RADAR ECHOES FROM A GROWING THUNDERSTORM

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ABSTRACT

The theoretical growth of hail and the behavior of the first radar echo in a cumulus cloud are analyzed. Good agreement is found between the theory and the observations of Workman and Reynolds.

1. Introduction

In an investigation of twelve thunderstorms in New Mexico by Workman and Reynolds (1949), it was found that the first radar echo received by an AN/APQ–13 radar from the growing cumulus was centered at about −10C. The radar echo then expanded upward at a rate ranging from 2 to 9 m/sec. At about 12 min after its first appearance, the echo reached its maximum height at about −30C.

Observations made by the Thunderstorm Project (U. S. Weather Bureau, 1949) in Ohio with an AN/TPS–10 radar give the most frequent appearance of the first radar echo as located 1000 ft above the freezing level (about −2C). The discrepancy between these data and those of Workman and Reynolds was ascribed to the difference in the detection capabilities of the two sets and to geographic differences. In a further analysis of showers observed by the Thunderstorm Project in Ohio, Battan (1952) found that many of the echoes first appeared at temperatures above 0C. It was concluded that the precipitation in such cases was initiated by large water droplets and did not involve the ice phase. Battan, however, found difficulty in explaining the simultaneous ascent and descent of the radar echo after its first appearance.

The New Mexico observations were made for the specific purpose of locating first echoes from cumuliform clouds, whereas the Ohio observations had other purposes. In addition, the New Mexico observations were made at ranges generally under 20 mi. The range at which the showers were observed in Ohio were not listed, but one shower, displayed in a photograph of a range-height indicator by Battan, had a range of 47 mi. In showers observed at such ranges, the accuracy in determining the height of the first echo is small, and in addition the stronger echoes from below the freezing level are often detectable while the weaker echoes from above are not detectable. Hence, the range factor may explain some of the discrepancy in results.

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It is the purpose of this paper to compute theoretically the growth of precipitation particles in cumuliform clouds, and to determine thereby the theoretical first appearance of the radar echoes. In analyzing this problem, it is important to distinguish between the processes of initiating precipitation growth in cumuliform clouds by the ice phase and by the introduction into the lower part of the cloud of large droplets of diameter about 50μ, the origin of which may be the giant sea-salt nuclei observed by Woodcock (1952) over ocean areas and coastal regions. It is not known whether these giant nuclei exist over continental interiors; to the writer's knowledge, no cases of precipitation from clouds entirely above freezing have been reported over continental interiors. Present observations and theories indicate that the ice phase must be involved in thunderstorms. Since the radar observations were made of thunderstorms over land areas, it will be assumed in this paper that the precipitation is derived from the ice phase. Because of the more detailed data available, atmospheric conditions appropriate to the observations made in New Mexico will be utilized. In a future paper, the writer expects to analyze the theoretical behavior of radar echoes in clouds in which the precipitation is derived from large water droplets.

2. Theory of hail growth

An ice crystal in a supercooled water cloud grows by sublimation and by collision with cloud droplets which become frozen to it. In its early stages of growth, the ice particle maintains its crystalline habit; but as soon as sufficient water has been caught, the crystal converts into graupel (or soft hail) of quasi-spherical shape. It is possible, by making suitable assumptions concerning the fall velocity and efficiency of catch, to compute in detail the initial growth of the crystal and its subsequent transformation into graupel. It is found, however, that these refinements add little accuracy, and that there is negligible error involved in assuming the ice particle to grow as a sphere from its initial stages. The rate of growth of an ice sphere is given by

$$dm/dt = 2\pi DcK\,\Delta\rho + \tfrac{1}{4}\pi D^2 EVW, \quad (1)$$

where D is the diameter, V the difference in fall velocity between the ice particle and the cloud droplets, W the liquid-water content, E the efficiency of catch, K the coefficient of diffusion of water vapor in air, and $\Delta\rho$ is the vapor-density difference between the ambient air and the surface of the particle, corrected for heat balance (see Wexler, 1952). The coefficient c modifies the diffusion equation due to the motion of the particle (Kinzer and Gunn, 1951). In the cloud of high liquid-water content which we shall be considering, the growth rate by diffusion becomes negligible compared to growth by coagulation with the cloud drops, when D exceeds about 0.1 mm. The rate of growth of the graupel then becomes

$$dD/dt = \tfrac{1}{2}EVW\sigma^{-1}, \quad (2)$$

where σ is the density of the added frozen water droplets.

