QUANTITATIVE DETERMINATION OF LONG WAVES AND THEIR TIME VARIATIONS

By Lt. Cmdr. Paul M. Wolff, U.S.N.

Bureau of Aeronautics Project AROWA[1]

(Original manuscript received 19 July 1954; revised manuscript received 29 April 1955)

ABSTRACT

A method of space averaging is presented for separating long and short waves in the 500-mb geostrophic flow. The long-wave features are further identified as perturbations. Some of the forecasting characteristics noted in the application of these computations to six years of historical and one year of current hemispheric 500-mb charts are included.

1. Introduction

Since the fundamental work of Rossby [1], a large group of meteorologists has adhered to the concept that long, quasi-barotropic waves extending through most of the troposphere govern the broad-scale weather patterns and their evolution. Other perturbations, however, especially short baroclinic waves, are superimposed on the basic long waves at any given instant. They often make it difficult to locate the long waves and to define them numerically from an instantaneous upper-air chart. Hence, some filtering technique is needed to eliminate the superimposed disturbances and to permit the long waves to stand out clearly.

Following early experiments at the Massachusetts Institute of Technology [2], five-day time averaging has been adopted as one such technique for extended forecasting [3]. But despite its well known usefulness, this method has two drawbacks: (1) the mean map, essentially most representative for the middle of a five-day interval, is 2½ days old at the time of forecast preparation (this lag certainly impairs its usefulness for application to, say, 48- to 72-hr forecasting); and (2) during periods of rapid motion of long waves, or large changes in intensity and changes in wave number, the five-day chart often becomes ill-defined and, containing much information that is far from up-to-date, is difficult to interpret.

For these reasons, a search for alternate methods of averaging has been going on for years. But as recently as 1952, Riehl, La Seur *et al* [4] still called for improved methods of long-wave determination. This article presents a technique believed to provide substantial improvement.

2. Long-wave determination

The principal suggestion for the proposed method has come from a paper by Fjörtoft [5], who, following earlier attempts to average circulation parameters either over latitude or longitude, suggested averaging on a square grid to produce space-mean charts.[2] This attack raised the question whether, independent of the original objectives of Fjörtoft's work, some technique of space averaging could be found that accurately represented the long waves. If so, the drawbacks of time-averaged charts would be eliminated.

An empirical investigation was conducted at Project AROWA to settle this question. Five considerations governed the study:

1. The averaging technique should be representative of the scale of long waves.
2. The resulting hemispheric patterns should be reasonably sinusoidal, with wave numbers ranging from three to six in middle latitudes.
3. After averaging, the 24-hr time variations of pressure or contour height at any fixed point should be much less, preferably an order of magnitude less, than the total 24-hr height changes.
4. The 24-hr time variations so computed should not oscillate rapidly in sign, but should maintain the same sign for much longer periods than the total changes. Requirements (3) and (4) follow from the concept that the long waves move slowly and undergo only slow changes in intensity.
5. The space-mean chart should be capable of rapid calculation by machines for day-to-day practical use.

The following averaging procedures were tried out on three months of daily 500-mb hemispheric charts:

a. averages over 10, 20 and 30 deg lat;
b. averages over 10, 20 and 30 deg long;
c. averages over "squares" bounded by 10, 20 and 30 deg lat and long; and
d. averages over "diamonds" with diameters of 10, 20 and 30 deg lat and long; fig. 4 illustrates the 20-deg diamond.

From the viewpoint of criterion (2) above, it turned out, somewhat unexpectedly, that averaging with respect to latitude alone is superior to longitudinal averaging. Neither method (a) or (b), however, proved nearly as satisfactory as (c) or (d). Considering the grid sizes under (c) and (d), one might expect that the smallest size should not filter out the short waves entirely, because the average half-wavelength of the

---

[1] Present duty station: U.S.S. Bennington (CVA-20).

[2] *Editor's note.* For a brief discussion of practical application of this technique, see U. S. Air Weather Service, "Forecasting upper-level winds, Part 1, Forecasting by vorticity techniques" (AWS Manual 105–50/1), Washington, Dept. of the Air Force, 50 pp.

