SOME CONSIDERATIONS ON NORMAL MONTHLY TEMPERATURES

By Capt. Robert W. Lenhard, USAF, and Werner A. Baum

Florida State University [1,2]

(Original manuscript received 2 February 1954; revised manuscript received 13 April 1954)

ABSTRACT

The concept of a satisfactory normal monthly temperature is considered. January and July mean temperature records at seven stations in the United States are examined. It is found that, except in the western United States in summer, such temperature records may be accepted as constituting a random sample. It is also found that, in general, mean monthly temperatures are not normally distributed.

The method of confidence limits is applied to determination of a satisfactory normal temperature. Seasonal and geographical variations in reliability of normal temperatures are observed. Consideration is given to the adequacy with which the normal characterizes the temperature record, and to the influence of trends and cyclic fluctuations on this adequacy. It is suggested that a normal of desired reliability be computed from the most recent portion of the record to be most representative.

In general, normals computed for different periods of record will differ; but these differences may have little practical significance when the magnitude of the effect is considered relative to the requirements of the application.

1. Introduction

Because normal monthly temperatures are a standard climatological item, there is need for them to be reliable. Most standard reference sources indicate that 30 to 40 years of record should be used to establish a satisfactory normal. The reasons for choice of this length of record are not clear; one suspects that belief in the Brückner cycle is a major reason.

In 1935, the then existing International Meteorological Organization [1] recommended a period of 30 years as appropriate for establishment of normal temperature conditions, and suggested 1901–1930 as a universal period for calculation of normals. During the discussions leading to this recommendation, the dependence of the reliability of means on the variability of climate was brought out.

In 1941, the U. S. Weather Bureau [2] employed the 40-year period 1899–1938 for the preparation of maps of normal temperatures. At other times, normals have been computed over other periods: the entire record, and the period 1901–1950.

Kendrew [3] notes the varying length of record necessary to establish normals in different latitudes, as does Landsberg [4], who also notes the seasonal change in variability of mean temperature at a middle-latitude station.

In general, a satisfactory normal temperature is incompletely specified but is considered to be one that is stable and representative of the record. At times, statistical measures of reliability have been used to establish such a satisfactory normal. Hann's [5] formula and computations have been most widely repeated and quoted. In this formula, the probable error of the mean is expressed in terms of the number of items n and the mean of the deviations (departures) of these items from their average value, disregarding algebraic sign:

$$\text{Probable error} = 1.1955\ (2n-1)^{-\frac{1}{2}} \times \text{mean departure}.$$

This formula is credited by Hann to Fechner, without restriction as to its applicability and without specific reference. Study of some of Fechner's [6] work indicates that the formula was developed to show the probable error of a series of physiological and psychological measurements. No success was encountered in attempts to convert this formula into that for standard error currently accepted in statistical theory.

The simplest modern statistical procedure in dealing with random distributions is to use the standard deviation of the mean to specify confidence limits. This procedure is suggested by Grisollet [7] and by Poncelet and Martin [8], and will be examined here in some detail. The primary objective of the study is to provide information and criteria for use in selection of a procedure for computing a normal that will be appropriate to the application that is to be made of it.

The assumption is made that instrumental errors do not bias the data.

2. Definitions and data

Terminology that will differentiate between mean temperatures is needed. Daily mean temperatures are averaged to form a mean temperature for an individual month, designated the monthly mean temperature.

[1] This work has been supported in part by the Office of Naval Research, under Contract Nonr-988(01), Project NR-082-071.

[2] Captain Lenhard participated while assigned to Florida State University by the U. S. Air Force Institute of Technology as a student in Advanced Meteorology (Climatology).

The latter are averaged over a period of record to form another mean, usually spoken of as the normal temperature for the month. This designation has some misleading connotations, as is amply demonstrated by Landsberg [9]; but a clear differentiation of the two means is necessary when both are being discussed, and the term "long-period average" is unwieldy. When symbols are used, T will designate monthly mean temperature and $\bar{T}$ normal monthly temperature. When the discussion is solely of statistical technique, the word "mean" will be used in a general sense.

