AN APPLICATION OF ABSOLUTE-VORTICITY CHARTS

By George P. Cressman

Air Weather Service

(Manuscript received 21 August 1952)

ABSTRACT

The analysis and use of charts of absolute vorticity are described. Measurements from a winter and a summer series of maps gave 608 and 641 mb, respectively, as the pressure at the mean equivalent barotropic surface. It is shown that, on charts near the equivalent barotropic surface, the absolute-vorticity patterns give indications useful in short-range forecasting, since the lines of constant absolute vorticity are advected with nearly the speed of the wind. Examples are presented, showing typical contour and vorticity patterns for a rapidly moving pattern, a stationary pattern, and a situation of rapid trough development.

1. Introduction

Widespread interest has been aroused recently by the good results obtained in computations of the height tendencies near the *equivalent barotropic surface*. Results of such computations have been published by a group of workers at Princeton under the leadership of Charney and by a group at Stockholm under the leadership of Rossby [1; 3; 10]. Since hand computations of height tendency consume considerable time, and since machine computations on a regular basis have not appeared to be imminent, it seemed desirable to make use of the basic principle responsible for the success of the computations in a form adaptable to forecasting routine. In an effort to accomplish this objective, charts of the vertical component of absolute vorticity (referred to here as absolute vorticity) have been computed and analyzed on an experimental basis since February 1952. The following discussion is a report on the results obtained.

It is easy to make the fatal error of presenting synoptic charts of a few situations and then attempting to generalize. The examples presented below are intended rather to *illustrate* the experience gained and the lessons learned in the regular analysis of absolute-vorticity charts over a period of five months.

2. Method of computation and presentation

The computations consist of determining the absolute vorticity at and between significant points on a constant-pressure surface. The significant points are selected by inspection, and are points of maximum and minimum absolute vorticity and zero relative vorticity. Lines of equal absolute vorticity are then drawn. The computations of absolute vorticity at approximately fifty points will serve as an adequate basis for a vorticity analysis over an area slightly larger than the United States. The computations are made by the well-known method [1; 9] of reading the heights of the constant-pressure surface Z_1, Z_2, Z_3, Z_4 and Z_5 at points P_1, P_2, P_3, P_4 and P_5 in fig. 1. The absolute vorticity of the geostrophic wind, $\zeta_{a,g}$ (average value for the area $P_1P_2P_3P_4$, and plotted at P_5), is then given by

$$\zeta_{a,g} = f + (g/fS^2)(Z_1 + Z_2 + Z_3 + Z_4 - 4Z_5), \quad (1)$$

where f is the Coriolis parameter, g the acceleration of gravity, and S is the distance from P_5 to any of the other points. The value of S selected for the computations was 150 nautical miles at latitude $32\frac{1}{2}$°N, increasing slightly northward according to the change of map scale with latitude, since the grid of fig. 1 was copied on a piece of plexiglass and used as an overlay.

The vorticity of the geostrophic wind proved to be greatly different from the vorticity of the actual wind, when pronounced curvature was present. This fact explained several cases of unsatisfactory continuity of the absolute-vorticity patterns in and around intense cyclones and well-developed ridges. It is well known that in curved flow the actual wind speed differs systematically from the geostrophic wind speed, frequently by 30 per cent and occasionally by a much greater percentage. Therefore the policy was adopted of multiplying the relative vorticity, or the sum $(Z_1 + Z_2 + Z_3 + Z_4 - 4Z_5)$, by the ratio

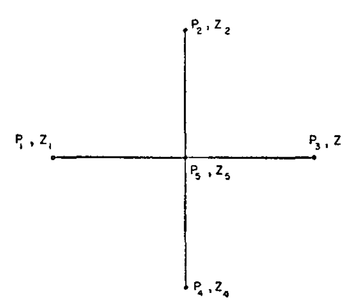

FIG. 1. Grid used in computation of absolute vorticity.

v/v_g, where v and v_g are the observed and geostrophic wind speeds. If no observed winds were available, the gradient wind speed was substituted for the observed wind speed. The justification of this procedure is evident from a consideration of the normal equation for the relative vorticity ζ_r, where

$$\zeta_r = v/r + \partial v/\partial r \tag{2}$$

where v is the wind speed and r is the radius of curvature. The above correction procedure involves the