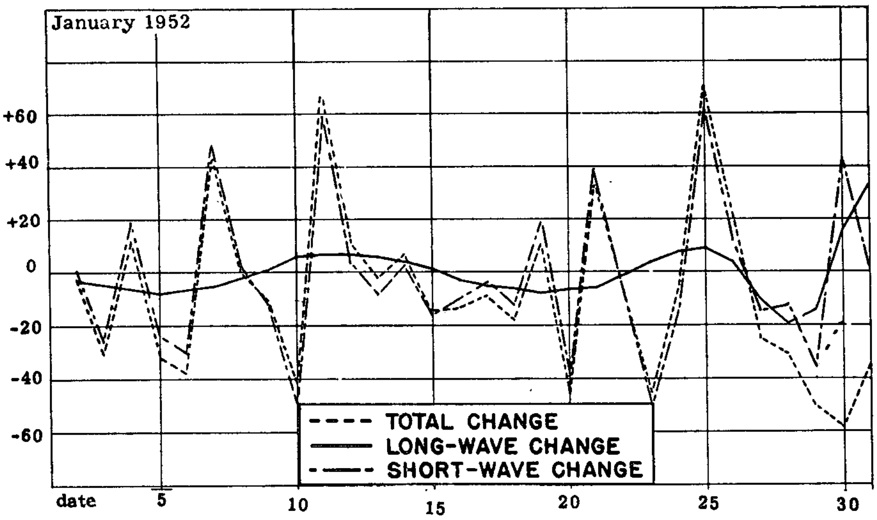

FIG. 1. Plot of daily values of three terms of equation: total height change = long-wave change + short-wave change (at 40°N and 80°W, in tens of feet per day), January 1952.

short waves exceeds 10 deg; this expectation proved correct. The largest grid size, on the other hand, corresponds to the half-wavelength of the long waves at wave number six and should therefore tend to obliterate the long waves; this outcome was also observed. It follows that the 20-deg square or diamond must give the most satisfactory result; this conclusion was verified.

Further experiments were carried out with the 20-deg square and diamond, such as to include values not only on the boundaries of the areas but also in the interior. These calculations tended to bring the short waves and other smaller circulation features back into the picture and were therefore abandoned.

There is little to choose between the diamond and the square. The diamond, however, verified slightly better from the viewpoint of criteria (3) and (4). It therefore provides the most satisfactory approximation to the long waves obtained from the experiment.

Figure 1 shows the total and space-averaged 500-mb 24-hr height changes at 40°N and 80°W, in time-section form, for January 1952. The result is very satisfactory. Changes in the space-averaged grid are essentially an order of magnitude less than the total tendency, and the sign of the changes persists for a week or more.

In January 1949 (fig. 2), the result was less satisfactory, especially in the first half of the month. The large and rapid long-wave changes suggested by this time section are relatively rare.

The space-averaged 500-mb contours for 13 April 1954 (fig. 3) reveal a beautifully simple and symmetrical pattern with four long waves. This pattern is quite similar to a model published by Palmén [6].

3. Long- and short-wave changes

We shall now assume that the space mean compiled from 20-deg diamonds represents the long-wave part of the flow. Then the difference between total and space-mean 24-hr height changes should represent

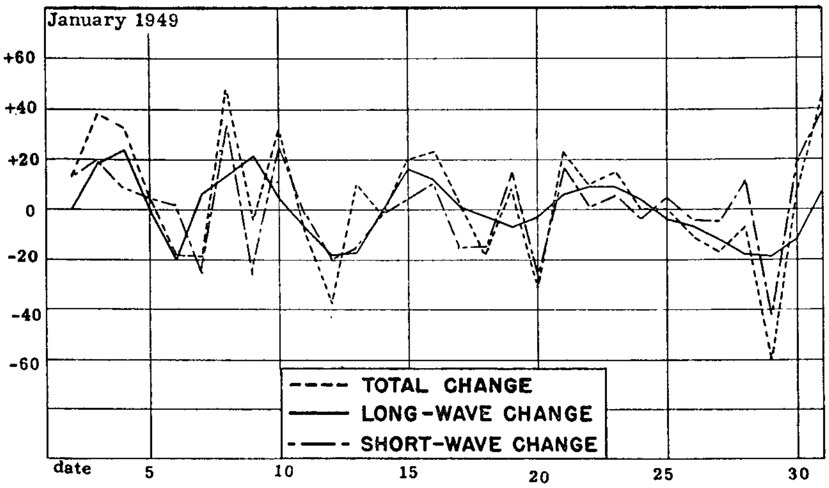

FIG. 2. Same as fig. 1, but for January 1949.

