain assumptions regarding the terms, C , $\bar{U}$, and L . In the over-simplified theoretical model used by Rossby in deriving the formula, the pressure waves are assumed to be sinusoidal, and L and $\bar{U}$ are assumed everywhere constant. In practice, it is not only difficult to decide what to use for L , due to the nonsinusoidal nature of observed pressure waves, but having decided this, it is then found that both L and $\bar{U}$ vary widely from place to place and from time to time. It is therefore necessary to adopt a more or less arbitrary definition for L and $\bar{U}$ to obtain best results. The definitions used in this study are modifications of those suggested by H. Wexler (12).

First it is necessary to select a certain latitude circle along which the displacements of the mean pressure troughs are to be measured. The displacements may then be conveniently expressed in degrees of longitude at the chosen latitude circle. In the first part of this study, the fixed latitude chosen is 45°N.

A trough or ridge line is defined as a line drawn through the minimum or maximum latitude points of the isobars in the troughs or ridges. The advantage of this definition over one involving maximum curvature points is mainly one of practical simplicity, and has been discussed more fully elsewhere (2). The position of a trough or ridge on the chosen latitude circle is simply the longitude of the point where the trough or ridge line intersects the latitude circle. The wave length is twice the difference in longitude between a given trough and the ridge to its west.

The definition of $\bar{U}$, the average zonal velocity, was originally taken as the mean west-to-east geostrophic wind between trough and ridge and in the latitude zone extending 5 degrees to the north and south of the

chosen latitude circle. Later, as will be explained below, it was found more satisfactory to use a value of $\bar{U}$ covering the entire range of longitudes included in the available data, or from about 130°W to 30°W.

On substituting the so-obtained values of $\bar{U}$ and L in formula (2) an estimate of the instantaneous speed of the trough is obtained. To obtain the displacement over a given time interval, it is necessary to make some assumption regarding the variation of speed with time. The most obvious assumption is that the speed of the trough will remain constant. The estimated displacement is then given by the simple formula:

$$S = CT, \quad (3)$$

where S is the displacement, expressed in degrees of longitude at the chosen latitude circle, C is the speed in degrees of longitude per week obtained from formula (2), and T is the time interval in weeks.

B. Statistical treatment and division of data. Before discussing the results of applying this method, it is desirable to make a few general remarks concerning the nature of the available data and the statistical methods employed.

The method of comparing estimated displacements with observed displacements is by means of the well-known simple correlation coefficient, reference to which may be found in any standard textbook on statistics. Other equally elementary statistical techniques, such as determination of linear regression equations, are used throughout the report. The principal objective of the study is to find the general order of magnitude of such coefficients, but from time to time comparison is made between one coefficient and another. In making such comparisons, no attempt is made to use statistical tests of significance.

The data consist of 2½ to 3 years of 5-day mean 10,000-foot charts, covering the period from January 1941 to April 1943. These were first divided according to season, the standard seasonal divisions being used (December, January and February being counted as winter, for example). The troughs selected for the tests were taken within the range of available data, from about 130°W to 30°W and, for the present study, at latitude 45°N. Cases of uncertain continuity were first eliminated. As explained in a previous publication (2) these constitute up to 10 per cent of the available cases. The remaining data for each season were then divided in accordance with certain criteria suggested in the earlier study of J. Namias, mentioned above. These criteria will be discussed in some detail in the following paragraphs.

It was found as a result of that study that one of the most important causes for the failure of formula (3) is the development of new troughs or the disappearance of old ones during the time interval covered by the forecast. This is evident when we realize that

such occurrences in the vicinity of the trough position which we wish to extrapolate, constitute a major violation of many of the assumptions upon which formula (2) is derived. Furthermore, they cause sudden changes in wave length which in turn result in changes of the speed of the troughs. Thus it is desirable to eliminate such cases when testing the Rossby formula. However, because of the spacial limitation of the data and because of the fact that only two or three mean maps are constructed a week, it is necessary to adopt an arbitrary definition of trough development or disappearance. A case is defined as one in which a trough develops or disappears during the week if on the chart a half week later than the current one there is evidence that such an event caused a sudden modification of the wave length as measured between the trough concerned and the ridge to the west. Cases of the appearance or disappearance of troughs on the chart a full week later, but not on the half week chart, are not eliminated, as it is assumed that the occurrence took place too late to greatly affect the motion of the trough.

Another suggestion arising from the preliminary study is that those computations which work best are made well within the main belt of the westerlies, as implied in the derivation of formula (2).