For working data, January and July have been selected, to provide the maximum seasonal difference. To provide a variety of physical conditions, the following stations have been selected: Portland, Oregon; San Diego, California; Salt Lake City, Utah; Bismark, North Dakota; Cairo, Illinois; Blue Hill, Massachusetts; and Jacksonville, Florida. This selection provides a representation of some of the major features of normal temperature conditions in the United States, and of most of the features of temperature variability as revealed by Sumner's [10] maps of the standard deviations.

As an example, fig. 1 presents a summary of the record at Jacksonville. Running curves for estimated normal January and July temperatures, and for corresponding standard deviations of the normals, are shown. The abscissa gives the number of items that entered into the computation, and the effect on the normal and its standard deviation of increasing sample size may be observed directly. The samples were accumulated chronologically, moving backward in time, beginning with the year 1952. Results for less than 10 years of record are not shown, due to the extreme fluctuations that occur.

3. Randomness of sample

Interpretation of the confidence limits of a sample mean rests on the theoretical distribution of sample means, which, in turn, assumes random selection of samples. It would seem that a climatic record cannot be a random sample, since it is sequential. Because of this time sequence, specification of the initial item of the record delimits the entire sample.

An essential feature of a random sample is the mutual independence of the values of the component items. When applied to a sequential sample, this means that the value of one item does not determine the value of the next item. A fundamental precept of weather forecasting is just the opposite: this moment's weather determines subsequent weather. Qualitatively it is clear that the time scale is an important consideration with respect to randomness of samples; certainly one would expect a different order of serial dependence in a sequence of 1-min mean temperatures than in a sequence of annual mean temperatures. In meteorology, in general, a sequential sample should not be regarded as a random sample without further investigation.

That independence of the values in a climatological series does not always exist is shown by Reidat [11], who studied the monthly mean temperatures of Berlin, Germany. After computing deviations from the long-term means for each month, he examined the duration of periods of consecutive months with deviations of like sign. In a record of 80 years, only 95 positive and 98 negative anomalies had a duration limited to a single month. Three-hundred and ninety positive deviations formed 112 sequences, and 372 negative deviations formed 110 sequences, of anomalies of the same sign which extended over two or more months. The number of cases dropped off rapidly with increasing length of sequence; ⅔ of the sequences had a duration of but 2 months, and only two sequences showed a duration of an entire year. On the basis of such results, it is unlikely that the mean temperature of one January will be significantly interrelated with the mean of the subsequent January, but the possibility exists. It is therefore advisable to test the independence of items in a sequential temperature sample, if it is to be accepted as a random sample.

A test for randomness of sequence, equivalent to a test of the serial correlation coefficient with lag 1, is given by Wald and Wolfowitz [12]. If $X_1, X_2, \cdots, X_n$ is the sequence to be tested, the statistic

$$R = \sum_{i=1}^{n-1} X_i X_{i+1} + X_n X_1$$

is considered for all possible $n!$ permutations of the sequence. If all possible permutations of the sequence are treated as equally likely to occur, the statistic R is approximately normally distributed for large values of n. It may therefore be used to test the hypothesis

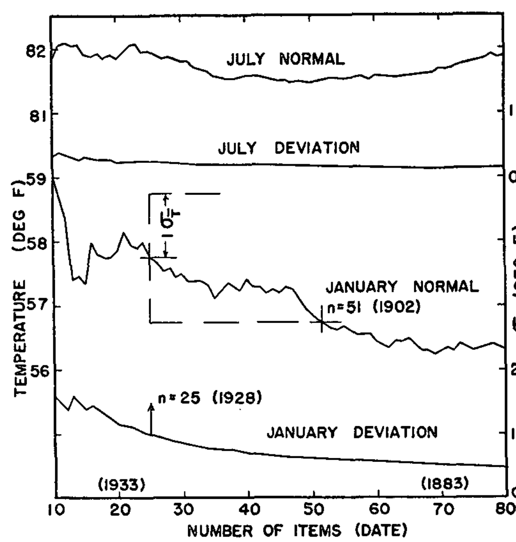

FIG. 1. Normal temperature and its standard deviation, January and July, Jacksonville. Note that samples are accumulated backward in time.

TABLE 1. Probability of obtaining value of $|M_R - R|$, as large or larger than computed, from population with no serial correlation.