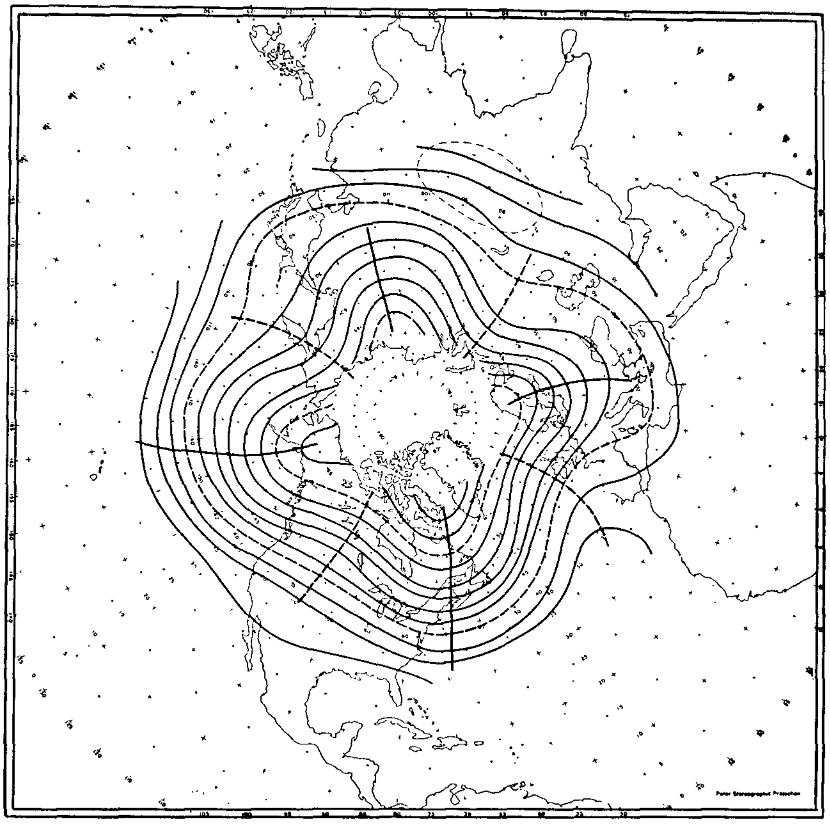

FIG. 3. Space-mean contours, 1500 GCT 13 April 1954.

variations in the mass field due to short baroclinic waves — if such waves are the major residual upper-air disturbances. In the past, techniques to separate long and short waves have not always produced clear-cut results. Nyberg [7], for instance, defines the short-wave number as the number of sign alternations around latitude circles on 24-hr 500-mb height-change charts. This definition gives hemispheric short-wave numbers of only seven to nine [8], whereas, in order that short waves may be readily distinguishable from long waves, the short-wave number should be at least double the long-wave number.

The question of separation of long and short waves may now be studied by drawing charts of space-mean or long-wave 24-hr tendencies, and charts of the difference between total and long-wave tendencies. As an example, fig. 4 shows the 500-mb contours of a large-amplitude pattern. This chart permits us to locate the long waves without difficulty. Superficially, the space-mean chart (fig. 5) adds little except more precise location of the trough line and simplification and increased symmetry of the contour pattern. The 24-hr height change chart (fig. 6), however, though clear

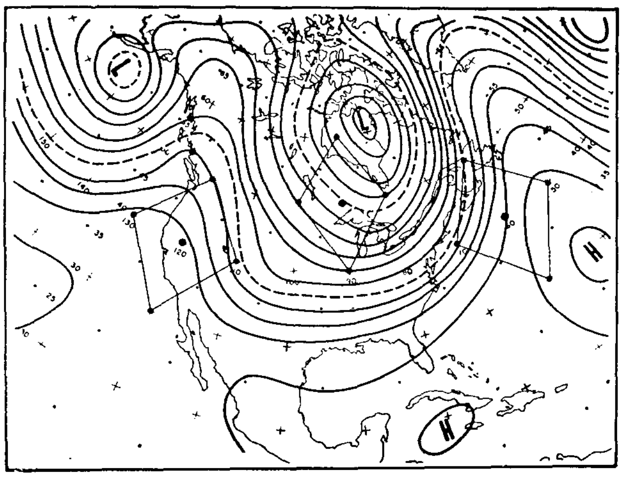

FIG. 4. 500-mb contours, 1500 GCT 5 November 1951. 18,400-ft and 17,200-ft contours dashed.

