CORRESPONDENCE

On the computation of horizontal divergence

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In a recent article[2] in the JOURNAL, Dr. Cressman presented a convenient equation for the computation of horizontal divergence:

$$(V' - V)\cdot\nabla_h \ln \eta = \nabla_h \cdot V, \tag{1}$$

where V is the horizontal velocity, η the vertical component of the absolute vorticity, and primed quantities are to be evaluated at the surface of non-divergence. I would like to comment upon two of the assumptions made in deriving (1).

The first of these is easily missed by the reader in Dr. Cressman's derivation, which starts from the following equation (taken from an earlier article[3]):

$$(V - C)\cdot\nabla_h \ln \eta \approx \nabla_h \cdot V. \tag{2}$$

Here C is the velocity of propagation along an arbitrary direction of the η lines; the solenoidal, frictional, and vertical-velocity[4] terms have been neglected; and the local time derivative of $\ln \eta$ has been replaced by $-C\cdot\nabla_h \ln \eta$. To make clear the assumption here being discussed, we repeat his derivation, but without his substitution for $\partial(\ln \eta)/\partial t$.

We have, with his assumptions and notation,

$$\partial(\ln \eta)/\partial t + V\cdot\nabla_h \ln \eta = -\nabla_h \cdot V. \tag{3}$$

And for the surface of non-divergence,

$$\partial(\ln \eta')/\partial t + V'\cdot\nabla_h \ln \eta' = 0. \tag{4}$$

We here accept his empirical result that, to a good approximation,

$$A\ \nabla_h \eta' = \nabla_h \eta,\ A \text{ a scalar}, \tag{5}$$

i.e., that the absolute-vorticity *patterns* are of the same shape at all levels. If we multiply (4) by A, and subtract the resulting equation from (3), we obtain

$$\nabla_h \cdot V = (V' - V)\cdot\nabla_h \ln \eta - [\partial(\ln \eta)/\partial t - A\ \partial(\ln \eta')/\partial t]. \tag{6}$$

Assumption (5) implies that the velocity of propagation is the same at all levels for centers, troughs, *etc.*, of the field. This independence of elevation of the velocity of propagation is not to be assumed for individual isolines of η, unless the patterns are moving without change. Dr. Cressman makes this assumption and so fails to get the terms of (6) which are enclosed in brackets. These terms represent the effect of differential intensification of the vorticity field at the surface of non-divergence and at the level for which $\nabla_h \cdot V$ is being computed. Such differential intensifications are most surely present in developing situations — precisely those which present the most difficult forecast problem. Hence, this assumption implicit in (1) should be borne in mind when one considers Dr. Cressman's suggestion that (1) can be used to simplify the "2½ dimensional" numerical prediction model.

A second assumption involved in (1) is the neglect of the vertical-velocity term of the vorticity equation. Dr. Cressman's use of this assumption is shared by most workers in the numerical prediction field. I cannot desist from re-registering a doubt as to its validity, particularly for the surface of non-divergence.[5]

[1] On leave of absence from Florida State University, to which the writer has returned.

[2] G. P. Cressman, "An approximate method of divergence measurement," *J. Meteor.*, 11, 83–90, 1954.

[3] G. P. Cressman, "An application of absolute-vorticity charts," *J. Meteor.*, 10, 17–24, 1953.

[4] In Dr. Cressman's terms: the vertical transport of vorticity and "rotation of the vorticity" from a horizontal to a vertical axis.

[5] See, among others, R. J. Reed and F. Sanders, "An investigation of the development of a mid-tropospheric frontal zone and its associated vorticity field," *J. Meteor.*, 10, 338–349, 1953, or L. Sherman, "Estimates of the vertical velocity based on the vorticity equation," *J. Meteor.*, 10, 399–400, 1953.

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