The water caught by the hail during a given time interval is frozen to it only if it is capable of disposing of the heat of fusion during that interval. Due to the addition of this latent heat, the temperature of the hail becomes higher than its surroundings. When the temperature of the hail is 0C, the maximum amount of heat is lost to the air. The hail becomes wet when the heat of fusion of the water caught exceeds the heat lost by conduction and evaporation to the air (Ludlam, 1950), *i.e.*, when

$$\tfrac{1}{4}\pi D^2 EVW(L_f + s\,\Delta T) > 2\pi Dck\,\Delta T + L_v 2\pi DcK\,\Delta\rho, \quad (3)$$

radiative heat loss and heat conduction to the interior of the hail being negligible by comparison. Here ΔT represents the difference between the hail temperature at 0C and the air temperature T; $\Delta\rho$ denotes the difference between the vapor densities at water saturation at 0C and T; L_v and L_f are the latent heats of vaporization and of fusion, respectively; k is the heat conductivity of air, and s is the specific heat per gram of ice.

As a result, during this "wet" stage of hail growth, a portion of the water caught will freeze, forming clear ice; some will evaporate; some will remain as a film on the hail; and the remainder will stream off the lee side of the hailstones, probably in the form of small raindrops. The rate of growth of the ice portion of the wet hailstone becomes

$$\frac{dm}{dt} = \frac{2\pi cDk\,\Delta T + 2\pi cDK\,\Delta\rho\,L_v}{(L_f + s\,\Delta T)}. \quad (4)$$

The above theory can account for three layers of the hail: an inner crystal core, a layer of rime ice, and an outer layer of clear ice. As suggested by Ludlam, hail of many more layers could be produced in a cloud of variable liquid-water content, such that the hail alternately undergoes dry and wet growth.

TABLE 1. Mean concentration of ice nuclei for expansion rate equivalent to 5 m/sec updraft (Findeisen and Schulz, 1944).

Temperature (deg C)	Concentration (cm^{-3})
−6	10^{-7} (extrapolated)
−8	10^{-6}
−10	10^{-5}
−14	10^{-4}
−23	10^{-3}

3. Assumptions

Ice nuclei.—It has been found by various investigators that the concentration of freezing nuclei in supercooled water clouds increases as the temperature decreases. An experiment considered most representative of atmospheric processes is that of Findeisen and Schulz (1944), who observed the formation of ice particles in a 2-m^3 expansion chamber, using uncleaned surface air. With an expansion rate equivalent to a 5 m/sec updraft, the mean concentration of ice nuclei at different temperatures was found to be as shown in table 1. For comparison with the radar observations, these concentrations will be used as representative of ice-particle concentrations to be expected in cumuliform clouds. The effect of a variation in the concentration by a factor of ten, which essentially covers the range of observations of different experimenters, will be discussed.

The cloud.—The composite cloud of Workman and Reynolds in New Mexico has a base at 8C and 650 mb, and extends along the moist adiabat above that level. For the purpose of computing hail growth in this cloud, the distribution of liquid-water content is determined on the basis of the parcel method; the liquid-water content is 2.1 g/m^3 at 0C, and rises to a maximum of 3.8 g/m^3 at about −25C. The parcel method may be approximately valid (Ludlam, 1952) in the central core of the thunderstorm, where, due to the high updraft velocities, the effect of entrainment is negligible. Results, however, are not dependent on this assumption, since calculations show that the behavior of the radar echo is similar over a large range in liquid-water content. Some computations of hail growth were also made for a cloud with base at 15C and 780 mb, and extending along the same moist adiabat. The liquid-water content in this cloud had a maximum value of 5.5 g/m^3. For the same updraft velocity and distribution of ice nuclei, the theoretical behavior of the radar echoes for the two clouds is found to be very similar (see section 5).

Table 2. Assumed densities of hail growth.

Diameter interval (mm)	Density (g/cm³)	Comments
0–0.1	0.9	Diffusion predominant
0.1–0.2	0.6	Coagulation predominant
>0.2 (dry)	0.6 and 0.2	Rime ice
>0.2 (wet)	0.9	Clear ice

Hail properties.—No reliable measurements exist on the densities of the different layers of hail, and estimates may only be made indirectly. It is assumed that the crystal has a density of 0.9 while growth by diffusion is predominant. According to icing data from multicylinder runs, the density of rime ice may vary from 0.1 to 0.9. For this reason, separate computations are made for densities 0.6 and 0.2 during the graupel or dry stage of hail growth; but because the computed trajectories of the hail for the two densities are found to be almost identical (see section 4), giving very similar radar-echo behaviors, the results for hail of density 0.2 will be discussed only briefly. The density of clear ice during the wet stage is probably close to 0.9. For the computations of hail growth, assumptions concerning the growth density of hail are made, as shown in table 2.