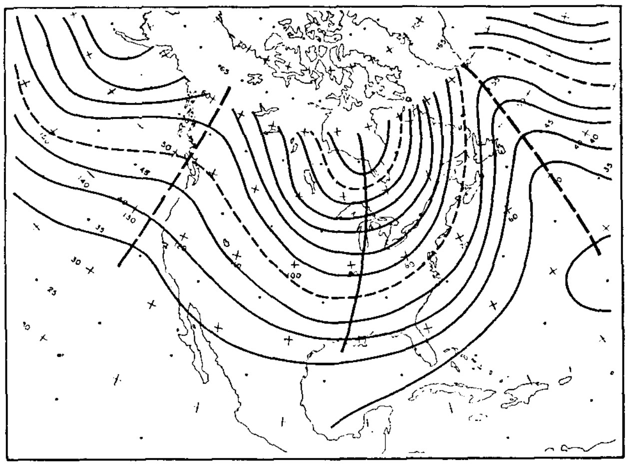

FIG. 5. Space-mean contours, 1500 GCT 5 November 1951. 18,400-ft and 17,200-ft contours dashed. Long-wave trough and ridge lines indicated.

enough in the Pacific, has a complicated configuration over the continent and the Atlantic that is not readily interpreted without manipulation.

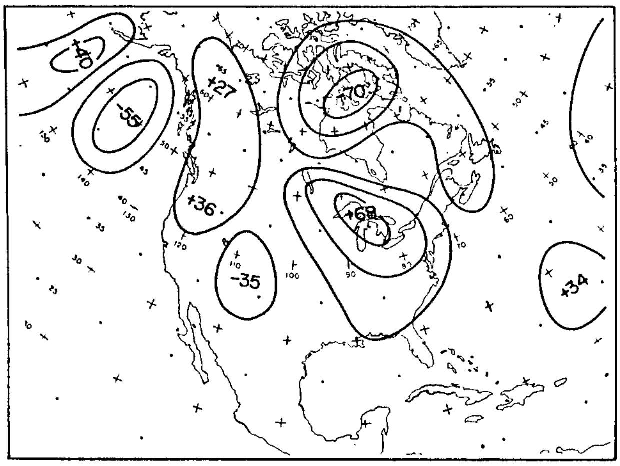

Fig. 6. Isopleths of total 24-hr height changes (200-ft increments), 1500 GCT 5 November 1951. Maximum values in tens of feet for each center. Zero isopleths omitted.

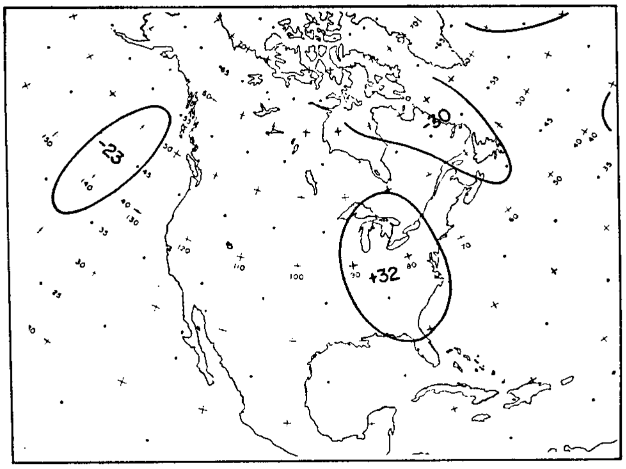

Fig. 7. Isopleths of 24-hr long-wave height changes (200-ft increments), 1500 GCT 5 November 1951. Maximum values in tens of feet for each center. Zero isopleths omitted.

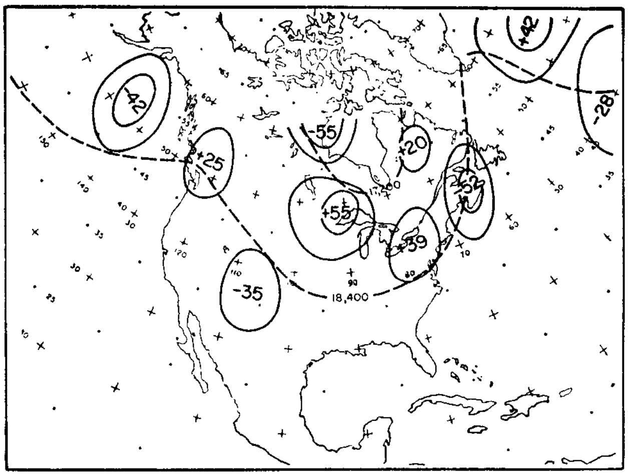

Fig. 8. Isopleths of 24-hr short-wave height changes (200-ft increments), 1500 GCT 5 November 1951. Maximum values in tens of feet for each center. Zero isopleths omitted.