Station	20	30	40	50	60	70	80
				Years of record			
			January				
Portland	0.43	0.48	0.57	0.77	0.77	0.94	0.80
San Diego	0.52	0.65	0.47	0.81	0.84	0.62	0.42
Salt Lake City	0.24	0.45	0.15	0.20	0.22	0.48	0.52*
Bismark	0.34	0.45	0.75	0.96	0.78	0.56	0.76*
Cairo	0.85	0.68	0.49	0.33	0.12	0.80	0.42
Blue Hill	0.28	0.26	0.24	0.78	0.96	0.72**	—
Jacksonville	0.94	0.87	0.81	0.75	0.81	0.89	0.85
			July				
Portland	0.03	0.15	0.12	0.10	0.02	0.02	<0.01
San Diego	0.82	0.87	0.85	0.28	0.05	0.06	0.05
Salt Lake City	0.77	0.90	0.10	<0.01	<0.01	<0.01	<0.01*
Bismark	0.83	0.74	0.94	0.80	0.72	0.71	0.67*
Cairo	0.71	0.95	0.37	0.52	0.29	0.12	0.31
Blue Hill	0.94	0.93	0.71	0.83	0.92	0.77**	—
Jacksonville	0.98	0.91	0.67	0.69	0.64	0.55	0.51

* 78 years of record.
** 67 years of record.

of zero serial correlation. The only quantities necessary are the mean and variance of R, given by

$$M_R = \frac{S_1^2 - S_2}{n - 1},$$

and

$$\sigma_R^2 = \frac{S_2^2 - S_4}{n-1} + \frac{S_1^4 - 4S_1^2S_2 + 4S_1S_3 + S_2^2 - 2S_4}{(n-1)(n-2)} - M_R^2,$$

where

$$S_k = \sum_{i=1}^{n} X_i^k.$$

The analysis of serial correlation was performed at points 10 years apart. Each point at which the analysis is made, then, is a sample that is larger by ten items than the preceding sample. Table 1 shows the probability of obtaining a value of $|M_R - R|$, as large or larger than that computed, from a sample drawn from a population of zero serial correlation.

In January, no probability is less than 0.12 and most are considerably greater. Within the climatic regions represented, it appears valid to consider records of January mean temperature as acceptable random samples.

In July, the probabilities are generally smaller. Nevertheless, in the eastern and midwestern United States, it appears valid to treat records of July mean temperature as random samples. In the west, however, particularly in the regions represented by Salt Lake City and Portland, one would be inclined to reject the hypothesis that the records of July mean temperature are random samples. This statistical result poses a meteorological problem that warrants further investigation. It should be pointed out that selected periods from the records at Salt Lake City and Portland do test out as random samples. At Salt Lake City, for example, the period from 1952 to 1933 may be accepted as such.

4. Applicability of Gaussian law

In many problems, the normality of the frequency distribution of monthly mean temperatures is an important consideration. Croxton and Cowden [13] present several tests for the normality of a frequency distribution. The most useful and practicable one for the present problem is Fisher's test, wherein two statistics are computed: g_1, a measure of skewness, and g_2, a measure of kurtosis. For samples drawn from normal populations, these statistics are normally distributed with mean values of zero and variances depending solely on the size of the sample. They may therefore be used to test the hypotheses of zero skewness and normal kurtosis.

As in the test for randomness, the analysis was performed at points 10 years apart. Table 2 shows the probability of obtaining a value of g_1, as large or larger than that computed, from a population of zero skewness. It is quite clear that the hypothesis of zero skewness in the population cannot be accepted generally.

Sufficient evidence is already provided that January and July mean temperatures at some of the selected stations may not be treated as forming a normal distribution. However, for the sake of completeness, table 3 shows the probability of obtaining a value of g_2, as large or larger than that computed, from a population with kurtosis equal to that of a Gaussian frequency distribution.

The general lack of normality in the frequency distributions of T precludes the possibility of using normal probability tables for determination of the likelihood of a specific T from a knowledge of $\bar{T}$ and σ_T, the standard deviation of the monthly mean temperatures. However, even though the original population is not normal, $\bar{T}$ will be normally distributed with standard deviation $\sigma_{\bar{T}} = \sigma_T/n^{\frac{1}{2}}$, if the population is random and homogeneous.

