RESOLUTION IN HEIGHT OF A RADAR PULSE

By V. G. Miles

Meteorological Office, Southern Rhodesia

(Original manuscript received 4 May 1954; revised manuscript received 4 October 1954)

ABSTRACT

The resolution in height of a radar pulse is of interest in connection with the observation and interpretation of horizontally stratified echoes, and also in the comparison of values of echo intensity measured at different ranges and elevation angles.

An analysis is made of the ability of a radar pulse to resolve, either completely or partially, the echoes from two closely-spaced horizontal reflecting layers situated close to the radar. The general expressions obtained are illustrated graphically.

1. Introduction

In the study of physical processes in the atmosphere by means of narrow-beam pulsed radar, observations are often made of reflecting layers which are very nearly horizontal. For the making and analyzing of such observations, it is worthwhile to know how the resolution in height of the radar pulse varies with the radar parameters and with the elevation angle of the beam. Also for the study of echo-intensity measurements made at different ranges and elevation angles, the height resolution is of interest, since it effectively specifies the vertical thickness of the space contributing to the echo.

The present analysis was prompted by the observation of an apparent splitting of the well-known "bright band" precipitation echo. It was found that, when the radar beam was directed vertically, the bright band showed on the range-amplitude display (A-scope) as the usual single fluctuating maximum of echo deflection. As the elevation angle of the beam was reduced, however, the band echo split up into two maxima appearing side by side on the A-scope, and the spacing in range between the two maxima increased in a regular way with decreasing elevation of the beam. At a low elevation, both maxima disappeared, leaving only the normal rain echo. When the beam was again raised, the two maxima reappeared, moved closer together and shortly before 90-deg elevation merged once more into a single maximum of echo deflection. The splitting of the band echo was also to be seen on the PPI display which has an independent range-sweep circuit, so it seemed unlikely that the splitting was due to an irregularity in the range sweep; but the splitting was apparent on the PPI only when the receiver gain had been reduced sufficiently to prevent the echo from saturating the PPI screen. This apparent splitting or doubling of the bright band echo has been observed on only two occasions, and no photographs or readings have yet been obtained.

From the information available, we are unable to find any difference between the meteorological conditions of the splitting bright band and those of the normal bright band. These conditions usually are northwesterly upper winds and thick altostratus cloud, from which light to moderate rain falls. Fronts are rarely observed in this region, at 18°S.

It was felt that the double-band echo might be due to the existence in the cloud of two closely spaced, nearly horizontal layers of high reflectivity, and that the behavior of the echo at different elevations of the radar beam could be a result of a variation, with elevation angle, in the ability of the radar pulse to resolve the echoes from the two layers.

2. Method of analysis

It is shown in textbooks on the subject [1] that the intensity of echo received at any instant is that which is returned by a certain volume, conveniently called the "echo volume." This volume is given by the following expression:

$$\tfrac{1}{2}\tau c\phi\theta R^2. \tag{1}$$

Here τ is the pulse duration, c the speed of travel of electromagnetic waves, ϕ and θ are angles defining the effective width of the beam in two perpendicular planes such that, when the beam axis is horizontal, ϕ is the beamwidth in the horizontal plane and θ the beamwidth in the vertical plane. The echo volume is thus a "solid" of length $\tfrac{1}{2}\tau c$, and of sides $R\phi$ and $R\theta$.

To derive a quantitative expression for the resolution in height of the echo volume, it is convenient to consider the case of two horizontal reflecting layers separated by a gap wherein the reflectivity is zero. For simplicity, it is assumed that the layers are of equal reflectivity, and that at the upper and lower boundaries of the gap the reflectivity rises abruptly to the full value. Then, to determine the form of echoes produced at a given elevation angle of the beam,

