INTERPRETATION OF THE HEIGHT-VERSUS-TIME PRESENTATION OF RADAR ECHOES

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ABSTRACT

The height-*versus*-time presentation of radar echoes (used with radars of 1.25- and 0.86-centimeter wavelength) is a record of the precipitation and of some of the clouds passing directly over the radar. Such a record corresponds to a "surface of observation" in the atmosphere. For non-precipitating clouds, the shape of the surface depends upon the upper-wind patterns; for precipitating clouds, the surface is a vertical plane.

Many of the recorded precipitation echoes have a marked slope, due to the effect of wind shear on falling precipitation particles. The magnitude of slope appears to be determined mainly by the speed of travel of the precipitation column, the wind shear, and the fall speed of the particles; lesser influences are the change of wind direction with height, horizontal dimension of the column, and the path of the column relative to the radar position. The horizontal distribution of particle sizes may be very important, but not enough is known about this factor to permit an estimate of its effect on the shapes of echo boundaries.

In practical use of this radar system for weather observation, it does not appear that any purpose would be served by making measurements of echo slope. Probably the chief value of the system lies in the qualitative information it furnishes about the current physical processes of cloud and precipitation. An important question arises as to the size of geographical area adequately represented by the height-time observations.

1. Introduction

In a recent article, Plank *et al* [1] have presented a large amount of information concerning the detection of cloud and precipitation by means of the AN/APS-34 radar, which operates at a wavelength of 1.25 cm. A few years previously, Swingle [2] reported on the results of a comparison of the cloud-detection characteristics of this radar with those of the AN/TPQ-6 radar, which operates at a wavelength of 0.86 cm.

Both of these radars are used with a fixed, vertically-directed beam, and the echoes are continuously recorded against a scale of height by a facsimile recorder. The result, over a period of time, is a chart of echoes on a diagram whose ordinate is height above the radar and whose abscissa is time.

This type of echo presentation is an unfamiliar one, and the exact form in which echoes appear on the record is influenced by a number of factors which are not immediately obvious. It is the purpose of this article to draw attention to these factors.

A typical record obtained from the AN/APS-34 radar is shown in fig. 1. This time-section of radar echoes does not, in general, correspond with a plane vertical space-section through the atmosphere for two reasons:

1. The cloud structure changes with time.
2. The cloud elements at different levels may move in different directions and/or with different speeds.

The importance of changes in cloud structure depends upon how rapidly they proceed with respect to the speed of horizontal travel. Between ordinates which are close together on the record, changes in cloud structure are small, and the record effectively shows space variations; but for large time intervals, structural changes may be very important.

Within the limits imposed by structural changes, the time section may be converted to a space section, or "surface of observation."

2. Relation between time and space sections

In general, the cloud or precipitation producing the echoes may have speeds and directions of travel which vary with height. The surface of observation is defined by the horizontal paths of the clouds at the various levels, and the correspondence between time and space sections is illustrated in fig. 2, where the arrows on the vertical axis indicate the motions.

If the speeds of movement are uniform with height, AC = BD; otherwise AC and BD are unequal. When the directions of movement are uniform with height, the surface of observation becomes a vertical plane.

In the above, we have referred only to the motions of cloud or precipitation, and these motions may differ considerably from the upper-air flow.