The fall velocities of spherical hail have been computed with the aid of drag coefficients for spheres, as listed by Langmuir (1948), and are shown in table 3 for hail of density 0.6 at the −10C level of the cloud.

The efficiency of catch of cloud droplets may be computed from the theory of Langmuir (1948), although there is still considerable question as to the proper values when the ice particles are of a size comparable to that of the cloud drops. In our computations, E is assumed to be 0.8 for hail diameters less than 0.2 mm; for diameters greater than this value, E is assumed to be 0.9 for hail of density 0.6, and $E = 0.8$ for density 0.2. These values are in approximate agreement with Langmuir for cloud drops of average size in potential flow.

4. Hail trajectories

To determine the hail trajectories for a given updraft velocity, assumed constant throughout the cloud, an ice particle of diameter 20μ is assumed to originate at a given level within the cloud. The time of growth between successive diameters is then computed, and the new position of the particle within the cloud is calculated, with use of the updraft velocity and the average fall velocity of the particle. For $D \leq 0.2$ mm, where diffusion is an important growth factor, (1) is used; for $D > 0.2$ mm, where diffusion may be neglected, (2) is used for the dry stage and (4) for the wet stage of hail growth.

Table 3. Terminal fall velocities of hail of density 0.6 at 425 mb and −10C.

Diameter (mm)	Velocity (m/sec)
0.2	0.6
1.0	3.9
2.0	7.3
3.0	9.8
4.0	12.0

Fig. 1 shows the computed trajectories of hail of density 0.6 (during the dry stage), for an ice particle originating at the −8C level, in a cloud with updrafts of 2, 5 and 8 m/sec. The hail diameters in millimeters are shown along the trajectories. The wet stage of hail growth is indicated by the dashed line. For the three updraft velocities, the respective temperatures at which wetness occurs are −7, −12, and −18C.

The trajectories for hail of density 0.2 are found to be almost identical to those in fig. 1. The hail is larger, and wetness occurs at a somewhat higher level in the cloud. In the 8 m/sec updraft, for example, the hail becomes wet with a diameter of 1.9 cm at −20C; the maximum hail diameter at 0C is 2.2 mm.

Similar trajectories may be computed for ice crystals originating at other levels. At temperatures warmer than −8C, a crystal has a slower rate of growth, due to lower vapor-density differences and lower liquid-water content. Likewise, crystals originating at temperatures between −8C and about −25C have faster rates of growth. This variation may be of importance in determining which nuclei first reach detectable sizes.

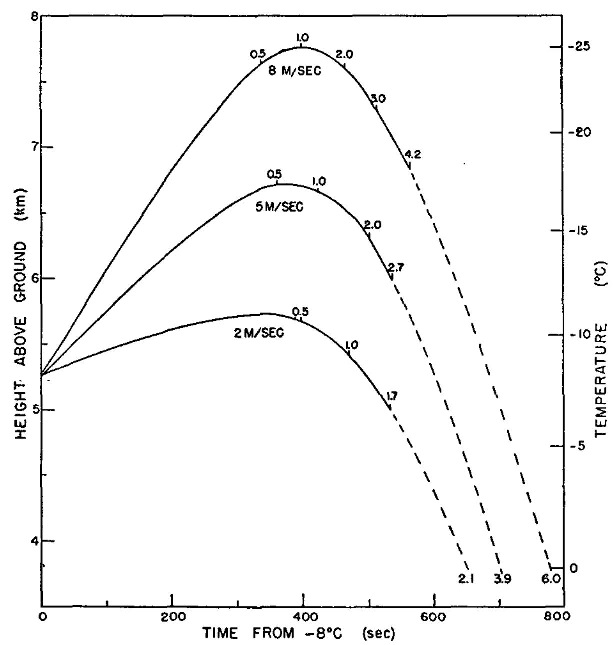

Fig. 1. Trajectories of hail originating at −8C level in updrafts of 2, 5, and 8 m/sec. Dashed line represents wet stage of of hail growth. Numbers represent hail diameters in mm.