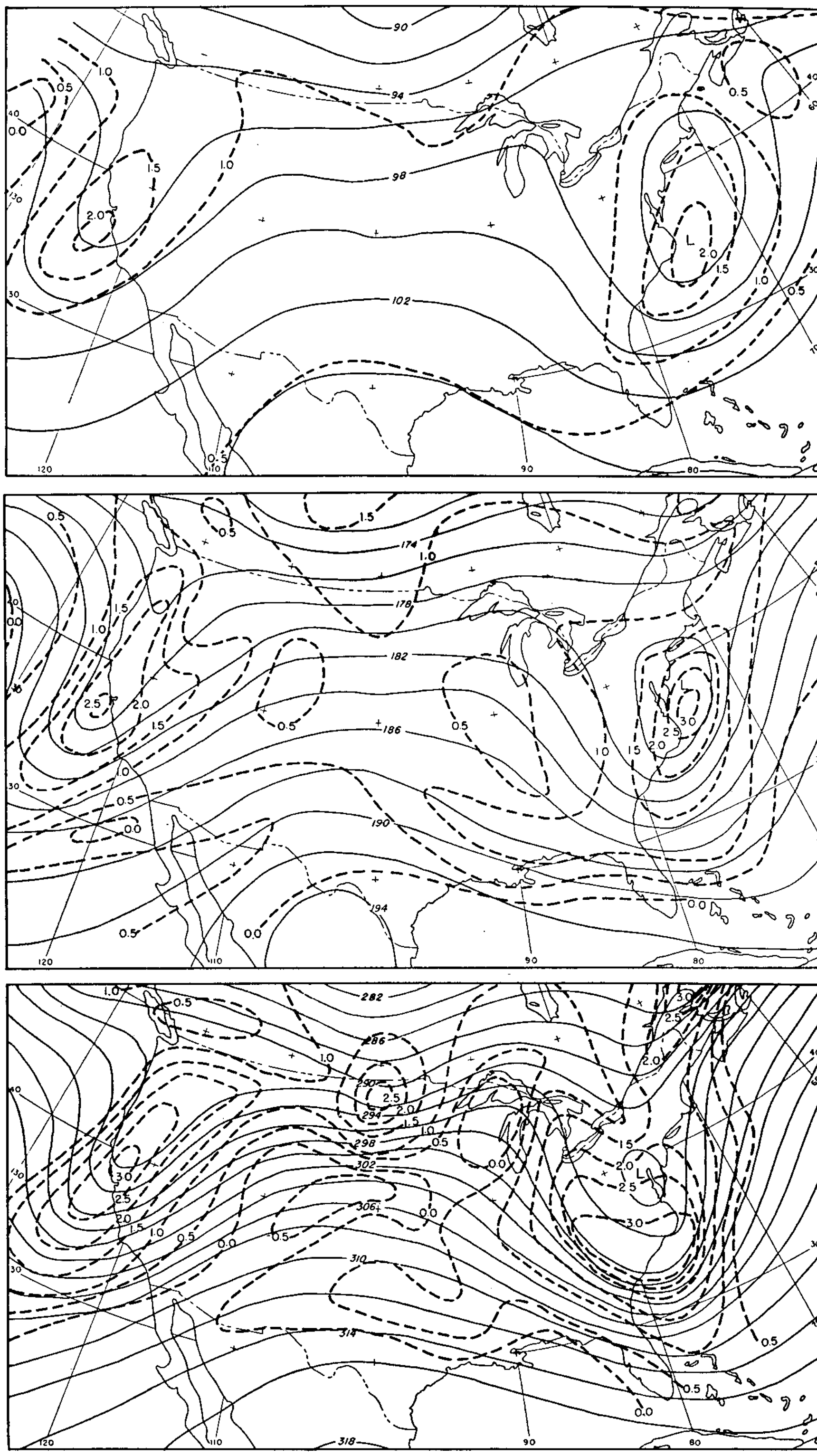

Fig. 2. Maps for 0300 GCT 7 January 1952. Solid lines: contours, labelled in hundreds of ft; dashed lines: lines of constant absolute vorticity, labelled in units of 10^{-4} sec^{-1}. Top: 700 mb; center: 500 mb; bottom: 300 mb.

assumption that the lateral shear is proportional to the wind speed.