As a first example, take the case of two rather thin

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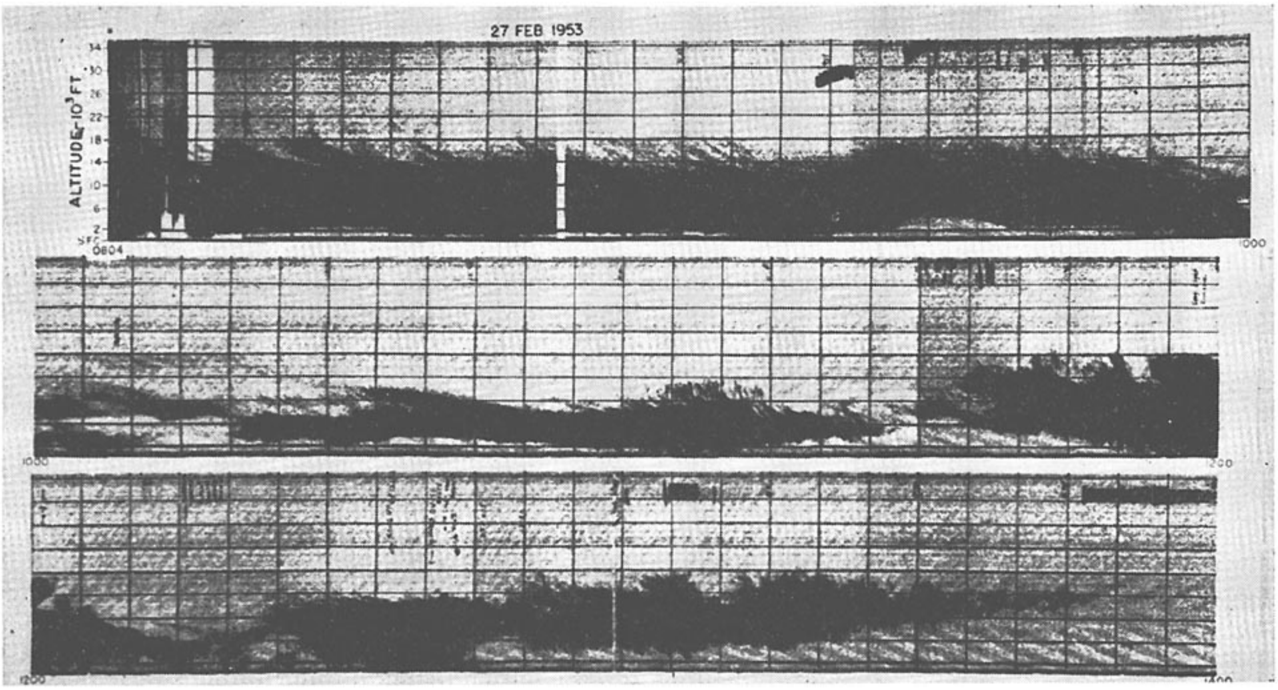

FIG. 1. Height *versus* time record of echoes on AN/APS-34 radar. Vertical lines at intervals of 5 min. Sharp-edged vertical gaps due to reduction of receiver gain. Occasional marks of uniform thickness at high levels are calibrating signals.

and stable cloud layers, sufficiently dense to produce detectable echoes. Each layer moves in the direction of the wind at its own level and at about the same speed, so that fig. 2 illustrates the situation, the arrows on the Z-axis now indicating the distribution of upper-air flow over the point of observation.

As a second example, suppose that a shower is "generated" at some high level, and that the precipitation falls down through winds of varying speeds and directions. Then, according to Gunn and Marshall [3] and Lhermitte [4], the whole trail of precipitation moves with the speed and in the direction of the air flow at the generating level, and the surface of observation is plane (fig. 2) with AC = BD. (According to Plank *et al* [1], the disturbance responsible for the generation of precipitation may on occasion travel at a speed exceeding that of the wind at any level.)

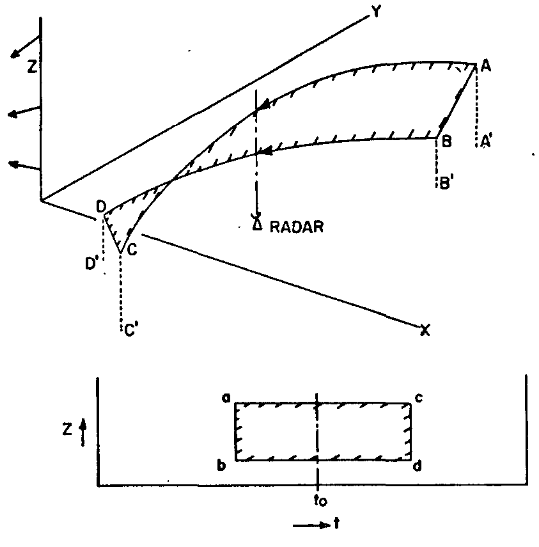

FIG. 2. Relation between time and space sections: ABCD is surface of observation corresponding to portion abcd of height-time record. Arrows on Z-axis indicate cloud motions.