In this study certain criteria were used in order to distinguish whether a particular trough or ridge should be classed as being in the belt of westerlies or not. These criteria were not entirely satisfactory because they could not be made rigid. However an effort was made to consider such factors as the amplitude of the pressure waves and the presence or absence of closed centers at 45°N, as well as others. As will be shown later, cases classed as "not in the westerlies" actually give better results than the others, and so were no longer considered as a separate group in subsequent computations.

The division of the data in accordance with the above criteria is shown in the following table:

TABLE 3
Division of Data (Figures are Number of Cases)

	Troughs develop or disappear	Troughs not in westerlies	In westerlies, no new development	Total
Winter	12	15	37	64
Spring	6	22	36	64
Summer	7	12	29	48
Fall	6	6	35	47
Totals	31	55	137	223

C. Results of statistical tests. The first test of formula (3) was made using a value of the strength of the westerlies between the trough and the ridge to the west. The extrapolations were made for a half week and a full week in advance. The correlations

are shown in Table 4. The column headings show the type of data used, the same division of data being used as in Table 3. Thus, column 1 of Table 4 refers to those troughs which are within the main belt of westerlies and in the vicinity of which no troughs developed or disappeared. The number of cases used in each correlation is also indicated.

The correlations in column 1 of Table 4 suggest that there is a definite relationship, especially in winter, between the observed displacements for a full week and those computed by the Rossby formula. The correlations are not remarkably high, but this may be in part due to the inadequate nature of the data. Thus, while the location of troughs and ridges at the beginning of the weekly interval is fairly definite, this is not true at the end of the week, as they are then often well out in the Atlantic, where, until the latter half of 1942, data were very scarce. This unavoidable difficulty results in poor definition of the final trough positions and, therefore, a poor estimate of observed trough displacement.

Another factor which may account for the low correlations is the length of time covered by the extrapolation. Thus, since formula (2) only gives us at best the instantaneous velocity of the troughs, it is not a fair test of its validity to extrapolate over too great an interval of time, since there may be significant changes in velocity during that interval. Therefore, if the extrapolation covers a shorter time interval, we should expect to find higher correlations. To test this conclusion extrapolations were made for a half week (3 or 4 days) in advance. The resulting correlations are shown in column 3 of Table 4. These correlations were made using essentially the same cases as for the full-week interval. However, there are a few additional cases due to greater ease in determining continuity. It will be noted that for all seasons the correlations are higher for the half-week extrapolations, as we should expect.

Column 5 shows the correlations for those cases which were not in the main belt of the westerlies, but for which there was no new trough development. Except for the winter season, these coefficients are considerably higher than those in column 1. The reason for this rather surprising result might possibly be found in the inadequate definition of the cases "in the westerlies," but the most probable explanation is that troughs and ridges of large amplitudes (which are classed as "not in the westerlies") are better defined than the others. From the standpoint of continuity, the advantage of this better definition is probably enough to counteract the more limited applicability of formula (2).

Column 7 shows the results of those cases where new troughs formed or old ones disappeared during the interval of a week. As we should expect, these correla-

TABLE 4
Correlations for Motion at 45°N, Rossby Trough Formula, Using Trough-to-Ridge Wave Length and Westerlies

	1 Full week		3 Half week		5 Full week		7 Full week	
	In westerlies, no new development	Number pairs	In westerlies, no new development	Number pairs	Not in westerlies	Number pairs	Development or disappearance	Number pairs
Winter	0.59	37	0.62	38	0.26	15	-0.15	11
Spring	0.38	35	0.46	38	0.71	20	-0.67	6
Summer	0.27	29	0.29	31	0.42	12	0.07	7
Fall	0.39	35	0.52	37	0.63	6	0.12	5

tions are low or even negative. However, it should be pointed out that this result is somewhat inconclusive because of the extremely small number of cases.

In an attempt to find a more representative value of the zonal westerlies ($\bar{U}$) than that given by the mean geostrophic wind between trough and ridge, a few other definitions were tried. One of these was a value of the geostrophic wind taken between 40° and 50°N, and covering the entire range of longitudes included in the data rather than the more limited range. Such a value of the mean zonal westerlies is known as the 10,000-foot index (I_3). The correlations obtained using this definition of the westerlies are shown in Table 5. Only the cases "in the westerlies" for which there was no new trough development are shown.

It will be noted by comparing these correlations with those in columns 1 and 3 of Table 4 that they are larger in every case, suggesting that the total zonal westerlies are more representative than the rather limited values obtained between trough and ridge.

The linear regression equations between observed and computed full-week displacement corresponding to the cases just discussed are shown in Table 6. In this table S' is the estimate of the observed motion and S is that computed by means of the Rossby formula. Both are expressed in degrees of longitude at 45°N.