5. Homogeneity of data

The importance of computing a mean from a portion of the record that is homogeneous is emphasized by Conrad and Pollak [14]. Homogeneity of weather records is questionable on the grounds that the locations of instruments are known to have been changed, as have the methods of computing the mean temperature, and that, in many cases, the local environment is known to have been changed radically over a period of time as a result of man's activities.

Dixon and Massey [15] present a number of tests for homogeneity. Application of these tests is compli-

TABLE 2. Probability of obtaining value of g_1, as large or larger than computed, from population with no skewness.

Station	Years of record 20	30	40	50	60	70	80
			January				
Portland	0.08	0.03	0.03	0.01	<0.01	<0.01	<0.01
San Diego	0.01	<0.01	0.01	<0.01	<0.01	<0.01	0.01
Salt Lake City	0.01	0.03	0.04	0.01	<0.01	0.02	0.02*
Bismark	0.25	0.21	0.25	0.47	0.40	0.90	0.74*
Cairo	0.35	0.60	0.01	0.01	0.01	0.04	0.27
Blue Hill	0.67	0.55	0.87	0.94	0.75	0.92**	—
Jacksonville	0.90	0.54	0.52	0.35	0.17	0.10	0.08
			July				
Portland	0.03	0.13	0.54	0.94	0.70	0.70	0.94
San Diego	0.20	0.28	0.27	0.12	0.07	0.14	0.25
Salt Lake City	<0.01	<0.01	<0.01	<0.01	0.12	0.24	0.39*
Bismark	0.13	0.64	0.62	0.02	<0.01	<0.01	<0.01*
Cairo	0.38	0.31	0.60	0.84	0.92	0.92	0.94
Blue Hill	0.02	0.01	0.40	0.23	0.45	0.66**	—
Jacksonville	0.04	0.20	0.60	0.70	0.40	0.14	0.02

* 78 years of record.
** 67 years of record.

TABLE 3. Probability of obtaining value of g_2, as large or larger than computed, from population with normal kurtosis.

Station	Years of record 20	30	40	50	60	70	80
			January				
Portland	0.97	0.89	0.84	0.83	0.56	0.62	0.45
San Diego	0.11	<0.01	0.41	0.15	0.29	0.47	0.92
Salt Lake City	0.16	0.42	0.86	0.31	0.29	0.67	0.44*
Bismark	0.65	0.23	0.29	0.24	0.30	0.11	0.05*
Cairo	0.98	0.51	0.05	0.09	0.11	0.83	0.69
Blue Hill	0.31	0.34	0.43	0.61	0.89	0.79**	—
Jacksonville	0.32	0.22	0.34	0.48	0.39	0.42	0.34
			July				
Portland	0.31	0.52	0.17	0.12	0.48	0.76	0.83
San Diego	0.69	0.09	0.10	0.08	0.12	0.05	0.03
Salt Lake City	0.76	0.79	0.77	0.34	0.28	0.27	0.29*
Bismark	0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01*
Cairo	0.54	0.84	0.13	0.28	0.27	0.38	0.32
Blue Hill	0.12	0.01	0.01	0.01	0.01	0.05**	—
Jacksonville	0.01	0.63	0.75	0.62	0.84	0.90	0.74

* 78 years of record.
** 67 years of record.

cated by the restrictions on the conditions under which they may be used. For example, the well-known analysis-of-variance test requires that the observations are from normally distributed populations and that the variance of each group is the same.

Insofar as the limitations on the tests made it possible, the temperature records were broken down into samples by locations and samples by decades, and tested by one or more methods. With very few exceptions, the records were found to be statistically acceptable as homogeneous.

An interesting anomaly was provided by Jacksonville. The entire January record was found to be statistically acceptable as homogeneous. But when the July record was tested for homogeneity by locations, it proved acceptable only when the period 1903–1932 was dropped from the record; the years which remain are all those in which locations of the instruments were less than 100 ft above the surface.

6. Confidence limits

The above considerations lead to the conclusion that, in general but not always, satisfactory temperature normals can be obtained by the use of confidence limits of reliability.