For a third example, consider showers due to well-developed convection cells extending through a deep layer of the atmosphere. In this case, the precipitation moves at about the speed and in the direction of the mean value of the air flow through the layer concerned. The surface of observation is again a plane, as for the case of high-level shower generation.

3. Slope of echoes

On many of the height-time records, the most striking feature is that of sloping "streamers" of echo. The right part of fig. 3 shows a good example. Gunn and Marshall [3] suggest that such streamers are produced by precipitation which is being generated at a high level (near the top of the observed echo) and which is falling down through air where there is marked wind shear. This explanation may be supported by reference to the distribution of upper winds and to ordinary radar plan-position-indicator (PPI) pictures.

The record shown in the right part of fig. 3 was obtained from an AN/APS-34 radar situated near Boston, Mass. An AN/CPS-9 radar (3.2-cm wavelength) is located 11 mi away from the AN/APS-34 radar site, and the left part of fig. 3 shows a PPI picture obtained from the AN/CPS-9 about 20 min before the sloping echo appeared on the height-time record. The arrow indicates a small echo which, when the PPI film is viewed as a moving picture, is first seen at a range of nearly 100 mi, and subsequently can be followed continuously as it moves eastward and passes over the site of the AN/APS-34 radar, to disappear finally into the permanent echo pattern of the PPI picture. This echo arrives at the site of the AN/APS-34 radar at 1000 EST, according to the PPI record. The height-time record shows the sloping echo to have been present, close to the ground, from 1001 to 1004 EST. It is clear, therefore, that both radars were observing the same precipitation.

The times quoted above for the presence of the echo on the height-time record have been read at a very low altitude, because it is only the low-level part of the precipitation which contributes to the echo seen on the PPI. Since the elevation angle of the axis of the AN/CPS-9 beam is $\frac{1}{2}$ deg, and since the beam has a full spread in elevation of about 6 deg, at 11-mi range the power of the beam extends almost from the ground up to about a 3600-ft altitude; however, most of the contribution to the echo is obtained from the portion of the beam between the half-power directions, *i.e.*, from the precipitation below about 1100 ft.

From the PPI film, the speed of travel of the echo was measured over three intervals in the period from 0950 to 1025 EST; the mean value of the speed was 87 mi/hr. Upper-wind observations extended only to 10,500 ft; up to about 6500 ft, there was constant wind shear (apart from minor irregularities); and between 6500 and 10,500 ft, the shear was again constant, but at a somewhat lower value. On the assumption that this constant value of wind shear continued above 10,500 ft, it was found that a wind speed of 87 mi/hr was attained at a height of about 15,000 ft. Thus the PPI echo, observed at a height of about 1100 ft and less, was moving at the same speed (and in the same direction) as that of the air flow at about 15,000 ft.

To estimate the level of origin of precipitation from the height-time record, we note that, according to the theory of particle trajectories, the trajectory is vertical at the level of origin. ("Trajectory" is here taken to mean the path followed by a falling particle in a coordinate system which moves with the speed of the air flow at the generating level, and in the same direction [3].) In the right part of fig. 3, it will be seen that the leading edge of the echo is vertical at a height of about 16,500 ft, in fair agreement with the height of 15,000 ft estimated from PPI pictures and upper winds. These figures, although somewhat rough, do lend support to the idea that the shower was "generated" at — and hence its movement controlled from — a level of about 15,000 or 16,000 ft.