These equations are currently used in the Extended Forecast Section in obtaining estimates of displacement by means of the Rossby formula.

A point of considerable interest in regard to these equations is that they by no means indicate an exact correspondence between observed and computed displacement, in spite of the fact that the correlations indicate that the theory has a fair degree of validity. In the case of the equation for winter, it will be noted that on the average the Rossby formula indicates a retrogression (westward motion) of almost 100 degrees when the observed motion is zero. Turning to equation (2) we see that this must mean that $\bar{U}$ is on the average too small in comparison to $\beta L^2/4\pi^2$. However, due to the normal increase of the westerlies with

height, one can presumably find a level such that the value of $\bar{U}$ will be large enough. This level at which the Rossby formula is satisfied is known as the level of nondivergence, for the formula was derived on the assumption of no divergence in the westerly flow. Two years ago Namias attempted to find this level of nondivergence by computing the $\bar{U}$ necessary to make the Rossby formula fit the observed results, and locating the level of this zonal velocity through the observed distribution of $\bar{U}$ with elevation. His results (unpublished) showed that most of the time the level of nondivergence lies between 1 and 6 km, although the scatter is great and numerous cases of levels above and below this interval are found. It is not the purpose of this report to go into detail concerning the theory involved in the concept of the level of nondivergence. Those interested may consult a forthcoming book on dynamic meteorology by J. Holmboe and others (5). Nevertheless, it is suggested here that the reason for the lack of a one-to-one relationship between observed and computed displacements is the fact that the 10,000-foot level lies on the average below the level of nondivergence.

D. Conclusions. It may be helpful to summarize the principal conclusions from the material presented above. They are as follows:

1. A fairly good relationship is indicated between observed displacements, as far as a week in advance, and those computed from the Rossby formula (assuming constant velocity), provided that cases of trough formation and disappearance are eliminated.

2. Computations for shorter intervals of time show even better correlations, thus further establishing the validity of the formula.

3. The total 10,000-foot index is a more representative value to use for the speed of the westerlies than a local index taken between trough and ridge.

4. The regression equations indicate that the 10,000-foot level may be, on the average, below the level of nondivergence.

3. **Empirical study of trough displacements.** Having completed the statistical study described above, it was thought desirable to attempt to find a purely empirical formula for trough displacement based on

TABLE 5

Correlations for Full-Week and Half-Week Movement at 45°N, Rossby Trough Formula, Using Total I_3 and Trough to Ridge Wave Length, and Eliminating Cases of New Trough Development.

	Full week		Half week	
	r	n	r	n
Winter	0.66	37	0.67	36
Spring	0.44	35	0.51	35
Summer	0.35	29	0.42	29
Fall	0.42	35	0.63	35

TABLE 6

Regression Equations of Observed on Computed Full-Week Displacement at 45°N, Rossby Trough Formula, Using Total I_3 and Trough to Ridge Wave Length, and Eliminating Cases of New Trough Development.

Winter	$S' = 14.78 + 0.156 S$
Spring	$S' = 17.14 + 0.114 S$
Summer	$S' = 9.23 + 0.139 S$
Fall	$S' = 12.23 + 0.179 S$

the variables which were indicated by the theoretical studies as being most important. Besides the variables $\bar{U}$ and L , another variable was added, the amplitude A of the pressure waves.

Thus, it is assumed that the displacement S can be expressed as a function of these three variables, or:

$$S = F(\bar{U}, L, A). \quad (4)$$

The problem, then, is to determine this functional relationship by means of the data alone. Unfortunately, in meteorological studies unlike experimental aerodynamics, it is not possible to hold all but one of the variables constant and study the effect of the variation of the remaining one. Nevertheless, there are two statistical methods which can be used to obtain the desired functional relationship, if any exists, provided a sufficient amount of data is available. These are the method of graphical correlation, and the closely related but more objective method of curve fitting by least squares. It is not the purpose of this report to do more than mention the use of such methods. Details may be found in most standard statistical textbooks (3). However, it must be emphasized again that for conclusive results by either of these two methods, a large amount of data must be available, and this was hardly true in the case of the present study, as can be seen from Table 3.

Nevertheless, application of both methods indicates the same result, namely, that the only important variable related to the displacement is the wave length, and in the case of the least squares method the first power of the wave length is the important factor. No other combination of the three variables seems to produce significantly better results.