As has been emphasized by Rossby and collaborators [10], it is necessary to have a very accurate analysis of the contours before computations are made. Once the contour analysis is completed, an experienced analyst can compute and analyze the absolute vorticity field at 500 mb in 45 min or less for an area slightly larger than the United States. A graphical solution of (1) was used but is not presented here, since it applies only to the polar-stereographic map projection because of the variation of S with latitude.

3. Variation of the vorticity patterns with height

The vorticity equation can be written in the form, with reference to horizontal motion,

$$(\mathbf{V} - \mathbf{C}) \cdot \nabla_H \zeta_a = - \zeta_a \nabla_H \cdot \mathbf{V}, \tag{3}$$

where $\mathbf{V}$ is the horizontal wind vector and $\mathbf{C}$ is the horizontal movement of the lines of absolute vorticity. Since troughs and ridges move at about the same speed at all elevations in the middle and upper troposphere, while the wind speed and vorticity gradient increase with height, the term $(\mathbf{V} - \mathbf{C}) \cdot \nabla_H \zeta_a$ increases with height up to the tropopause. At the level where, in the mean, $(\mathbf{V} - \mathbf{C}) \cdot \nabla_H \zeta_a = 0$, the equivalent barotropic surface is found. It is true that solenoid action, vertical transport of vorticity, and the turning of vorticity from about the horizontal to the vertical contribute to changes of absolute vorticity about the vertical [7]. However, it seems from the work at Princeton and at Stockholm that, at least for short-term forecasting, these effects are of secondary importance and that useful results can be obtained when they are neglected.

The analysis of a number of charts of absolute vorticity at 700, 500 and 300 mb has indicated that the absolute-vorticity patterns have about the same position and wavelength, but increasing amplitude and increasing horizontal vorticity gradients with increasing height. The product $\mathbf{V} \cdot \nabla_H \zeta_a$ ordinarily increases strongly in magnitude with height without changing sign, except along strongly sloping ridge or trough lines. This is of considerable significance in view of the work of Riehl, Norquest and Sugg [8], who use the product on the 300-mb chart to evaluate the chances of precipitation. Since this product does not as a rule change sign between 500 and 300 mb, its value at 500 mb can be used as a good indication of its value and also of the value of $(\mathbf{V} - \mathbf{C}) \cdot \nabla_H \zeta_a$ at 300 mb, thus indicating the divergence.

An example of absolute-vorticity charts at different elevations is shown in fig. 2. These charts furnish a good example of the observations discussed above. For example, in and on the east side of the west-coast trough, one can see the increase with height of the central value of absolute vorticity, and the strong increase with height of the product $\mathbf{V} \cdot \nabla_H \zeta_a$. At 700 mb the extreme values of ζ_a on the map are about -0.1×10^{-4} and $+2.2 \times 10^{-4}$, while at 300 mb they are -0.9×10^{-4} and $+3.2 \times 10^{-4}$. It can also be seen that the sign of the product $\mathbf{V} \cdot \nabla_H \zeta_a$ does not ordinarily change with height. (The negative values of absolute vorticity shown on these and subsequent charts check with the reported winds, where available.)

TABLE 1. Comparison of 24-hr movement of absolute-vorticity lines with wind speed normal to lines.