The leading edge of echo on the height-time record is vertical also at about 12,000 ft; and since the echo edge here is ahead of the "generating element" at 16,500 ft (the time scale of the record may be converted to a scale of horizontal distance by multiplying by the speed of travel as found from the PPI pictures), this forward part of the shower cannot be directly associated with that same element — unless the element is in reality more extensive in the horizontal direction than the radar indicates. There is not sufficient information available to resolve this question.

It is interesting to note that the diameter of the echo seen on the PPI was 2.7 mi, so that it would take 1.9 min to traverse its own diameter. The duration of echo on the height-time record, at a height of about 1000 ft, was 3.5 min; this would represent, at the speed of 87 mi/hr, a horizontal echo dimension of 5.1 mi. The difference between the two dimensions may be due to greater sensitivity of the AN/APS-34 (at a range of 1000 ft) than of the AN/CPS-9 (at a range of 11 mi) to a fringe of small particles around the boundary of the shower.

The difference is not due to beam-width of the AN/APS-34. Consider a point target at height H, moving horizontally at speed V; if θ is the effective width of the radar beam, the time t taken by the target to traverse the beam is $t = 0.198\ \theta H/V$, where the units are minutes, degrees, thousands of feet, and miles per hour. When a target of appreciable horizontal extent passes through the beam, the time t represents the amount by which the apparent duration of echo

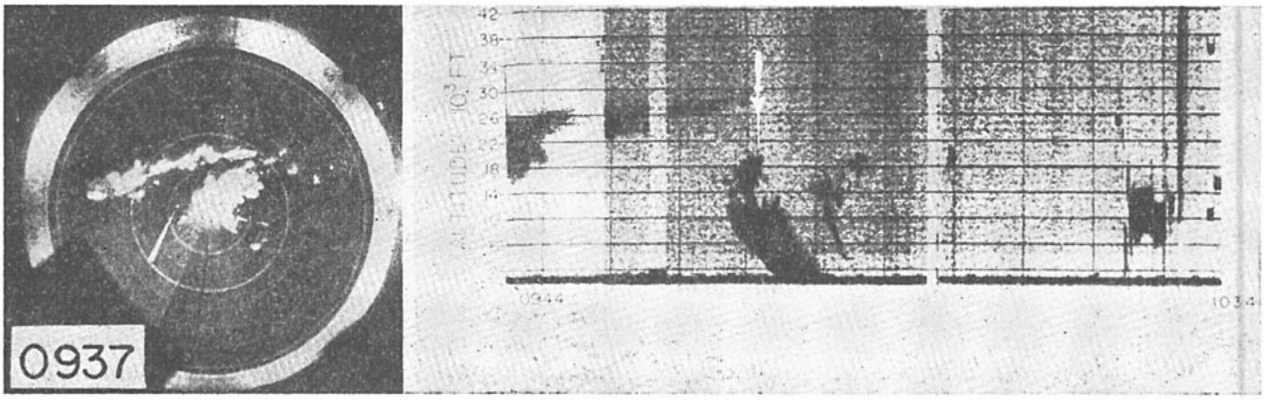

FIG. 3. Right: sloping shower echo on height-time presentation. Left: same shower as seen on PPI presentation (echo indicated by arrow).

on the record is increased due to radar beam-width. Even with the extreme values of $\theta = 1$, $H = 30$, and $V = 10$, we find $t = 1.6$ min only. For the situation discussed above, $t = 0.002$ min, which is entirely negligible. (The half-power beam-width of the AN/APS-34 radar is stated to be 0.37 deg, but the power of the whole beam has an angular spread which is considerably larger; see [5], for example.)

4. Magnitude of echo slope

The slope of a precipitation echo on the height-time presentation depends first on the speed at which the precipitation column passes over the radar. If this speed is known, the "time" slope can be converted to the "space" slope.