Surprisingly enough, this appears to suggest that the zonal velocity $\bar{U}$, which enters Rossby's formula,

TABLE 7

Correlations for Full-Week and Half-Week Movement at 45°N, Using Current Wave Length Alone, and Eliminating Cases of New Trough Development.

	Full week		Half week	
	r	n	r	n
Winter	-0.64	52	-0.71	36
Spring	-0.54	54	-0.46	35
Summer	-0.38	39	-0.33	29
Fall	-0.51	40	-0.55	35

is not an important variable. However, it should be stated that in studying the problem by means of graphical correlation methods, it was found that on eliminating certain questionable data, $\bar{U}$ became important (9). Furthermore, the trough-to-ridge value of $\bar{U}$ was used in this particular study, and was probably somewhat unrepresentative, as indicated previously. Therefore, these results cannot be interpreted to mean that $\bar{U}$ is not important in determining trough displacement.

The correlations of observed displacement with wave length are shown in Table 7. The correlations show a negative relationship, as is to be expected from the Rossby formula.

These correlations were made using essentially the same data as in the case of the Rossby formula (Table 5). The greater number of cases for a full week (Table 7) is due to the fact that cases classed as "not in the westerlies" were included.

In comparing Tables 5 and 7, it is interesting to note that, while correlations for a full week using wave length alone average slightly higher numerically than the corresponding ones for the Rossby formula, the half week correlations average slightly lower. Furthermore, unlike the case of the Rossby formula there does not seem to be a systematic decrease in the correlations with time.

4. Statistical study of the Rossby formula assuming changing velocity.

A. Formula for trough displacement. It is now of interest to consider a slightly different approach to the problem of trough displacement which may throw light on the reasons for the differences in the correlations of Tables 5 and 7, and may also help to explain other observed peculiarities of trough behavior.

It will be remembered that one of the assumptions made in regard to the use of the Rossby formula in an earlier portion of this report was that the speed of the troughs remains constant during the period covered by the extrapolation. However, the behavior of individual troughs at 10,000 feet as shown on observed mean five-day 10,000-foot charts, reveals that troughs seldom move with constant velocity, but rather may move eastward for three or four days or perhaps even a week, and then come to a stop or

perhaps retrograde. It is particularly true in the summer season that troughs move in an apparently irregular fashion, with rapid changes in both speed and direction of motion. Even in the winter season, when the simple assumption of constant velocity seems more justified, troughs do not continue to move in the same direction for very long periods of time. It seems desirable, then, to develop some method to account for the changing velocity of the troughs.

According to formula (2) the velocity of a trough may change either because of a change in speed of the westerlies ($\bar{U}$) or because of a change in wave length (L). It is obvious from a study of observed charts that such changes actually occur, but in order to make use of them in obtaining a better forecast of displacement, it is necessary to show that they can be expressed as a systematic function of time. At present, there is no known theory to account for a systematic change in the speed of the westerlies, but there is a very simple scheme to account for a systematic change in wave length. In fact, this scheme follows directly from the theory of trough formation. According to ideas expressed by Rossby and others, troughs in the westerlies form when an approximately west-to-east current of air strikes a region where there is a pronounced meridional concentration of solenoids. Such a region is to be found off the east coast of Asia and tends to remain fixed for long periods of time. The air current acquires vorticity in passing through this region and a wave train is set up to leeward which, in a completely balanced state, will have a stationary wave length L_s given by formula (2), where C is set equal to zero. Let us suppose, however, that at some initial moment the wave train is not completely balanced, but has a wave length shorter than the stationary wave length (see Fig. 12). All the troughs except the one fixed by the solenoid field will then start to move eastward with a velocity given by formula (2). Let us now consider the particular trough which, at the initial moment, is separated from the solenoid field by N whole wave lengths. In other words, its distance from the solenoid field is

$$x_0 = NL_0, \quad (5)$$

where x_0 is the coordinate of the trough line on the west-east axis, N is the number of whole wave lengths between trough and solenoid field, and L_0 is the wave length of the wave train at the initial moment. We will assume that as this trough moves eastward there will be no change in the speed of the westerlies, and there will be no formation of new troughs or disappearance of old ones. In this case, it is obvious that since the field of solenoids remains fixed, the wave length must increase as the trough moves eastward, and therefore the speed of the trough must decrease in accordance with formula (2). The trough will

FIG. 12. Schematic streamline pattern in the vicinity of a solenoid field.

continue to move eastward, however, until the wave length is equal to the stationary wave length (L_s) at which time the distance of the trough from the solenoid field will be

$$x_s = NL_s. \quad (6)$$

The total displacement of the trough will then be

$$S_t = NL_s - NL_0 = N(L_s - L_0). \quad (7)$$

This very simple and obvious formula states that the trough cannot go on moving indefinitely, but can only move a maximum distance given by formula (7). Furthermore, it is clear that this formula does not depend on any particular theory of trough motion but depends merely on the assumption that there is some stationary wave length at which the wave train will be completely balanced. It is interesting to point out, in connection with formula (7) that if N and L_s do not vary greatly from one case to the next, then the displacement will be a fixed linear, negative function of the wave length, as suggested by the results of the previous discussion.