5. Radar detection of hail

On the assumption of Rayleigh scattering and a single particle size, the maximum range R (in km) to which an ice sphere of diameter D (in cm) and density σ (g/cm^3) with a concentration N (cm^{-3}) can be detected by the following 3.2-cm radars is given by[2]

AN/APQ-13 (0.5-μsec pulse): $R^2 = 1.85 \times 10^{13} ND^6\sigma^2 \times 10^{-0.2aR}$,
AN/TPS-10: $R^2 = 12.6 \times 10^{13} ND^6\sigma^2 \times 10^{-0.2aR}$,

where a is the path attenuation in db/km.

A wet hailstone has a greater radar cross-section than a dry hailstone of the same size, due to the higher dielectric constant of the water and the higher density. According to results given by Langleben and Gunn (1952), a sphere in the Rayleigh scattering region, consisting of an inner core of ice and an outer concentric layer of water with thickness one-tenth the radius, has a radar cross-section about 4.5 times that of an ice sphere of the same size. In our computations of hail growth during the wet stage, the computed size of the hailstone was that of the ice; the wet film may be 0.1 mm or more in thickness. In view of these factors, and the variation from Rayleigh scattering for wet hail of sizes with which we shall be concerned, a value of the radar cross-section of wet hail has been chosen as 10 times that of a dry ice sphere of the same dimensions as the ice portion of wet hail. In general, any reasonable value of the radar cross-section of wet hail gives approximately the same results for the behavior of the first radar echo; but as will be discussed later, the value may be critical for first radar echoes located near 0C.

Most of the storms observed by Workman and Reynolds were at a range near 25 km, and this range is assumed initially. Assuming a path of 23 km through clear air with an attenuation of 0.015 db/km, and 2 km through a cloud containing 3.5 g/m^3 (with an attenuation at -15C of about 0.6 db/km), we find a value of $10^{-0.2aR} = 0.49$. The equations for the detection of hail at 25 km then become

	Dry hail	Wet hail
AN/APQ-13:	$ND^6 = 191 \times 10^{-12}$,	$ND^6 = 19.1 \times 10^{-12}$,
AN/TPS-10:	$ND^6 = 28.2 \times 10^{-12}$,	$ND^6 = 2.82 \times 10^{-12}$.

For a given concentration of hail, the minimum size detectable at 25 km may then be computed. The results of such computations are shown in table 4.

It may be seen from fig. 1 and table 4 that, for ice particles originating at -8C with a concentration of 1 m^{-3}, the location of the first radar echo depends largely on the updraft velocity; at 2 m/sec, the first echo is located at about -8C; for 5 and 8 m/sec, the echoes are at about -16 and -24C, respectively.

[2] Computed from values of the radar constants communicated to the writer by Mr. D. Atlas.

TABLE 4. Minimum hail diameters detectable at 25 km.

Concentration (cm^{-3})	AN/APQ-13 Dry hail (mm)	AN/APQ-13 Wet hail (mm)	AN/TPS-10 Dry hail (mm)	AN/TPS-10 Wet hail (mm)
10^{-8}	—	3.5	—	2.6
10^{-7}	3.5	2.4	2.6	1.8
10^{-6}	2.4	1.6	1.8	1.2
10^{-5}	1.6	1.1	1.2	0.81
10^{-4}	1.1	—	0.81	—
10^{-3}	0.76	—	0.55	—

Moreover, if the concentration of ice nuclei (or the radar parameters, or range and attenuation) were varied by a factor of ten, the location of the first radar echo would change only by about 1C.

For the 2 m/sec updraft, fig. 2 illustrates the method by which the behavior of the first radar echo is determined. Trajectories are computed for ice crystals originating at -6, -8, -10, -14 and -23C (the last not shown). Assuming the Findeisen-Schulz distribution of ice nuclei (table 1), we may draw the location of the first radar echo at different levels with the aid of table 4. Results show that the first echo to the AN/APQ-13 occurs at -6C, when the hail at the lowest level first becomes wet; the first echo to the AN/TPS-10 also occurs at about -6C, from dry hail just before the wet stage occurs. Subsequently the echo ascends, as hail at higher elevations becomes detectable. The echo simultaneously descends, follow-