To illustrate, a sample of the temperature record consisting of the last 25 years at Jacksonville in January gives $\bar{T} = 57.8$F with $\sigma_{\bar{T}} = 1.0$F, or a normal temperature of 57.8 ± 1.0F with a confidence of 0.68. That is to say that if 100 randomly selected samples of 25 years of data each were taken, on the average 68 of them would include the true mean in the range $\bar{T} \pm \sigma_{\bar{T}}$. Obviously this does not specify which of the 100 samples do include the true mean; a single sample is more likely to be one of 68 than one of 32, but it can belong to either group and there is no way of telling to which group it belongs.

The limitation of single sampling is discussed by Shewhart [16], and may be illustrated with the January record at Jacksonville. When the sample is accumulated backward in time from 1952, an accuracy of 1F with confidence of 0.68 is reached with 1928, at $n = 25$; the normal at that point is 57.8F. The minimum expected value, at the 68 per cent confidence level, for the normal temperature would be 56.8F. If the population is the entire record, the normal temperature is found to be 56.2F; the normal, computed as the sample increased in size, dropped below 56.8F at $n = 52$. Another sample may be obtained by accumulating backward in time from 1941. The two samples are shown in fig. 2. In the latter sample, an accuracy of 1F with confidence of 0.68 is reached with 1922, at $n = 20$, when the normal temperature is

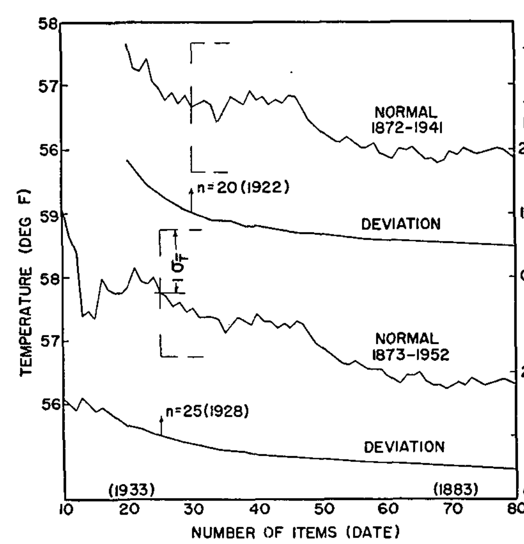

FIG. 2. Comparison of samples of January normal temperature and its standard deviation at Jacksonville. Note that samples are accumulated backward in time.

56.7F. If the sample is extended back to the beginning of the record, the normal changes but remains always within the range 56.7 ± 1.0F. Thus, if the population is the entire record, the sample 1922–1941 would be satisfactory whereas the 1928–1952 sample would not be. That this is not an unusual occurrence is demonstrable in two ways. First, the example was not specifically selected, but was obtained at the first attempt to find an illustration; 1941 was the starting year resulting from choosing a sample whose possible final size was seventy. Secondly, the mean of the 1928–1952 sample is but 1.4 $\sigma_{\bar{T}}$ away from the mean of the entire record; deviations as large or larger could be expected in 17 per cent of random samples of 25 from this population. Finally, it is clear that the choice of one standard deviation, corresponding to a confidence of 0.68, is an arbitrary base for discussion.

7. Comparison of normal temperatures

Table 4 presents for each station the normal January and July temperatures, with their standard deviations, for four periods: 1901–1930, the period recommended by the International Meteorological Organization; 1901–1952, a period nearly the same as the one that has found some recent use by the U. S. Weather Bureau; the entire period of record; and the minimum record required to obtain a normal of specified reliability. This reliability is 1F with confidence of 0.68 for January normals. Where this reliability was achieved with less than ten items in the sample, the normal and its standard deviation for a sample of ten years were used; slightly larger samples may actually be necessary for statistical techniques to be applicable. In the computation of July normals, a reliability of 0.45F with a confidence of 0.68 was used, so that as few stations as possible would have a record of but ten items.

To a limited extent, table 4 may be used to examine the seasonal and geographic variation of reliability of normal January and July temperatures in the United States. In general, July standard deviations of normal temperatures are about one-half as large as January values. The reliability in summer, then, will be greater than in winter. This increased reliability in summer could appear either as a narrower tolerance range or as greater confidence in the same range as for winter. The table indicates, as would be expected, that there is a northward decrease of reliability in middle latitudes, and that maritime control of climate leads to greater reliability than does continental control of climate.