However, we are interested in knowing what the displacement will be for any given time interval t . In order to obtain a formula for displacement as a function of time, it is necessary to adopt some theory of wave motion. Such a theory is expressed in formula (2). Before we use formula (2) in this connection, it is necessary to point out that in so doing we are violating one of the assumptions involved in its derivation. This is the assumption that there will be no change in shape of the wave perturbations, and the waves we are considering definitely change their shape as the wave length changes. However, we will assume that changes in wave length are so slow that they will not constitute a serious violation of this assumption.

Considering a given trough at any time after the initial moment, we have then,

$$x = NL; \quad (8)$$

$$C = \frac{dx}{dt} = \bar{U} - \frac{\beta L^2}{4\pi^2}. \quad (9)$$

Substituting (8) in (9) we get,

$$\frac{dx}{dt} = \bar{U} - \frac{\beta x^2}{4\pi^2 N^2}. \quad (10)$$

of the theory, but it must be pointed out here that it cannot be done in the simple manner outlined above. Not only is it not possible to determine the average value of N in this way, but it will be remembered from an earlier portion of this report that, while the Rossby formula seems to be of the correct form, it does not give results of the correct magnitude. Therefore, it is necessary to determine the three constants N , L_s , and B from the regression curves themselves. Furthermore, since there are three unknown constants, while only two can be determined from each regression curve, it is necessary to make use of both curves (see Fig. 13b) to determine them. It is also obvious that, in order to get a unique solution for the constants, it is necessary that both curves pass through the same point on the x -axis. Therefore, the full-week regression, it being assumed the more unreliable of the two, has been arbitrarily adjusted by shifting it parallel to itself along the y -axis until it passes through the point representing the x intercept of the half-week curve. This is illustrated by the dotted line in Figure 13b, and it will be noted how small an adjustment is necessary. The final arithmetical process of determining the constants, using formula (12) and the two regression equations for a half week and a full week, is perfectly straightforward although somewhat tedious, and will not be discussed here. The constants for all seasons of the year are shown in Table 8.

TABLE 8
Constants in Displacement Formula (Formula 12) Obtained
from Two Regression Curves
Lat. 45°N

	N	B	$T_c = 1/B$	L_s
Winter	0.62	1.50	0.69	98
Spring	0.49	2.10	0.48	100
Summer	1.10	0.55	1.80	75
Fall	1.10	0.95	1.10	81

The values of N are shown in column 1. They average close to $1/2$ for winter and spring. This result can probably be explained by the effect of the semipermanent pressure ridge found over the Rocky Mountain area. Thus in deriving formulae (7) and (11) it was assumed that a trough is the fixed point of the wave train, and that therefore in an individual case N is always a whole number. However, there are several regions in the atmosphere where ridges, rather than troughs, tend to remain fixed for long periods of time. Such a region is the Rocky Mountain area of the North American continent. Here, orographic influences, as well as solenoidal fields created by differential heating, tend to the establishment of a semipermanent ridge. Therefore it is clear that for the bulk of the troughs considered in this study, initially located in the region east of the mountain

TABLE 9
Selected Values of Wave Length Obtained from
Mean 10,000-Foot Charts

	L_s	L_n	L_o
Winter	98	110	83
Spring	100	100	75
Summer	75	82	65
Fall	81	94	76

area, N will be close to $1/2$ under circumstances leading to a semipermanent ridge over the plateau. Thus it is probably the Rocky Mountain ridge which is responsible for the average value of N being close to $1/2$.

It is not so easy to explain the larger values of N for the summer and fall seasons. Actually, there is every reason to think that these values should be smaller, if anything. It was pointed out before that motions of troughs seem to be more irregular in the warmer season than in the colder. When we realize that the motion will more nearly approximate motion at constant velocity the larger the values of N , then we should expect N to be larger during the winter season. We must then conclude that the large values in summer and fall may be fictitious. The possible reason for fictitiously large values may be due to the poorer determination of continuity for the ill-defined troughs and ridges of summer and early fall.