Constant-pressure surface (mb):	700	500	300
3–7 January 1952			
Mean movement of lines (per cent of normal wind speed)	121	79	50
Standard deviation (percentage)	24	19	12
Standard deviation of mean (percentage)	3.7	3.0	1.8
Number of measurements	43	43	43
16–20 June 1952			
Mean movement of lines (per cent of normal wind speed)	113	74	59
Standard deviation (percentage)	29	15	18
Standard deviation of mean (percentage)	4.9	2.6	3.0
Number of measurements	34	34	34

4. Selection of the best level

For forecasting changes of the flow pattern, as well as for computing tendencies, it is desirable to select a constant-pressure surface where, in the mean, the lines of constant absolute vorticity move with the wind. ($[\mathbf{V} - \mathbf{C}] \cdot \nabla_H \zeta_a = 0$.) Most of the literature points to the 600-mb surface as the most suitable [2; 4; 5; 6; 10]. An additional evaluation of the location of the surface of non-divergence was made in this study by selecting two five-day series of charts of absolute vorticity for 700, 500 and 300 mb. Then the 24-hr movements of the absolute-vorticity lines were compared with the wind speeds normal to the lines at the beginning and end of each 24-hr period, for each place where a reasonably accurate determination could be made. The 24-hr movement of the line was then expressed in terms of the percentage of the average normal wind speed for the same period. The results are given in table 1.

From table 1, it can be seen that the lines of constant absolute vorticity moved at 75 to 80 per cent of the wind speed normal to these lines at 500 mb. The meteorological significance of the standard deviations

TABLE 2. Pressure at mean equivalent barotropic surface.

Series	Mean equivalent barotropic surface (mb)	Standard deviation of mean (mb)
3–7 January 1952	608	15
16–20 June 1952	641	19

is not clear, since they include errors of measurements as well as anomalous movements. As a result of the certain existence of significant (but, it is hoped, random) errors of measurement, it would be highly desirable to see further tests made by this method as a check on the results. The decrease of the standard deviation with height is due to the increase of wind speed with height, since the standard deviations are given as percentages of wind speed.

The mean equivalent barotropic surface for each period was obtained graphically from the data in table 1, giving the results shown in table 2.

The results of table 2 confirm the work of the earlier studies cited, in giving 600 mb as being close to the equivalent barotropic surface. The lowering of this surface from winter to summer was unexpected, but is probably significant for the two series studied.

Although the 500-mb surface is widely used as a standard surface for upper-air analysis, the above data along with the many other similar results found in the literature point to the desirability of analysis of the 600-mb surface. The 600-mb surface has special significance in that it is the mean equivalent barotropic surface, in which the absolute vorticity is effectively conserved. It is adaptable to many forecasting methods, such as long-wave computations [4], tendency computations [3] and vorticity analysis. This does not mean that one should abandon analysis of higher levels, but rather that a slight rearrangement of standard analysis levels is desirable. Because of the lack of 600-mb charts, the 500 mb surface has been used in the following discussion.

5. Examples of synoptic situations

Maxima or minima of absolute vorticity are associated with specific features of the tropospheric flow, such as troughs or ridges. When the lines of absolute vorticity lie parallel to the wind, one should expect little or no change of the flow pattern. If, on the other hand, the lines are lying across the streamlines, motion of the vorticity pattern is indicated. The motion of the vorticity pattern is accompanied by a corresponding motion of the associated flow pattern.