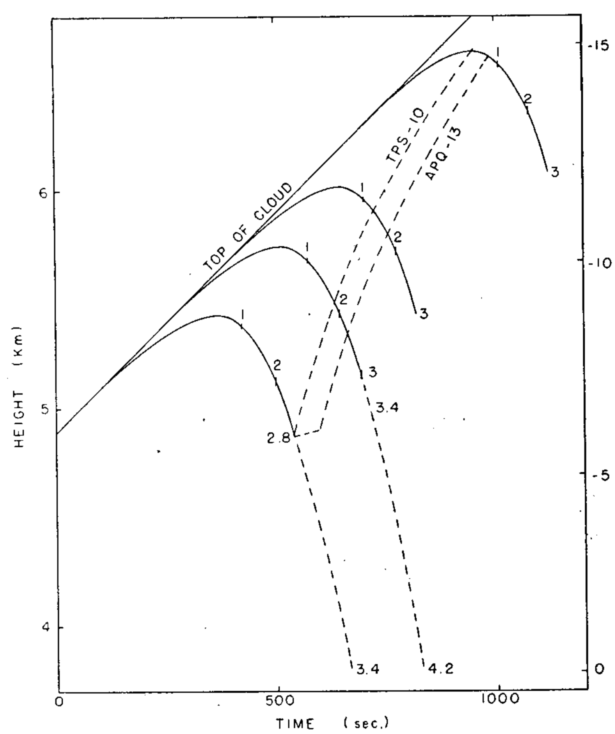

FIG. 2. Trajectories of hail originating from -6, -8, -10 and -14C in updraft of 2 m/sec. Upward motions of radar echoes to AN/APQ-13 and AN/TPS-10 are shown for distribution of ice nuclei of table 1. Descent of echo follows trajectory of hail originating at -6C.

TABLE 5. Approximate relationship between updraft velocity and location of first radar echo.

Updraft velocity (m/sec)	First radar echo (deg C)
2	0 to − 7
5	−14 to −17
8	−21 to −24

ing the trajectory of the hail which first became detectable.

If the concentration of ice particles at each level were decreased by a factor of ten, the first echo to the AN/TPQ–13 would be from the wet hail at 0C; the first echo to the AN/TPS–10 would be at −6C, when the hail first becomes wet. If the concentration of ice particles were increased by a factor of ten, the first echo to both radars would come from near −7C.

Fig. 3 shows the behavior of the first radar echoes from a cloud with a 5 m/sec updraft; the first echo to the AN/APQ–13 and AN/TPS–10 occurs almost simultaneously from about −14 to −17C. Subsequently the echo ascends and descends, the latter following the trajectory of the hail originating at −6C. For the 8 m/sec updraft, it is found that the first echo to both radars occurs almost simultaneously from about −21 to −24C. The results for the different updraft velocities are summarized in table 5.

The following conclusions concerning the behavior of the radar echo after its first appearance may be made:

1. Initially the upward velocity of the radar echo is much greater than that of the cloud top (or updraft). Subsequently its velocity decreases to that of the cloud top;
2. Initially the radar echo may be as much as 1500 m below the top of the cloud. At about the −30C level, the echo is only 300 to 500 m below the cloud top;
3. The greater the power of the radar, or the shorter the range of detection, the higher is the location in the cloud of the first radar echo;
4. For updraft velocities less than about 10 m/sec, the radar first detects hail that is descending through the cloud. The upward motion of the echo is caused by the detection at higher levels of hail which may be ascending or descending.

Some computations of hail growth were also carried out for the cloud with base at 15C. The level of the first echo is found to be about 200 m higher (with temperatures lower by about 1C) than in the cloud with base at 8C. It is thus evident that the results derived above are approximately valid for clouds with relatively large variation in liquid-water content.

For clouds in which the updraft velocity increases with altitude, the first echo appears higher in the cloud. This may easily be seen by comparing the trajectory of a −6C nucleus for a 2 m/sec updraft with that of a −8C nucleus for 5 m/sec: The smaller time interval between the initial growths of the two nuclei causes the first echo to come from the higher concentration of hailstones derived from the −8C nuclei.

6. Comparison of theory and observation

Of ten observations made by Workman and Reynolds, the initial radar echoes for seven storms were