In column 2 of Table 8 are listed the values of B . This constant is not of such immediate interest as its reciprocal, listed in column 3. The reciprocal of B is a measure of the "effective" length of time necessary to complete a displacement. Obviously, since the displacement reaches its final value exponentially, the time necessary to complete a displacement is theoretically infinite. In such cases, however, it is customary to choose, as a measure of the effective time, the time necessary to complete $2/3$ of the final displacement. This is known as the time constant. In this case the time constant (T_c) is given by the reciprocal of B . It is seen to be close to half a week for winter and spring and somewhat larger for the warmer months. These latter values may again be fictitious due to the poor continuity.

In the last column of Table 8 are listed the stationary wave lengths obtained from the x intercepts of the half-week regression lines. It will be noted that the spring value is slightly larger than that for winter, which may be due to the shortness of the record. The summer value is the smallest, as is to be expected. It is interesting to compare these values of the stationary wave length with some other wave length values obtained from mean charts. These are indicated in Table 9.

In the first column are found the stationary wave lengths obtained from the regression curves as listed

FIGURE 14.

in Table 8. The second column contains the normal wave lengths (L_n) obtained from normal monthly pressure charts at 10,000 feet (7). These values are obtained by taking twice the difference in longitude between the trough and ridge to its west on the normal charts. There is no reason to think that these normal wave lengths, which represent stationary conditions, should differ materially from the stationary wave lengths obtained from the regression curves. The slight differences indicated in Table 9 can probably be accounted for by the small amount of data from which L_s was computed. More specifically, if we assume that the stationary wave length is a direct function of the zonal velocity, as is indicated by the Rossby formula, then the somewhat smaller values in column 1 might be accounted for by lower than normal values of the zonal velocities. Actually the strength of the westerlies was below normal for our sample of data, particularly during the summer season.

The last column of Table 9 shows the average wave length (L_o) obtained by simply averaging the individual wave lengths for the $2\frac{1}{2}$ years of data. These values are all considerably smaller than either L_s or L_n , as we should expect, for this accounts for the observed average eastward displacement of the mean troughs.

D. Empirical displacement formulae. Let us now return to the constants in Table 8. If we assume that these represent good first approximations to the constants in the complete formula for displacement (formula 11), then we may substitute them into that formula and obtain a final complete expression for displacement. The final displacement formula for the winter season is indicated graphically in Figure 14. Numbers inserted in the curves of Figure 14 represent the time interval in days. However in the formula itself, also shown in the figure, the factor t is the time

interval in weeks. Thus the displacement for two days is obtained by substituting $2/7$ for t . This set of curves as well as similar ones for the other seasons and for latitudes 35°N and 55°N , are currently used in the Extended Forecast Section as an aid in forecasting mean trough motions. Because of lack of data the curves for 55°N are based on only two seasons, the warm season (April to September) and cold season (October to March).

In view of the possible usefulness of such formulae to others, the constants in formula (11) for latitudes 35°N and 55°N are listed in Table 10. Also included in the same table are the correlations and number of observations upon which the corresponding constants are based.

In order to obtain the complete formula for displacement, the constants in Table 10 are simply substituted for the corresponding values in formula (11). It should be remembered that L_s , as well as trough displacement (S) and current wave length (L_o), is always expressed in degrees longitude at the appropriate latitude. The factor t is the time interval in weeks. It is of course obvious that the practical usefulness of the formulae is limited by the magnitude of the correlations upon which they are based. Thus, it will be noted that the correlations are particularly low for all seasons at 35°N and in the summer season at 45°N and 55°N . It should be also noted that the correlations at 55°N are based on a small amount of data. More will be said in the following paragraphs about the use of the displacement formulae in practice.

E. Example of application of formulae. An individual example of the application of the fall displacement graph is illustrated in Figure 15. In this figure the abscissa is the date corresponding to the last day of the 5-day period, and the ordinate is the longitudinal position of the trough. The observed trough positions are indicated by dots, while those obtained from the fall displacement graph, using the observed wave length for the period ending November 7, are indicated by crosses. This case was not included in the data from which the graph was computed. It was chosen

TABLE 10
Constants in the Formula for Trough Displacement and
Other Pertinent Data, for Latitudes 35°N and 55°N

	Full week		Half week		Constants		
	r	n	r	n	N	B	L_s
35°N							
Winter	-0.32	47	-0.39	75	0.28	2.41	82
Spring	-0.51	62	-0.50	80	0.57	1.36	74
Summer	-0.25	59	-0.19	71	0.81	0.29	54
Fall	-0.54	29	-0.61	41	0.56	4.80	67
55°N							
Cold Season	-0.43	18	-0.61	26	0.32	5.12	101
Warm Season	-0.61	28	-0.53	38	0.47	0.80	113

because it was a fairly simple case, with N approximately equal to $1/2$, as shown by the fixed position of the plateau ridge throughout the period. The largest discrepancy between computed and observed positions is 4 degrees, on November 10. The other three points are within two degrees of the observed position.