Included in table 4, for each period of record, are the January and July normals averaged over the seven stations. The range of these averages is 0.5F in January and 1.2F in July. To evaluate the significance of the observed variations, an analysis of variance was applied to the table. At the 5 per cent level of significance, the test ratio has the value $F = 3.16$. The ratios obtained are: January, $F = 1.2$; July, $F = 22.5$. Since the seven stations were not selected at random, the test may not be exactly applicable. But since the stations were selected to provide a representative sample, the test results should at least be indicative. It seems, statistically, that it makes no difference which of the four periods is used to compute the normal temperature in January; the difference may be considered considerable in July.

If the magnitude of differences at individual stations is compared, it would seem that at some places the period over which the normal is computed would have an appreciable effect on the value obtained. In January the normals differ by as much as 1.9F at Jacksonville, in July by 1.6F in San Diego. Knoch [17] has compared the period 1901–1930 with longer records

TABLE 4. Normal temperatures and their standard deviations.

Station	1901–1930 $\bar{T}$	1901–1930 $\sigma_{\bar{T}}$	1901–1952 $\bar{T}$	1901–1952 $\sigma_{\bar{T}}$	Entire record $\bar{T}$	Entire record $\sigma_{\bar{T}}$	Minimum record $\bar{T}$	Minimum record $\sigma_{\bar{T}}$	Minimum record n
January									
Portland	39.3	0.79	39.5	0.63	39.3	0.47	39.1	1.00	26
San Diego	55.0	0.38	55.1	0.32	54.8	0.26	54.4	0.87	10
Salt Lake City	30.1	0.97	29.6	0.62	29.1	0.52	28.8	0.98	28
Bismark	9.0	1.40	9.4	1.18	8.2	0.97	8.3	1.00	73
Cairo	36.1	0.86	37.1	0.68	36.5	0.60	37.7	1.00	18
Blue Hill	25.4	0.85	25.9	0.67	25.7	0.58	27.1	1.00	25
Jacksonville	55.9	0.67	56.7	0.62	56.3	0.48	57.8	1.00	25
Average	35.8		36.2		35.7		36.2		
July									
Portland	67.6	0.38	68.1	0.27	67.6	0.22	69.0	0.44	14
San Diego	67.3	0.26	67.9	0.24	67.7	0.18	68.9	0.38	10
Salt Lake City	76.5	0.39	77.3	0.30	76.7	0.25	77.6	0.38	10
Bismark	70.0	0.57	71.3	0.53	70.7	0.40	71.0	0.45	65
Cairo	79.5	0.38	80.0	0.28	79.6	0.23	80.5	0.43	18
Blue Hill	68.6	0.34	69.1	0.27	68.8	0.24	69.8	0.44	19
Jacksonville	81.1	0.19	81.5	0.17	81.9	0.15	81.8	0.33	10
Average	72.9		73.6		73.3		74.1		

for Europe and found a corresponding difference in winter. Further, he found that the winter differences show the period 1901–1930 to be warmer at nearly every station examined. Were the present United States sample to be enlarged, the same phenomenon might be observed, in which case the analysis of variance should indicate a significant difference between normals based on different periods for January as well as for July.

Some conclusions can be reached. It should be noted that the practical significance of each of these will depend upon the precision demanded of the normal by the application to be made of it. It may be concluded that the period chosen can, and probably will, affect the value of the normal when an individual station is considered. It may also be concluded that, when a group of stations is involved, the normal will be dependent on the period of record in July and may be dependent upon it in January. In general, then, the value of the normal is not independent of the period of record; but the connection may have little practical significance when the magnitude of the effect is considered relative to the requirements of the application.