Separation of long-wave and residual tendencies (figs. 7 and 8) does much to clarify the pattern of fig. 6. The residual changes break down into two strings of centers. One of these starts in the Pacific and traces the space-mean pattern in a very regular manner. The average wave length of this train is 40 deg long, compared to almost 80 deg long for the long waves (fig. 5). Fall and rise centers succeed each other with considerable regularity and without excessive variations of central tendency. These facts all support interpretation of the isallobaric chain as true short waves.

A second train is indicated as circling the periphery of the Hudson Bay low. The two trains interfere with each other over the region of the eastern Great Lakes and St. Lawrence Valley, where, perhaps in consequence, the largest long-wave changes are also found.

Comparison of figs. 6 and 7 reveals that only a relatively small portion of the total height change is accounted for by the long-wave changes, in spite of the importance of the breakdown of the total tendency field for the pattern of fig. 8. Little experimentation has been done so far with interpretation of the long-wave tendencies, and comments must therefore remain very brief. The fall centers in the eastern Pacific and over Labrador, located west of the two long-wave ridges, indicate progression. Over the continent, however, we note the lack of a rise center over the Rockies and the existence of a rise center east of the trough in the central United States. These two features indicate weakening of both trough and Pacific ridge, foreshadowing a decrease in the amplitude of the upper-air flow.

The pattern and behavior of the change centers of figs. 7 and 8 are typical of what has been encountered in a sample covering several years. Short-wave change centers are small, intense in gradient, and symmetrical. They are always progressive, and their day-to-day continuity is excellent. The centers are lined up along the strongest flow, or jet stream. Occasionally two strings of centers coexist in high and low latitudes, when the westerlies are split. The intensity of the centers ranges up to 1500 ft; the hemispheric short-wave number as deduced from them averages ten to fifteen. The relation of these centers to surface synoptic events is quite definite.

In contrast, the long-wave changes frequently cover large areas from 20 to 60°N. Gradients are weak. Day-to-day continuity is poor, and the centers progress only occasionally. A good correlation exists, however, between the dimension of these centers and that of the long waves.

Frequently, large areas of the hemisphere are devoid of long-wave changes exceeding 200 ft — a stable condition. During increases or decreases of wave number, however, the change may amount to as much as 750 ft.

4. Perturbation analysis

Although charts such as fig. 3 clearly outline the long waves, it is essential to forecasting purposes to develop quantitative measures of such features as intensity. This may be done by applying to the hemispheric space-mean charts a method of perturbation analysis proposed by Riehl [9]. The average space-mean height for all values around a latitude circle is subtracted from each individual height on that circle. In this way, the mean zonal flow is removed; instead of long waves, definite cyclonic and anticyclonic perturbation centers appear on the space-mean chart.

Fig. 9 shows the perturbation analysis of fig. 3. The existence of perturbation centers is evident. They may be assigned central-intensity numbers, just like surface high- and low-pressure centers. Both position and intensity of the centers may be tracked from day to day. Further, the lines of equal perturbation strength represent approximately the stream function of the geostrophic perturbation windfield. Consequently, the latter can be calculated from fig. 9, as can also the geostrophic vorticity of the perturbations.

At times, the pertubation charts reveal a curious deficiency in the concept that the broad-scale flow aloft consists of a basic zonal current with long waves. On one side of the hemisphere, usually over the Atlantic and Asia, all perturbation values are positive. Troughs appear as centers of minimum positive perturbation. On the other side of the hemisphere, the reverse holds. This contrast suggests that a still larger feature is superimposed on the long waves, a general eccentric flow of the westerlies which rotate not about the North Pole but about a circulation pole located as much as 10 deg lat and more from the geographic pole. LaSeur [10] has described this eccentricity and has developed methods of calculating the circumpolar current with respect to the circulation pole. Attempts are currently underway at Project AROWA to effect a further separation of the perturbation height-field into the part due to the eccentric flow, which may be characterized as a wave number of one, and the part due to the long waves.