F. Refinement of extrapolation formulae. As explained previously, the derivation of the empirical displacement formulae was based on the assumption that one can obtain the correct average values of N and L_s and the correct value for B even though N and L_s vary in individual cases. If this assumption is justified, then it is clear that one can improve the estimate of displacement if it is possible to find the values of N and L_s in individual cases. It was mentioned previously that if the stationary wave length (L_s) were a direct function of the zonal velocity, then values of the zonal velocity greater than normal should be accompanied by greater than normal values of L_s . Therefore, this will result in values of the displacement greater than those obtained from the graphs. In order to test this idea the stationary wave length as given by the Rossby formula was used. However, it will be remembered that observed values of the 10,000-foot index are too small to satisfy the formula, and will therefore result in values of the stationary wave length which are also too small. In order to correct this scale error, the Rossby stationary wave length may be multiplied by the ratio between the observed average stationary wave length (Table 8) and that obtained by substituting the normal value of $\bar{U}$ in the Rossby formula. The resulting equation for the stationary wave length for winter is

$$L_s = 1.53L_{sr}; \quad L_{sr} = 2\pi\sqrt{\bar{U}/\beta}. \quad (13)$$

This formula was substituted for L_s in the empirical formula for winter and the resulting equation used to compute the half-week displacement. Unfortunately the results were not as good as those obtained using wave length alone. This may have been due to a poor sample in our particular case, for more recent tests suggest that the zonal velocity is important as a supplement to the wave length graphs (10). As a working rule it is assumed that greater than normal values of the zonal westerlies will result in somewhat greater displacements than those obtained from the graphs using wave length alone.

An attempt was also made to determine values of N in individual cases. General synoptic experience, obtained by inspection of the character and motion of mean waves, can often be used to tell which trough or ridge is likely to remain fixed. For example, it is well known that pronounced cold troughs or warm ridges, extending to high levels in the atmosphere,

FIG. 15. Displacement of an individual mean trough, November 7-17, 1943.

can be depended on to remain fixed for long periods of time.

G. Quantitative verification of displacement formulae. It is now desirable to discuss further the quantitative verification of formula (11). In deriving the empirical formula for trough displacement it was assumed that to the first approximation the relationship between displacement and wave length was linear. This was done in order to simplify the statistical problem. However, it is possible, although considerably more difficult, to fit the complete formula (11) to the data for a half week by the method of least squares. Since in doing so the term in the denominator is included in the process of curve fitting, all three constants will be obtained directly. One may then use the resulting empirical formula to obtain an estimate of the full week displacements and compare the results with those observed. However, a curve of the form of equation (11) can be treated in this way only with difficulty, and furthermore, due to the smallness of the term in the denominator, a large amount of data is necessary in order to obtain conclusive results. Therefore this method has not been used.

A verification of the empirical formulae already derived was made by obtaining the displacement for a different time interval from the one used in computing the coefficients. These hypothetical displacements were then compared with those observed. This was done, using a time interval of two days. Because the construction of mean maps two days apart was only begun about the first of April, 1943, the amount of data is necessarily limited. The correlation between the observed two-day displacement and that obtained from the empirical formula for winter, together with the number of observations, is:

$$r = 0.67; \quad n = 16.$$

Thus the correlation between computed and observed displacement is about what we should expect, and therefore seems to constitute a satisfactory verification of the empirical formula. However, when we examine a graph of observed displacement plotted against computed displacement, we see that the regression line runs parallel to the line representing perfect

TABLE 11
Correlations for Summer and Winter Seasons Based on Two Different Hypotheses of Trough Displacement

	Full week				Half week			
	"Null" method		"Wave-length" method		"Null" method		"Wave-length" method	
	<i>r</i>	<i>n</i>	<i>r</i>	<i>n</i>	<i>r</i>	<i>n</i>	<i>r</i>	<i>n</i>
Winter	0.51	36	-0.64	52	0.64	36	-0.71	36
Summer	0.17	29	-0.38	39	0.18	29	-0.33	29

agreement, but somewhat to one side, indicating that in this case the observed displacements averaged somewhat greater than the computed ones. This may indicate an error in the theory but it may also suggest that during the short period covered by the test the 3-km index was much higher than normal. This was actually found to be the case.