The slope in space depends on any variation of radar sensitivity with height, and on the structure of the precipitation column with respect to radar reflectivity. However, it will be shown that the variation of radar sensitivity with height is usually negligible in its effect on echo slope; then the effects of precipitation structure will be discussed.

Variation of radar sensitivity with height.—The radar equation [6] shows that, if the precipitation fills the radar beam, and if attenuation and deviations from the Rayleigh scattering law are neglected, $P \propto Z/R^2$, where P is the intensity of echo received, R the range, and $Z = \Sigma\, Nd^6$, N being the number per unit volume of precipitation particles of diameter d. According to the level of electronic noise in the radar receiver, there is a minimum detectable signal $P_{\min}$, and hence a minimum detectable Z-value, say $Z_{\min}$. In the normal way, $Z_{\min} \propto R^2$, so that the radar sensitivity decreases with increase in range, unless sensitivity-time control is used. Sensitivity-time control reduces the receiver gain at the time of each transmitter pulse and thereafter allows the gain to recover at a chosen rate.

Now, suppose that the distribution of Z-values in the column of precipitation is as shown in fig. 4, where $Z_3 > Z_2 > Z_1$. If $Z_{\min}$ for the radar is independent of range and equal to Z_2, the echo boundary corresponds exactly with the Z_2-contour in the precipitation. However, if $Z_{\min}$ varies with range in such a way that H_3, H_2 and H_1 are the heights at which $Z_{\min}$ becomes equal to Z_3, Z_2 and Z_1, respectively, the echo boundary corresponds with the curve AB. Thus, if the value of $Z_{\min}$ for the radar is closely comparable with the values of Z applicable to the precipitation, the echo boundary may be quite different in shape from the column of precipitation; if $Z_{\min}$ is much smaller than the Z-values in the precipitation, the shape is not distorted in this way.

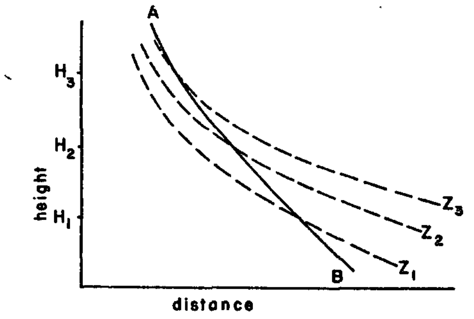

FIG. 4. Showing possible difference between echo boundary AB and precipitation boundary Z_1, when radar sensitivity decreases with height.

Gunn and Marshall [3] have calculated the distributions of Z-values for a particular shower and obtained values of 10 and greater in units of 10^3 mm^6/m^3. If the characteristics of the AN/APS-34 radar given by Plank *et al* [1] are inserted in the radar equation, we find that, even at a range of 20,000 ft, $Z_{\min}$ is about 6×10^{-5} in units of 10^3 mm^6/m^3. Since this value is so much smaller than those for the shower, the echo boundary would truly represent the shower boundary. Indeed, it appears from the graph of Z *versus* rate of rainfall, given by Gunn and Marshall in their fig. 8, that even in the most trifling rates of rainfall the values of Z are three or four orders of magnitude greater than $Z_{\min}$. Hence, we may conclude that the variation with height of radar sensitivity produces negligible distortion in the shape of the echo boundary.

Structure of precipitation column.—Gunn and Marshall [3] developed the theory of particle trajectories for the case of uniform wind shear, constant wind direction, and constant fall speed of the particles.

To suit the analysis to the present purpose we can proceed as follows. Take the origin of coordinates at the generating level and moving with the air flow at this level. Take the X-axis pointing into the wind at the generating level and the Z-axis vertically downward. For a particle at the point (x, y, z), let v = fall speed of particle, V = wind speed in the moving coordinate system, and α = angle between X-axis and direction of air flow in the moving coordinate system. Then, in a time interval dt,

$$dz = v\,dt,$$
$$dx = V\cos\alpha\,dt = (V/v)\cos\alpha\,dz,$$

and

$$dy = V\sin\alpha\,dt = (V/v)\sin\alpha\,dz.$$

Hence, the complete trajectory in the moving coordinate system is given by

$$x = \int_0^z (V/v)\cos\alpha\,dz + A,$$

and

$$y = \int_0^z (V/v)\sin\alpha\,dz + B,$$

where A and B are constants of integration.