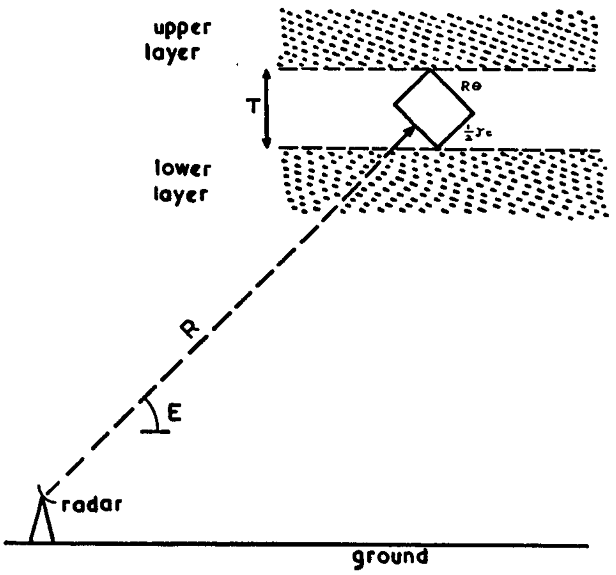

FIG. 1. Conditions of complete resolution of echoes from two reflecting layers.

we imagine the echo volume to pass through the layers along the direction of the beam axis. First we may note that, provided the cloud is not too far from the radar, the reflecting layers are of larger horizontal extent than the echo volume; hence, the analysis requires consideration only of the vertical section through the echo volume, and therefore involves only the beamwidth θ.

Fig. 1 illustrates the situation assumed for the analysis. With the beam at an elevation angle E, the vertical section of the echo volume is shown at the instant when the volume is situated exactly in the gap between the layers; at this instant, the volume is distance R from the radar. The vertical separation T between the layers is shown equal to the interval of height occupied by the echo volume. This is the condition of complete resolution of the echoes returned by the layers: the resulting deflection seen on the A-scope would fall to zero at the instant (range) represented.

Figs. 2, 3 and 4 show the three possibilities for partial resolution of the layer echoes. The vertical section of the echo volume is situated symmetrically across the gap between the layers. In these cases the echo would not fall to zero, but would reach a minimum at the instant represented.

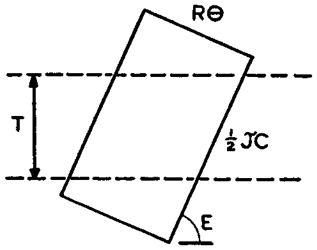

FIG. 2. Conditions of partial resolution, case A, showing echo volume straddling gap between two reflecting layers.

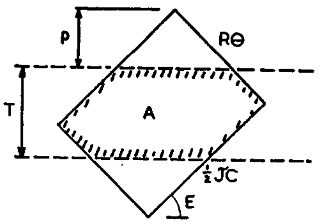

FIG. 3. Conditions of partial resolution, case B.

3. Complete resolution

This condition has been studied by Atlas and Katz [2]. Their analysis is a general one, and the results are presented in an elegant graphical form. In order that complete and partial resolution may be compared, the result is re-stated here and typical examples given.

From fig. 1 it will be seen that

$$T = \tfrac{1}{2}\tau c \sin E + H\theta \cot E, \qquad (2)$$

where H is the height above ground. For a given radar, differentiation with respect to E leads to the following condition for maximum or minimum T:

$$\sin^2 E \cos E = H\theta/\tfrac{1}{2}\tau c = P. \qquad (3)$$

This equation has real solutions only for values of P less than about 0.38. It can also be shown that when $P = 0.3$ approximately, the resolution in height is practically independent of beam elevation for elevation angles between about 30 and 90 deg. For larger values of P, best resolution is always obtained with the beam directed vertically.

These results are illustrated by four typical examples, in figs. 5 and 6.

4. Partial resolution

In fig. 3, let A be the area of the echo-volume section enclosed in the gap between the two layers. Then, for partial resolution,

$$A = n\tfrac{1}{2}\tau c R\theta, \qquad (4)$$

where n is a fraction. For example, if $n = \frac{1}{2}$, the echo intensity falls by one-half to its minimum value between the layer echoes.