An example of a rapidly moving trough and ridge

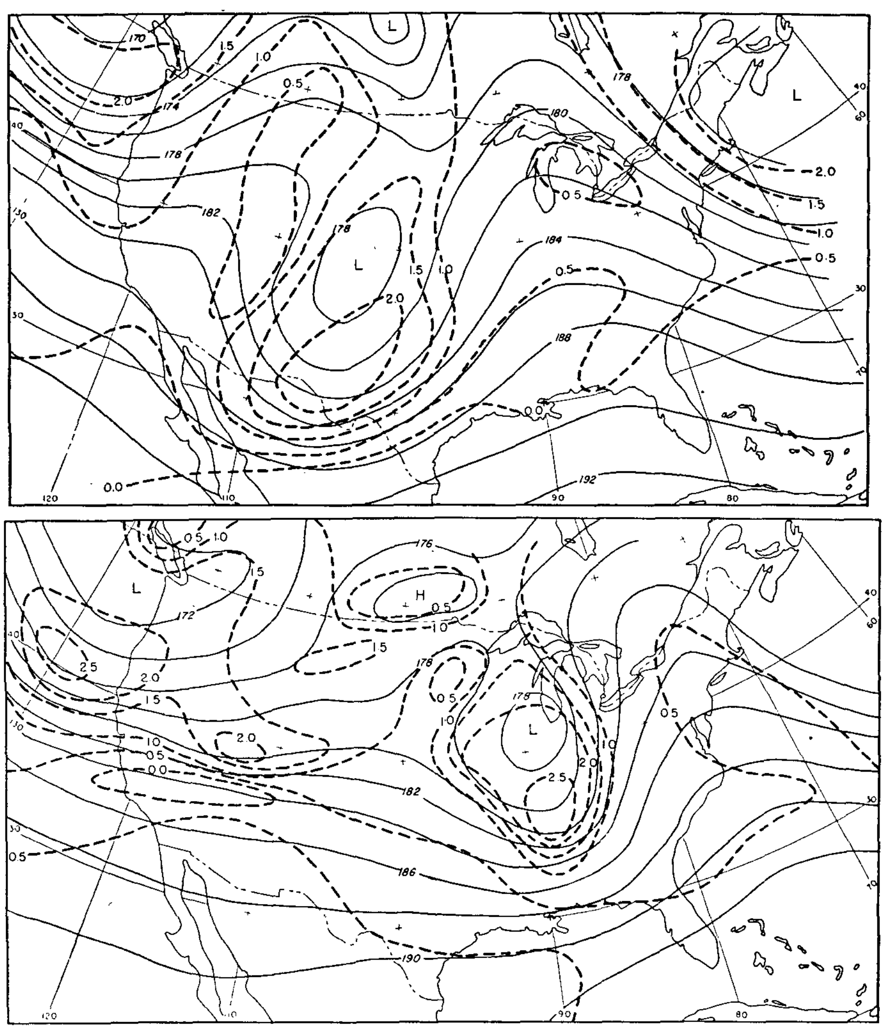

FIG. 3. 500-mb charts, 0300 GCT. Top: 18 March 1952; bottom: 19 March 1952.

pattern is shown in fig. 3. The close relation of the maxima and minima of absolute vorticity to the troughs and ridges is evident. The strong advection of the lines of absolute vorticity by the wind is clearly shown in both ridges and in the trough in the top portion of fig. 3. Additional evidence of movement of the trough is found in the Texas panhandle area, where the vorticity maximum is imbedded in a westerly flow. It is interesting to note that, on both maps, vorticity minima are found in association with strong lateral shear south of troughs. Comparison of the two maps in fig. 3 shows that many details of the vorticity pattern are not well preserved from one map to the next. This is partly due to the difficulties of making a sufficiently exact analysis, but must also reflect solenoid action, *etc.* However, the main indication, *i.e.*, that of rapid motion of the flow pattern, was reliable. In situations such as this, it is easy to compute trough and ridge motion by computing the eastward advection of the lines of absolute vorticity (with use of the factor of 75–80 per cent given earlier for the 500-mb surface). Good results from this type of computation will be obtained, provided a clear-cut and regular pattern of intersections is found on the starting map.

A search for an example of a stationary pattern led to the maps shown in fig. 4. It seems impossible to find a pattern which is completely stationary in all respects over an area as large as that shown, since there are always some moving wiggles or some slow motion in the flow patterns. However, the difference between figs. 3 and 4 is clearly evident. The chart for 7 March (fig. 4, top) shows very little vorticity advection, particularly in the ridge. The 1.0×10^{-4} line of absolute vorticity almost coincides with the 18,200-ft contour. The maximum of vorticity is found slightly west of the low center off the west coast, thus indicating a slow southeastward motion of the low. The chart for 8 March (fig. 4, bottom) shows the expected results. However, the pattern is about to lose its stationary characteristics, as shown by the increasing advection of absolute vorticity over the west half of the map.