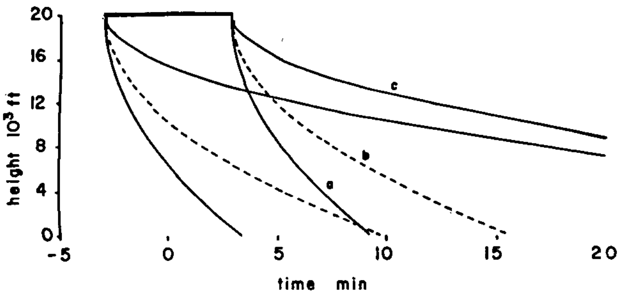

FIG. 5. Theoretical boundaries of shower. Particle fall-speeds: (a) 8 m/sec (water drops, 1-mm radius), (b) 4 m/sec (water drops, 0.5-mm radius), (c) 1 m/sec (snow). Wind shear 3 mi/hr per 10^3 ft, wind direction constant. Shower travelling at 60 mi/hr.

For the height-time record, observations are made at a fixed point on the ground, for which $x = V_g t$, V_g being the wind speed at the generating level, and time t being measured from the instant that the moving origin passes over the ground point. Also, as stated in section 2 above, the surface of observation of the zenith-pointing radar is a plane parallel to the X-axis, say $y = y_1$.

Thus, if the wind speed V and direction α, and particle fall velocity v are known as functions of z, we can obtain the equations of the particle trajectories.

The simplest case is that of uniform wind shear so that $V = az$ where a is constant, constant wind direction $\alpha = 0$, and constant fall speed v. Then

$$x = V_g t = az^2/2v,$$

i.e., the trajectory is parabolic in space (as observed in the moving coordinate system) and parabolic on the height-time record, as found by Gunn and Marshall. If the fall speed is not constant but varies with height under the influence of coalescence, accretion, evaporation, condensation or melting, we can put $v = bz^n$, where the values of b and n are determined by the physical processes at work. n is positive for coalescence, accretion, condensation and melting; for evaporation, n is negative. Then we get

$$V_g t = az^{(2-n)}/b(2 - n).$$

If the value of n is significantly different from zero, the trajectory is no longer parabolic in shape. Hence it is possible for changes in particle fall-speed to modify the slope of echo and the height-time record. Lhermitte, however, calculated particle trajectories [4] using observed winds and a series of assumed values of fall speed; from the close similarity of one calculated trajectory to the observed shape of echo boundary, Lhermitte concluded that there was no appreciable variation of fall speed during the descent of the particles.

To take a particular example, suppose that all the particles of a column of precipitation are of equal size, are all produced within a circle of radius r centered at the origin, and have constant fall speed. Then the horizontal cross-section of the shower at all levels is a circle of radius r. For uniform wind shear and constant wind direction, the equation to the axis of the shower is

$$x = az^2/2v.$$

The boundary surface of the shower is given by

$$(x - az^2/2v)^2 + y^2 = r^2.$$

For a given plane of observation, $y = y_1$, and $x = V_g t$, so that the shower edge in the height-time plane is given by

$$V_g t - az^2/2v = \pm (r^2 - y_1^2)^{\frac{1}{2}}.$$

The plus and minus signs correspond to the rear and front edges, respectively. Fig. 5 shows a plot of the equation when $V_g = 60$ mi/hr at 20,000 ft, $r = 3$ mi, $y_1 = 0.6$ mi. Curves are shown for three different values of fall speed.