Although work with the perturbation charts is only in its beginning stages, fig. 10 indicates the type of use to which the charts can be put. This figure shows a time plot of the maximum perturbation strength of an east Pacific ridge and a United States trough during a month with quasi-stationary long waves. Large rises in the ridge are followed within one to two days by pronounced deepening of the trough. This plot indicates that a quantitative measurement of the often-discussed downstream propagation of amplitude is possible. Indeed, as fig. 10 shows, the trough deepened markedly on only two occasions during the month — subsequent to building of the Pacific ridge. Other cases have led to the tentative conclusion that, when the perturbation strength of a trough or ridge increases by more than 200 ft in 24 hr to an absolute

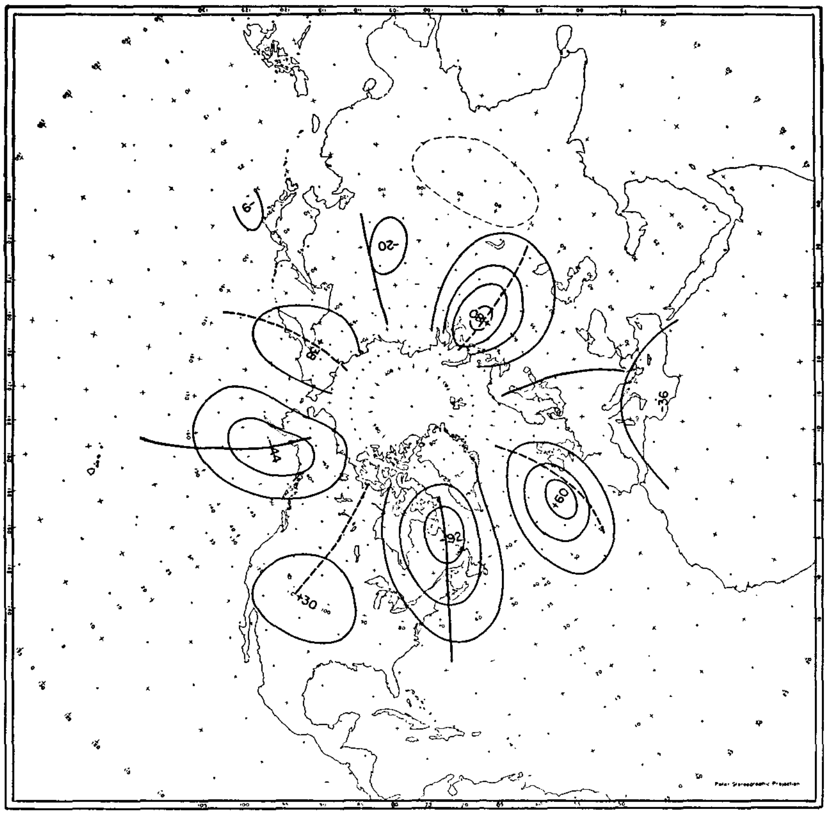

FIG. 9. Perturbation of space-mean heights, 1500 GCT 13 April 1954. Maximum perturbation strengths in tens of feet. Long-wave trough and ridge lines indicated.