Certain recognizable sequences of vorticity patterns occur repeatedly. These can often be recognized in their early stages and can be used as valuable forecast-

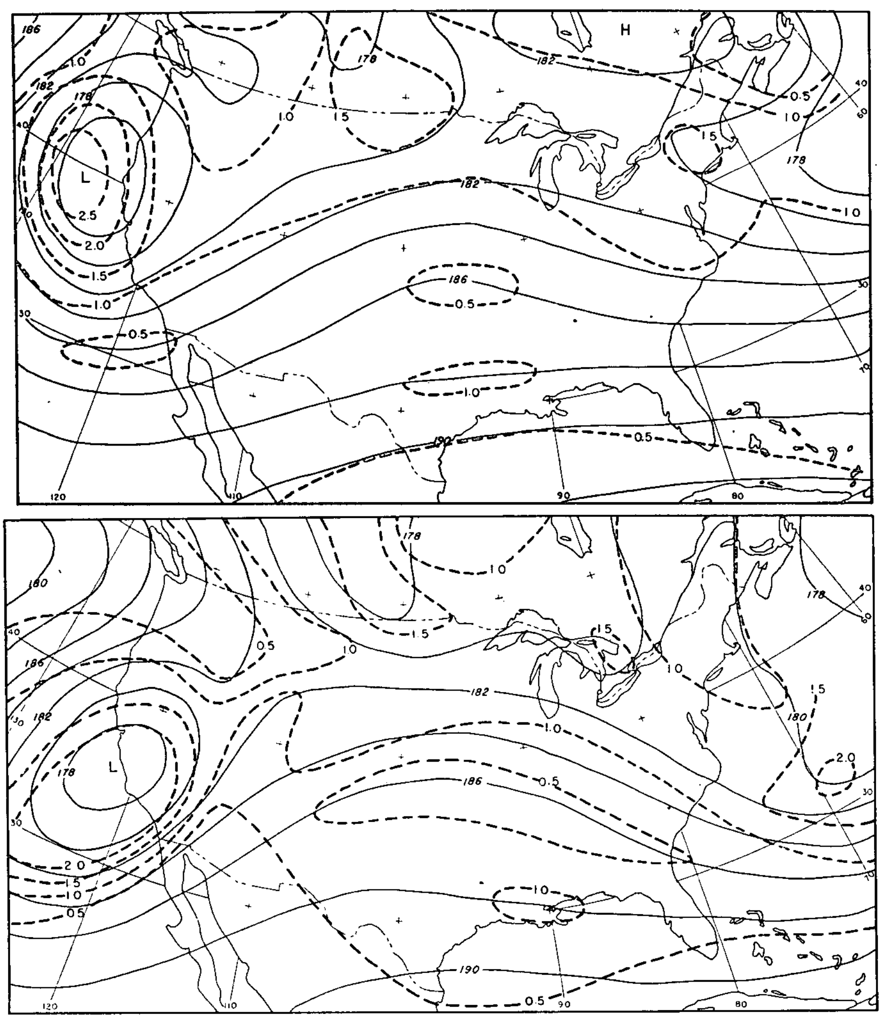

FIG. 4. 500-mb charts, 0300 GCT. Top: 7 March 1952; bottom: 8 March 1952.

ing indications. For example, a maximum of absolute vorticity having central values of 1.5×10^{-4} or more, which is being advected in a southerly direction, will be associated with a deepening trough which will often develop into a closed low at 500 mb. In the majority of cases of this type, the absolute-vorticity maximum will expand in area along with an increase of the central value. No immediate explanation for the intensification of the absolute-vorticity maximum itself can be offered. The possibility of solenoid action and conversion of potential to kinetic energy suggests itself, but more study of this effect is needed. However, even