If, in addition to the wind shear, we include a linear change of wind direction with height $\alpha = Kz$, where K is a positive constant, the equation of the shower edge in the height-time plane becomes

$$\begin{aligned} V_g t &- (a/vK)[(z \sin Kz) - (1 - \cos Kz)K^{-1}] \\ &= \pm (r^2 - [y_1 - (a/vK)(K^{-1} \sin Kz - z \cos Kz)]^2)^{\frac{1}{2}}. \end{aligned}$$

Fig. 6 shows (dotted line) a re-plot of the curve of fig. 5 for $v = 1$ m/sec, and (full line) the effect of allowing the wind direction to change at the rate of 2 deg per 1000 ft. While the slope of the shower echo in the height-time plane is not seriously altered, the lower part of the shower is carried out of the plane.

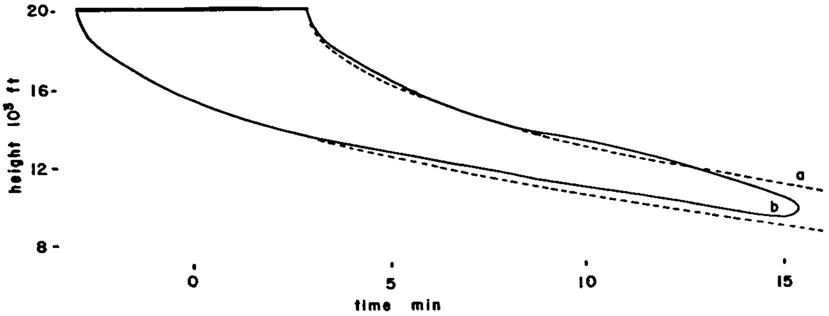

FIG. 6. (a) is curve (c) of fig. 5. (b) shows effect of including wind direction change at rate of 2 deg per 10^3 ft. Shower radius 3 mi, axis 0.6 mi from observation plane at 20,000 ft.

The above example is a shower in which all particles are of the same size. In general, the particle size may increase steadily with horizontal distance back from the leading edge of the shower source. If the rate of increase of size with distance is small, the shower edge is not affected, as illustrated in the top part of fig. 7. If the rate of increase is large, we may have the situation illustrated in the lower part of fig. 7. There is, in principle, an almost unlimited variety of combinations of trajectories — but probably not in practice. (Atlas and Plank [7] have described a very interesting series of observations which bears on the problem.)

To summarize this section, we can state the factors affecting the slope of a precipitation echo on the height-time presentation as follows:

1. Speed of travel of the column of precipitation; a higher speed results in smaller departure of the echo boundary from vertical on the record.
2. Wind shear; greater shear produces larger departure from vertical.
3. Fall speed of precipitation particles; higher fall speed results in an echo which is more nearly vertical.
4. Horizontal structure of the column of precipitation with respect to radar reflectivity.
5. Horizontal dimensions of the precipitation column, and distance between its axis and the plane of observation, in relation to changes of wind direction with height; the smaller the dimensions and the larger the spacing, the more sensitive is the echo slope to changes in wind direction.

5. Conclusion

The preceding discussion of the height-time display used with the AN/APS-34, 1.25-cm wavelength radar is far from complete; a large amount of work still remains to be done. However, it brings out the more important factors inherent in this type of echo display and so helps toward understanding the possibilities and limitations of the system.

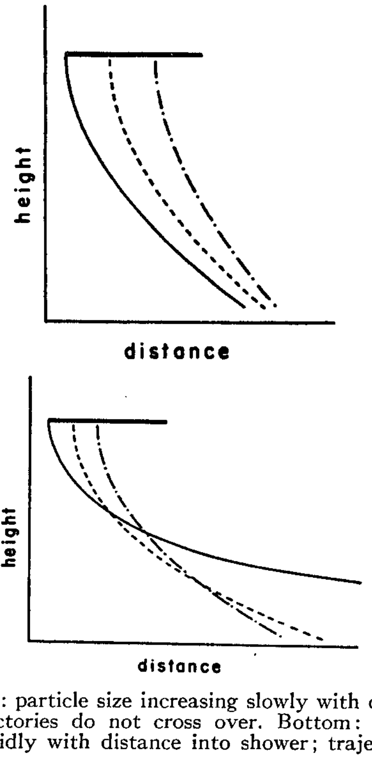

FIG. 7. Top: particle size increasing slowly with distance into shower; trajectories do not cross over. Bottom: particle size increasing rapidly with distance into shower; trajectories cross over.