8. Representativeness of the normal temperature

The extent to which the normal temperature represents the entire record of mean monthly temperatures is dependent on the variation of the mean temperature of a month from year to year. There are at least two classes of time variations that may limit the representativeness of the normal temperature, namely trends and cyclical fluctuations. To examine these variations, the monthly mean temperature records were plotted as exemplified by Jacksonville in fig. 3. The solid lines connect normal temperatures for decades, thus providing a smoothed trace of the time variation of mean monthly temperatures. The dashed lines connect the normal temperatures of the half-records and show the trend of mean monthly temperatures. Other, more accurate methods of measuring

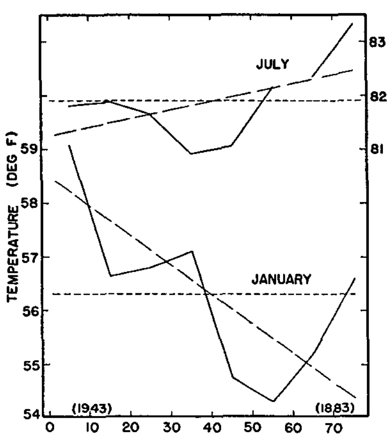

FIG. 3. Decade normal temperatures, January and July, Jacksonville. Note that time progresses from right to left along abscissa.

trend are available; this method of semi-averages is of sufficient accuracy for illustrative purposes. The dotted lines are drawn at the levels of the normal temperatures as computed from the entire records. Quasi-cyclical fluctuations of the decade normals may be observed either about this long-term normal line or about the trend line.

Trend.—The effect of trend on the representativeness of the normal can be illustrated with the January record at Jacksonville. To judge from the changing normal shown in fig. 1, the normal temperature of 57.8 ± 1.0F, established at $n = 25$, adequately characterizes the record for the period 1902–1952, but is not adequate for describing the entire record. Beyond 1902, the long-term normal continues to fall as it has throughout the record. This tendency to fall is consistent with the trend shown in fig. 3. In general, a normal determined from a sample at one end of the record will not be representative of the entire record when a trend is present.

To be considered satisfactory, a normal temperature must be not only reliable but representative. If trend exists, some compensation for that trend must be made in determining the normal. The proper method of determination will depend on the use to be made of the mean. If an analysis of the past record is the objective, the entire record should be used as fully as the analysis requires. The normal then obtained will be the true one for the population under consideration, but will be inadequate without specification of the trend if the model resulting from the analysis is to symbolize the record.

If extrapolation of the record is desired, inclusion of those portions of the record remote from the point of extrapolation will tend to bias the normal. If the attempt were to be made to extrapolate Jacksonville January temperatures, it being assumed for the moment that the trend will continue undisturbed, the normal at $n = 25$ would be more useful than that obtained from the entire record.

Projection of a trend is a dangerous process, unless the physical causes of the trend can be determined and the continuance of their effects can be assured. Consider two alternatives for the Jacksonville example: (1) the warming may be due to the growth of the city and increasing industrialization; (2) the warming may be due to atmospheric causes. If the warming is due to the first alternative, its future behavior is as unpredictable as are the growth rate of the city and the effects of industrial activity. If the warming is due to atmospheric causes, it might be expected to continue; or the apparent trend may just be a portion of a cycle having a long period. Which eventuality is more likely is largely a matter of speculation.

Most likely, both causes would be operating. Whatever the facts, extrapolation of the record is of dubious validity. Yet such extrapolation is implicit in almost all activities. Whenever a normal temperature is used as a design temperature, the assumption is made that the normal temperature used will be characteristic of times to come. Probably the best forecast that can be made is to use the normal from the part of the record nearest to the point of extrapolation, and for this normal to be established over the shortest period that will give a useful reliability. It was with this end in mind that samples were accumulated backward in time in this study.

Cyclical fluctuations.—With respect to the decade normals, if judgment as to representative ability of the Jacksonville January normal of 57.8 ± 1.0F established at $n = 25$ is based on fig. 3, it is adequate only for the decade 1913–1922. The range fails to include either of the first two decade normals (proceeding backward in time), barely includes the third decade normal, and includes the fourth but no more decades. These additional normals would be eliminated by the trend. The quasi-cyclical fluctuations bring the most remote decade normal, that for 1873–1882, almost up to inclusion within the reliability range of the normal temperature for $n = 25$. Whether or not a reliable normal temperature will be representative when quasi-cyclical fluctuations are present depends on the magnitude of the fluctuations and the portion of the cycle considered in computing the normal.