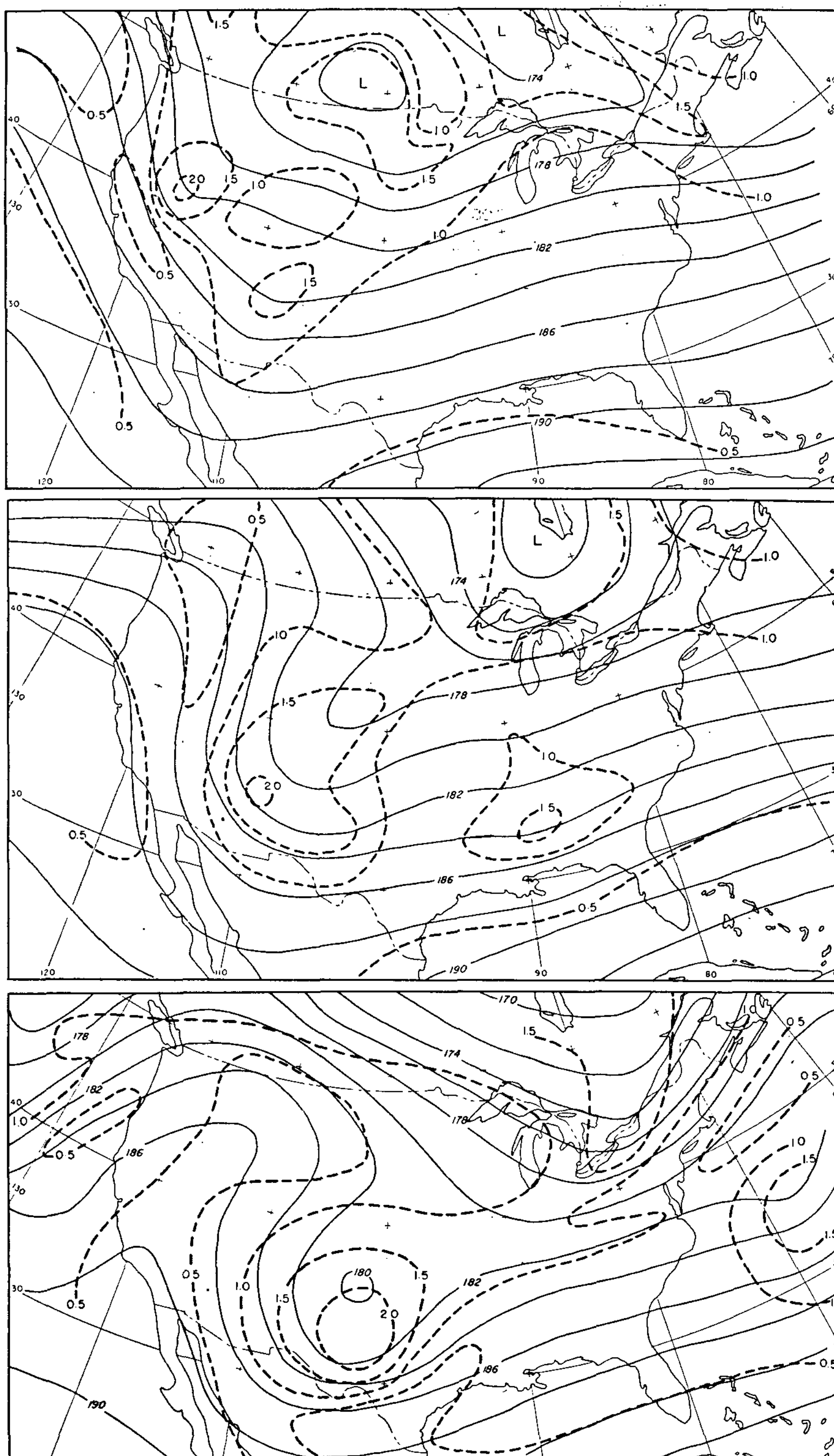

FIG. 5. 500-mb charts, 1500 GCT. Top: 23 February 1952; center: 24 February 1952; bottom: 25 February 1952.

when the maximum of absolute vorticity does not intensify, the deepening 500-mb trough and low are observed. A small part of this can be explained by the increase of relative vorticity as the system moves southward. The remainder of the deepening seems to occur in association with an increase in the cyclonic curvature of the flow, at the expense of the cyclonic shear.