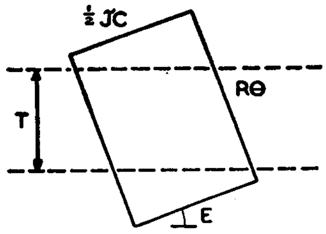

FIG. 4. Conditions of partial resolution, case C.

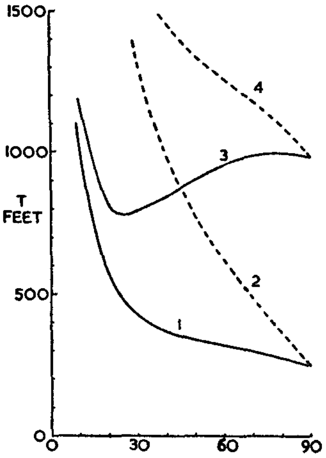

FIG. 5. Curves of complete resolution values for $H = 10{,}000$ ft. 1: 0.5-microsec pulse, 1-deg beam ($P = 0.71$); 2: 0.5-microsec pulse, 4-deg beam ($P = 2.84$); 3: 2.0-microsec pulse, 1-deg beam ($P = 0.18$); 4: 2.0-microsec pulse, 4-deg beam ($P = 0.71$).

Case A (*fig. 2*).—It is easily shown that

$$T = n\tfrac{1}{2}\tau c \sin E. \qquad (5)$$

Thus, the resolution in height is improved by reducing the elevation angle below 90 deg, but only to the point where

$$T = \tfrac{1}{2}\tau c \sin E - H\theta \cot E. \qquad (6)$$

At lower elevations, the conditions become those of case B below. By equating (5) and (6), we find that the lower limit of elevation for case A to apply is given by

$$\sin^2 E/\cos E = H\theta/\tfrac{1}{2}\tau c(1-n) = P/(1-n). \qquad (7)$$

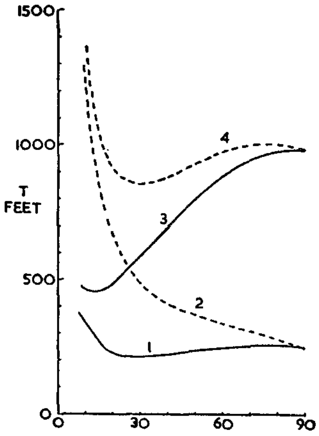

FIG. 6. Curves of complete resolution values for $H = 3000$ ft. 1: 0.5-microsec pulse, 1-deg beam ($P = 0.21$); 2: 0.5-microsec pulse, 4-deg beam ($P = 0.85$); 3: 2.0-microsec pulse; 1-deg beam ($P = 0.05$); 4: 2.0-microsec pulse, 4-deg beam ($P = 0.21$).

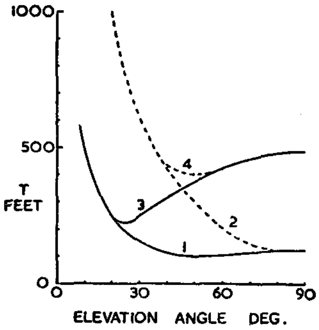

FIG. 7. Curves of partial resolution values for $H = 10{,}000$ ft. 1: 0.5-microsec pulse, 1-deg beam ($P = 0.71$); 2: 0.5-microsec pulse, 4-deg beam ($P = 2.84$); 3: 2.0-microsec pulse, 1-deg beam ($P = 0.18$); 4: 2.0-microsec pulse, 4-deg beam ($P = 0.71$).

Case B (*fig. 3*).—The area A is equal to the area of the echo-volume section less the two unshaded triangles which are equal and have a perpendicular height p. Following this through and applying (4), we obtain

$$T = \tfrac{1}{2}\tau c \sin E + H\theta \cot E - [(1-n)2\tau c H\theta \cos E]^{\frac{1}{2}}. \qquad (8)$$

The first two terms are those for complete resolution, when $n = 1$.

This result holds for elevation angles between that found from (7) above and that found from (10) below.