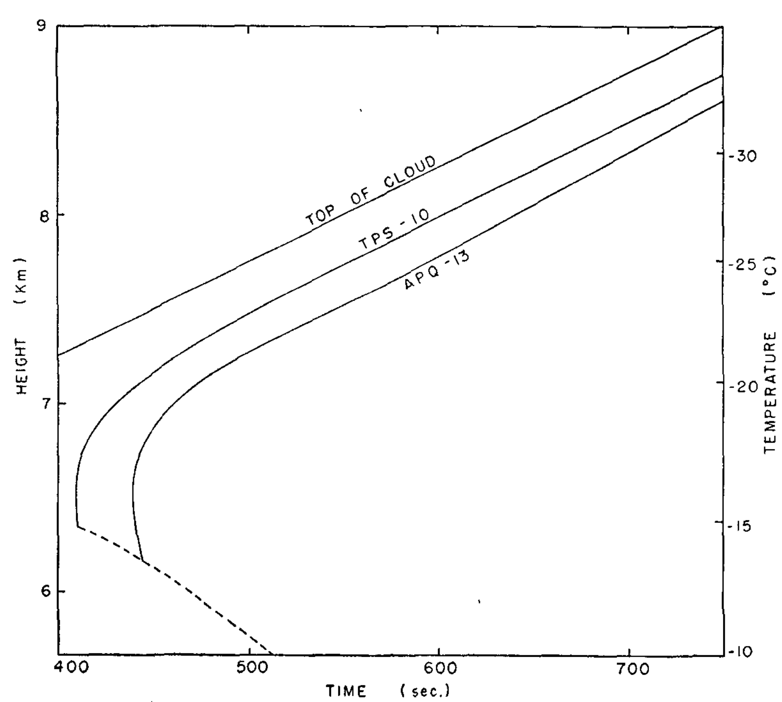

FIG. 3. Upward motions of radar echoes for 5 m/sec updraft. Dashed line represents echo descent.

located at temperatures from −8 to −11C; there were three storms with first echoes at −4 to −6C. The analysis of the previous section indicates that an updraft velocity of 2 m/sec produces initial echoes at −4 to −6C, and about 3 m/sec causes initial echoes at −8 to −11C. Visual measurements of the upward velocity of the top of the cloud, made in only four cases, show values between 2 and 3 m/sec, indicating good agreement with the theory.

About 80 per cent of the 54 observations by the Thunderstorm Project in Ohio locate the first radar echo between 0 and −7C, indicating a theoretical updraft velocity of about 2 m/sec. Measurements of the updrafts by aircraft in only six cases, at levels between 20,000 and 25,000 ft (temperatures between about −8 and −16C), give a mean velocity of 8 m/sec, which is considerably higher than the theoretical value. The discrepancy may be due to the fact that the observations were not representative of the upward motion of the top portion of the cloud, which is considered most significant in determining the location of the echo.

According to the theory, the radar echo should rise more rapidly than the cloud top. In fig. 2, the radar echo rises at an average rate of 4.7 m/sec (to the −15C level), while the top of the cloud rises at 2 m/sec. The composite cloud of Workman and Reynolds shows an average rate of ascent of the echo of 4.3 m/sec, as compared with 2 m/sec for the top of the cloud, hence showing excellent agreement with theory. However, in the six cases of the Thunderstorm Project, the mean updraft velocity within the cloud exceeded the mean rate of ascent of the radar echo by about 1 m/sec. The excess was interpreted as due to the fall velocity of the particles causing the echo.

After first detection, the radar echo should descend at a rate which is equal to the difference between the fall velocity of the hail and the updraft velocity. For the 2 m/sec updraft in fig. 2, the radar echo descends at the rate of about 8 m/sec; for the 5 m/sec updraft in fig. 3, the echo descends at about 6 m/sec. These descent rates were computed from fall velocities of the hail appropriate to clouds in New Mexico; but in regions such as Ohio, where the clouds are at lower altitudes, the hail velocities and the echo descent rates should be somewhat lower. No information concerning the descent of the echo was given by Workman and Reynolds; but in the Ohio showers studied by Battan, the echo descended at an average rate of 6.5 m/sec, in reasonable agreement with theory. In a single thunderstorm, investigated by Hilst and MacDowell (1950) in Massachusetts, the echo descended at a rate of 4 m/sec.

If the theory is valid, it would indicate that hail is responsible for the initiation of precipitation in thunderstorms. In experiments in Germany (Wichmann, 1952), in which glider planes were used to ascend into thunderstorms, hail was always encountered; but in the Thunderstorm Project, hail was reported by aircraft only on a small percentage of occasions. It is likely that soft hail would be mistaken from aircraft for snow, which was invariably reported. Hail of higher density would be encountered only after wetness occurs, and for average hail size should be most frequent at temperatures above about −10C.

In conclusion, the theory shows excellent agreement with the New Mexico observations with respect to the behavior of the first radar echo; it is compatible with the observed location of the first echo for an updraft velocity of 2 to 3 m/sec. The theory explains satisfactorily the simultaneous ascent and descent of the echo after its first appearance: the echo ascent is caused by the detection of hail at successively higher levels in the cloud; the echo descent is caused by the descent of the first detectable hail through the updraft.

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