An example of this type of development is shown by Charney, Fjørtoft and von Neumann [3], and by Bolin and Charney [1]. Their maps have the deficiency that there is an increase in the geostrophic vorticity, which does not reflect a corresponding increase in the actual vorticity as the curvature of the flow increases. This southward motion of a vorticity maximum is shown in fig. 5. On 23 February (fig. 5, top), the two vorticity maxima of interest are found over northern Nevada and northwestern New Mexico. The latter maximum has reached the southern part of its trajectory, and is about to move eastward. The maximum over Nevada is associated mainly with wind shear in a relatively insignificant-looking trough. However, it has a central value of 2.1×10^{-4}, and is being carried southeastward. On 24 February (fig. 5, center), a pronounced change can be seen. The first maximum was carried rapidly eastward with little change in intensity. The Nevada maximum moved to eastern Arizona, with no change in the central value. However, the 1.5×10^{-4} line encloses a much larger area than previously, and a notable development of the 500-mb trough has occurred. This is accentuated by the eastward advection of low absolute vorticity, along with height rises, in the northwest. Further development of the trough can be expected, since the maximum of vorticity over Arizona is still being carried southeastward. By 25 February (fig. 5, bottom), the easternmost of the two vorticity maxima has moved to a position west of Bermuda. The other maximum is now found over Texas, in association with a deep trough and closed low. Although the 1.5×10^{-4} line has stopped expanding by this time, the 2.0×10^{-4} line encloses a much greater area than before. The central value of absolute vorticity has increased only slightly. In the meantime, the ridge in the northwest states has moved eastward in association with extensive height rises, as was indicated by the negative advection of vorticity in that area earlier.

Again, it is evident that many details of the absolute vorticity pattern are not preserved, due to many complicated effects; but many of the main developments are clearly indicated.

6. Conclusions

The maxima and minima of absolute vorticity analyzed by the above methods are associated with individual troughs, ridges and regions of large lateral shear, and are usually of different dimensions from the long-wave patterns. It is not uncommon for a vorticity maximum to move completely through a long-wave pattern. When the maximum of absolute vorticity is superimposed on the long-wave ridge, the vorticity is mainly manifested in the form of lateral shear. As the maximum enters the long-wave trough, the shear decreases and the curvature increases. The converse type of development is often shown by minima of absolute vorticity.

The preparation of the charts requires an accurate and painstaking analysis of the 500-mb chart. There are sufficient data over the United States [1] and over the Atlantic and northwest Europe [10] to permit such an analysis. The successful preparation of vorticity charts over the Pacific has been shown to be practical by experience at the USAF Weather Central. However, this required a very careful preparation of the 500-mb charts by a two-stage differential analysis, building from 1000 to 700 mb and from 700 to 500 mb, with the use of every scrap of data available at all three levels. If the 500-mb charts over the Pacific are prepared hastily, poor results will be obtained from the vorticity analysis. This is not too surprising a result, since hastily analyzed charts are of little value for any purpose.

The disadvantages of the use of absolute vorticity charts are then seen to be:

1. The time consumed in their computation and analysis;
2. The necessity for spending more time in the preparation of the basic contour analysis. This, however, could be considered by many to be an advantage rather than a disadvantage, since a more accurate analysis is the result.

The advantages of using these charts are:

1. The current movement of the various parts of the flow pattern is immediately evident at a glance. Computations of movement yield excellent results in places where good cross-patterns of vorticity lines and contours exist. The motion of the north and south parts of troughs or ridges relative to each other is easily seen. These motions lead to such changes in shape of troughs or ridges, "cut-off lows," *etc.*, which are seen either not at all or with great difficulty from continuity methods of prognosis. The tendency computations which can be derived from absolute-vorticity charts are a relatively round-about method of prognosis, since the height tendencies which are derived from the charts do not give motion directly but must be used either in formulae or must be added back to the contour field to give motion;
2. Non-barotropic effects in the atmosphere can be readily observed from the larger-scale changes in the vorticity patterns;
3. A determination of the direction of vertical motion and the chances of precipitation can be made by the methods of Riehl, Norquest and Sugg [8], since the 500- or 600-mb charts are nearly as satisfactory as the 300-mb chart for this purpose.

The chief benefits derived to date from the analysis and study of absolute-vorticity charts appear to be in the realm of short-range (24-hr) forecasting. The possible uses of these charts for indicating developments for longer periods are not evident at present. The charts do present a clear and readily grasped picture of motion and development trends at map time, and thus enable the forecaster to make a good start on his 48-hr forecast.

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