In discussing the question of the validity of formula (11) as applied to trough displacement, it is appropriate to mention another hypothesis which is of quite a different nature but which could possibly lead to essentially the same results. This is a hypothesis based on statistical reasoning alone, and may be called the "null" hypothesis. It is based on the known fact that if one selects a series of random values of a normally distributed variate, then the probability that a given value will be followed by another closer to the average is greater than one half. In fact it can be shown that if the observed change from one value to the next is correlated with the departure of the first value from the average, a coefficient of 0.71 will be obtained. This reasoning may be applied to trough displacement if it is assumed that there is some preferred, or normal, region of trough concentration. Troughs to the west of this position, for example, would have a probability greater than 1/2 of moving eastward, and if there were no relation between the positions of a trough on two successive maps, the correlation between departure of trough position from normal and displacement from one map to the next would be 0.71. Due to the observed fact that there is serial correlation between successive positions of troughs the above coefficient would be somewhat less than 0.71. It can also easily be shown that if this "null" hypothesis is correct the regression lines of observed displacement on departure from normal, both for a full week and a half week, will have the same intercept on the *x*-axis, and the full week regression line will have the greater slope. Furthermore, since the ridge over western North America also tends to concentrate about a normal position, the wave length will be negatively correlated with the departure of the trough position from normal. This means that the above statistical hypothesis is not independent of the more physical one expressed in formula (11), which may be called the "wave-length" hypothesis. That is, if one hypothesis is correct, then positive results will

be obtained upon adopting the other. Thus it might be postulated that this accounts for the success obtained on correlating displacement with wave length. The only way to find out which hypothesis is closer to the truth is to see which one produces the better results.

A comparison of correlations obtained by these two hypotheses for approximately the same set of data is shown in Table 11. It will be seen that the correlations for the "wave-length" method (obtained from Table 7) are numerically higher in all cases.

H. Application to day-to-day trough displacements. It should be emphasized that the formulae for trough displacement presented above apply only to five-day mean 10,000-foot charts. No attempt has been made to find out how they should be modified for use on daily synoptic charts.

However, it is of interest to point out that the Rossby trough formula may have more success when applied to mean charts than when applied to daily charts. Thus, if we may assume that the formula is theoretically as valid for mean charts as for daily charts, then it should produce better results because of the much better approximation to ideal sinusoidal waves.

Statistical evidence apparently favoring this idea is contained in research recently completed at the Massachusetts Institute of Technology. Using different data, both Brooks (1) and Yehle (14) found extremely low correlations when applying the Rossby formula to the 24-hour motion of daily troughs.

A somewhat different approach to daily trough displacement was made at M.I.T. by Lt. R. A. Craig (see (4)). Instead of using for a value of wave length the distance between the trough concerned and the next trough to the west he used the distance between the trough and the next trough having approximately the same depth. On correlating this value of the wave length with the subsequent 24-hour motion he found considerably higher coefficients than those obtained by Brooks and Yehle. It is possible that this result is due to the fact that the selection of the most prominent troughs is comparable to smoothing the pressure patterns by constructing five-day means.

I. Summary of results.

The principal results of this portion of the report may be summarized as follows:

1. Statistics indicate a definite correlation between displacement and wave length, as indicated by the Rossby formula, but they also indicate a systematic variation of velocity with time.

2. These statistical results agree qualitatively with a theoretical formula for displacement based on the Rossby formula (formula 11).

3. Preliminary quantitative tests of the formula further verify the theory.

4. Although the results of an application of formula (11) based on wave length alone are not materially better than those assuming constant velocity, they may be improved qualitatively by consideration of the strength of the zonal westerlies and other factors.

5. **Conclusions.** The importance of the results outlined in this report does not lie in the correlation coefficients of Table 7, upon which the graphs of trough displacement depend, for even the largest of these coefficients accounts for only 50 per cent of the variability in trough displacement. The significance of the results lies in the apparent verification of the theory, and in the promise that if better methods can be developed for determining stationary wave lengths and areas of semipermanent troughs or ridges in individual cases, then correlations such as those in Tables 5 and 7 will be materially improved. Recently a theoretical study has been completed in the Extended Forecast Section (11), which suggests that amplitude, as well as zonal velocity, is important in determining the stationary wave length. Preliminary tests of this modified theory suggest that it may have some advantages over the Rossby method, but these are so doubtful and inconclusive that they have not been presented here.

It must also be admitted that any theory leading to improved methods of estimating stationary wave length, will without doubt also lead to different formulae for trough velocity, and therefore trough displacement. But it is not likely that the formulae will differ materially from formula (11).

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