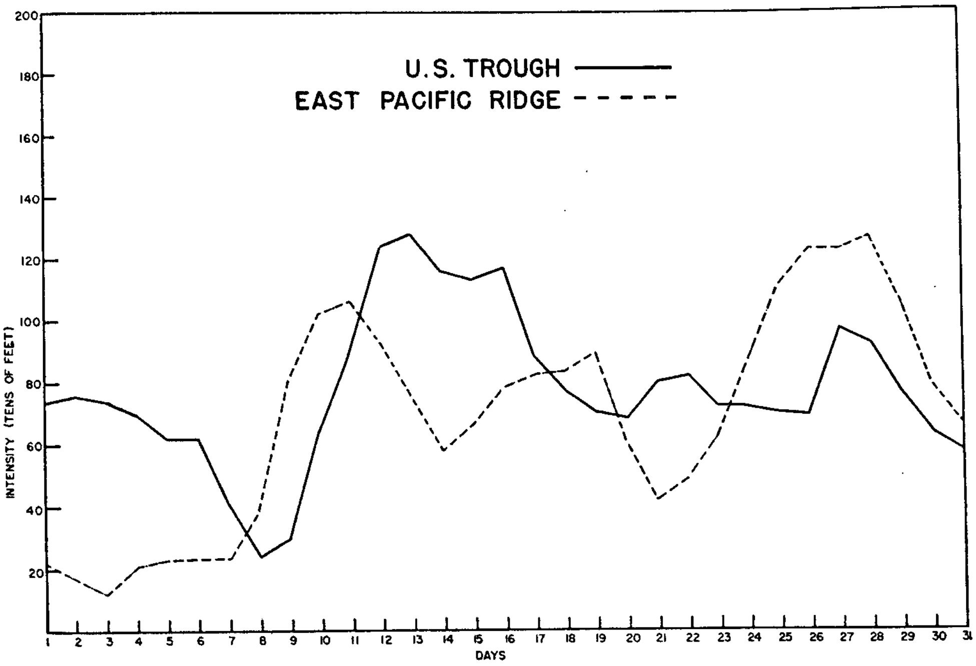

FIG. 10. Plot of maximum perturbation strengths in two adjacent long-wave features, December 1951.

value exceeding 600 ft, intensification of the next feature downstream can be expected to begin within 24 hr.

5. Conclusion

The technique of 500-mb analysis here described has been applied to hemispheric charts at Project AROWA on a daily basis since January 1954. All numerical steps are capable of rapid machine evaluation, if the heights of the analyzed 500-mb charts are read and tabulated at fixed latitude-longitude intersections. The time consumed for the preparation of all charts by two subprofessionals with the aid of International Business Machines equipment is less than 2 hr. (Hand calculation would require more than 7 hr.)

Similar charts have been prepared at Project AROWA for the series of historical 500-mb charts beginning in 1945.

The present article has been concerned mainly with techniques of analysis. Intensive research is now being carried out at Project AROWA to evaluate the space-mean, perturbation, and tendency charts. One of the purposes for which the series is suitable is a long-wave census. For instance, long-wave numbers ranging from three to six have been observed. The wave number of five predominates; only on occasional days do we encounter larger or smaller wave numbers. From consideration of Rossby's barotropic wave formula [11], it appears that the westerlies in middle latitudes are too weak to support small wave numbers. Whenever the wave number fell to three or four, it increased again within 24 hr in all cases noted. Blocking patterns also occur with a wave number of five; the blocking ridges are mainly situated at 0 and 160°W.

Acknowledgments.—The use of space means to locate long waves was suggested to the writer by Dr. H. Riehl, who gave much guidance during the investigation. The International Business Machines computations were accomplished at the National Weather Records Center in Asheville. Mr. R. A. Corvo assisted in the preparation of the manuscript.

REFERENCES

1. Rossby, C.-G., and collaborators, 1939: Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action. *J. marine Res.*, **2**, 38–55.
2. Willett, H. C., 1941: Report of the five-day forecasting procedure, verification, and research as conducted between July 1940 and August 1941. *Pap. phys. Ocean. Meteor., Mass. Inst. Tech. and Woods Hole ocean. Instn.*, **9**, No. 1, 88 pp.
3. Namias, J., 1947: *Extended forecasting by mean circulation methods.* Washington, U. S. Dept. Commerce, 89 pp.
4. Riehl, H., N. E. La Seur, *et al*, 1952: Forecasting in middle latitudes. *Meteor. Monogr.* **1**, No. 5, 80 pp.
5. Fjörtoft, R., 1952: On a numerical method of integrating the barotropic vorticity equation. *Tellus*, **4**, 179–195.
6. Palmén, E., 1951: The aerology of extratropical disturbances. *Compendium Meteor.*, Boston, Amer. meteor. Soc., 599–620.
7. Nyberg, A., 1949: An aerological study of large-scale atmospheric disturbances. *Tellus*, **1**, No. 1, 44–53.
8. Berry, F. A., W. H. Haggard, and P. M. Wolff, 1953: 500-millibar studies at Project AROWA. *Bull. Amer. meteor. Soc.*, **34**, 444–454.
9. Riehl, H., 1949: A method for quantitative determination of long waves. *J. Meteor.*, **6**, 207–211.
10. La Seur, N. E., 1954: On the asymmetry of the middle-latitude circumpolar current. *J. Meteor.*, **11**, 43–57.
11. Rossby, C.-G., 1940: Planetary flow patterns in the atmosphere. *Quart. J. r. meteor. Soc.*, **66** (Suppl.), 68–87.