In any attempt to compensate for cyclical fluctuations, they should be presented directly if the goal is analysis of the past record. In the event that such cycles are to be analyzed further and are desired as deviations from the normal temperature, care must be taken that the normal is computed over a number of years equal to a full cycle or multiple thereof. If the purpose is extrapolation, the cyclical variations rather than the normal should be extrapolated if they can be established as physically real and if there is an adequate length of record. Such a length of record will rarely be encountered.

Generally the evidence in support of cycles is questionable, so there will frequently be hesitancy in extrapolating them. The mean is then the best prognostic parameter. Ideally it should be computed over a period such that equal positive and negative deviations of the cyclical variations from the trend line are included. To meet the ideal condition, it is necessary to examine a longer record than that necessary to establish a reliable normal. Inaccuracies arising from a normal obtained from a period covering an unbalanced portion of a cycle will, of course, depend on the amplitude of the cycle.

9. Conclusions

Satisfactory temperature normals can be obtained by the use of confidence limits of reliability in many instances. The assumption of homogeneity of the temperature record must be validated, especially when the record involves a change of location.

The conclusions that monthly mean temperatures do not form normal frequency distributions and that the sequential record constitutes an acceptably random sample would seem to be suitable approximations. Since these conclusions are based on a "most usual" basis, they are not valid in every case.

The presence of secular and quasi-cyclical variations of temperature weakens the representativeness of the normal. This weakening can be minimized by computation of the normal over the shortest possible period adjacent to the point of extrapolation.

There exist wide ranges of reliability and representativeness of normal temperatures obtained from synchronous records. An alternative to synchronous records for the comparison of normals between stations is the use of normals of equal reliability obtained from records with synchronous origins in time.

REFERENCES

1. International Meteorological Organization, Climatology Commission, 1937: *Proceedings of the meetings in Danzig and Warsaw, 29–31 August and 12 September 1935.* Leyden, Secretariat of the I.M.O.
2. U. S. Weather Bureau, 1941: *Climate and man.* (U. S. Dept. Agriculture Yearbook). Washington, U. S. Govt. Printing Office, 1248 pp.
3. Kendrew, W. G., 1949: *Climatology.* Oxford, Clarendon Press, 383 pp.
4. Landsberg, H. E., 1950: *Physical climatology.* State College, Pennsylvania State Univ. Press, 277 pp.
5. Hann, J., 1908–1911: *Handbuch der Klimatologie.* Stuttgart, J. Engelham.
6. Fechner, G. T., 1907: *Elemente der Psychophysik.* Leipzig, Breitkopf.
7. Grisollet, H., 1948: Examen de la distribution des variables climatiques autour de leurs moyennes et de la valeur representative des "normales." *Ann. Inst. Hydrol. Climatol.*, 19, 38–56.
8. Poncelet, L., and H. Martin, 1947: Esquisse Climatographique de la Belgique. *Mem. Inst. roy. Meteor. Belgique,* No. 27, 265 pp.
9. Landsberg, H., 1947: Critique of certain climatological procedures. *Bull. Amer. meteor. Soc.*, **28**, 187–191.
10. Sumner, A. R., 1953: Standard deviations of mean monthly temperatures in Anglo-America. *Geogr. Rev.*, **43**, 50–59.
11. Reidat, R., 1951: Über die Abweichungen vom Monatsmittel in der 80-jährigen Berliner Temperaturreihe 1851–1930. *Z. Meteor.*, **5**, 224–229.
12. Wald, A., and J. Wolfowitz, 1943: An exact test for randomness in the non-parametric case based on serial correlation. *Ann. math. Statistics*, **14**, 378–388.
13. Croxton, F. E., and P. J. Cowden, 1939: *Applied general statistics.* New York, Prentice-Hall, Inc., 944 pp.
14. Conrad, V., and L. W. Pollak, 1950: *Methods in climatology.* Cambridge, Harvard Univ. Press, 459 pp.
15. Dixon, W. J., and F. J. Massey, 1951: *Introduction to statistical analysis.* New York, McGraw-Hill, 370 pp.
16. Shewhart, W. A., 1939: *Statistical method from the viewpoint of quality control.* Washington, Dept. of Agriculture Graduate School, 155 pp.
17. Knoch, K., 1947: Die "Normalperiode" 1901–1930 und ihre Verhältnisse zu längeren Perioden. *Meteor. Rundschau*, **1**, 10–23.