Case C (*fig. 4*).—Here we find that

$$T = nH\theta \cot E. \qquad (9)$$

This result is valid only for elevation angles below that given by

$$\sin^2 E/\cos^3 E = H\theta(1-n)/\tfrac{1}{2}\tau c = P(1-n). \qquad (10)$$

The same examples used previously have been worked out for a value of $n = \frac{1}{2}$, and the values of height resolution are shown in figs. 7 and 8.

5. Discussion

The value of n in the analysis of partial resolution was defined in terms of the vertical section through the echo volume; but this definition is really only convenient when the reflectivity varies abruptly with height, as was assumed in the first place. For practical purposes, it might be better to define n in terms of echo intensity, since this quantity is more directly observable.

Measurement of n, together with readings of range and elevation angle, enable one to determine from (7)

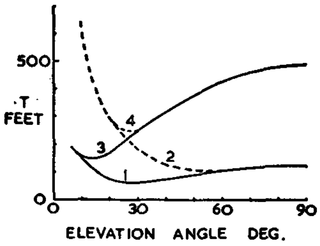

FIG. 8. Curves of partial resolution values for $H = 3000$ ft. 1: 0.5-microsec pulse, 1-deg beam ($P = 0.21$); 2: 0.5-microsec pulse, 4-deg beam ($P = 0.85$); 3: 2.0-microsec pulse, 1-deg beam ($P = 0.05$); 4: 2.0-microsec pulse, 4-deg beam ($P = 0.21$).

and (10) whether a given observation falls under case A, B or C. For this purpose, the two equations are more conveniently written in terms of range rather than height. The appropriate formula can then be used to calculate T, the vertical separation between the layers.

When the reflectivity in the gap between the layers is not zero, but equal to some fraction m of that above and below, we have new values A' and T'. Then, in case A for example, it is readily shown that

$$T' = T/(1 - m).$$

Thus, some allowance may be made for finite reflectivity in the gap.

The value of n is important in determining the actual figures expressing the resolution in height, but it is not critically important as far as the dependence of resolution on elevation angle is concerned. In case A, the form of the resolution-elevation curve does not depend on the value of n; in case B there is some dependence, and in case C the form of the curve is again independent of the value of n.

The analysis described above is capable of extension or variation to meet particular problems. One example is that of a single reflecting layer, such as occurs with the normal bright-band echo, the layer having a vertical thickness somewhat less than the dimensions of the echo volume. The band echo would have maximum intensity relative to that from above or below at the elevation angle for which T or T' has minimum value.

The curves shown in figs. 5 to 8 illustrate trends of height-resolution values applicable when the observations are confined to a fixed height above ground. With this restriction, it is clear that there is frequently a certain range of elevation angles within which the resolution curve rises or falls rapidly, and observations made within this range require careful interpretation. This range also depends on the particular height. Such effects should be borne in mind when studies are made of echo layers seen on a height-range display.

The curves are not applicable to studies in which observations are made within a vertical column of the atmosphere. However, in such cases the variation of the vertical interval T is not usually of any practical importance. For example, if observations are made at a horizontal distance of 10 mi from a radar of pulse length 2 microsec and beamwidth 1 deg, as the elevation angle is increased from 5 deg (height, 4600 ft) to 20 deg (height 19,200 ft) T varies only between 1010 and 1260 ft.

Acknowledgments.—The writer wishes to thank the Director, Meteorological Service, Southern Rhodesia, for permission to submit this article for publication. He also wishes to acknowledge the helpful comments and encouragement given him by Dr. M. G. H. Ligda and Dr. R. Wexler, both of the Weather Radar Research Project, Massachusetts Institute of Technology.

REFERENCES

1. Kerr, D. E. (ed.), 1951: *Propagation of short radio waves.* (Vol. 13, Mass. Inst. Tech. Radiation Lab. Series), New York, McGraw-Hill Book Co., p. 589.
2. Atlas, D., and L. Katz, 1951: Radar resolution in height. *Proc. Inst. Radio Engrs.*, **39**, 1341.