Without a conversion from the time section to the space section, the AN/APS-34 radar indicates the height of base of certain non-precipitating clouds ("non-precipitating" in the sense that any precipitation which may be formed does not fall very far before evaporating). It may be possible, as suggested by Plank *et al* [1], to make a useful classification of echo types in terms of cloud form; since the AN/APS-34 tends to detect the presence of precipitation rather than "pure" cloud, such a classification may have to be more along the lines of precipitation physics than is the case with the accepted classification of visual clouds. Boucher and Atlas [8] have already made a study of echo characteristics in relation to synoptic situations. The occurrence of sloping echoes gives a qualitative indication of the presence of wind shear; and where the echo extends over a rather large interval of height but disappears from the record at some level above ground, this would indicate and locate in height an important change of wind direction, provided there is no layer of dry air to account for the disappearance. The upper portion of the echo gives a reasonably good measure of the height of a generating level.

The conversion of the time section to the corresponding space section requires knowledge of the speed of travel of the echoing elements. This could be obtained from upper-wind observations or, in the case of precipitation reaching the ground, from a PPI display. Then the true slope of the shower can be measured. For this it is not necessary actually to draw the shower in space section, but only to measure the slope on the time section and divide by the speed of travel. Owing to small irregularities in the echo edge, it is not usually possible to make accurate measurements of slope at a particular level; but an average slope over an interval of height can be determined with fair accuracy. Interpretation of the slope involves the factors of wind shear, particle fall-speed, spatial distribution of particle sizes at the source, shower dimensions and position relative to the plane of observation, and change of wind direction with height. It appears that the AN/APS-34 radar is sufficiently sensitive that we can ignore any variation of sensitivity with height. The few examples illustrated in figs. 5 and 6 suggest that particle fall-speed and wind shear are the most important factors. The effect of the distribution in space of the particle sizes at the source could be very important but, on present knowledge, the importance of the effect cannot be estimated.

We may therefore conclude tentatively that, from measurements of echo slope, either a known wind shear may be used to infer a particle fall-speed, or a known particle fall-speed may be used to infer the wind shear. Whether such a procedure is worthwhile in practice seems doubtful.

It seems more likely that the chief value of the AN/APS-34 radar, with its height-time echo presentation, lies in the qualitative information it furnishes about physical processes — presence and height of generating level, presence of wind shear and large change of wind direction, and existence of an upper cloud layer above a lower one.

The height-time presentation used with a vertically pointing radar-beam is severely limited in being able to show only what passes directly overhead. This makes it desirable to investigate the question of the size of geographical area which is sufficiently well represented by a height-time record of radar echoes.

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REFERENCES

1. Plank, V. G., D. Atlas, and W. H. Paulsen, 1955: The nature and detectability of clouds and precipitation as determined by 1.25-centimeter radar. *J. Meteor.*, **12**, 358–378.
2. Swingle, D. M., 1952: An operational comparison of radar cloud detection at 1.25 cm and 0.86 cm. *Proc. 3rd radar wea. Conf.*, B Montreal, McGill Univ., 21–28.
3. Gunn, K. L. S., and J. S. Marshall, 1955: The effect of wind shear on falling precipitation. *J. Meteor.*, **12**, 339–349.
4. Lhermitte, R., 1954: Étude au radar de la structure des précipitations. *J. sci. Météor.*, **6**, 17–35.
5. Bent, A. E., P. M. Austin, and M. L. Stone, 1950: *Beam width and pulse length in radar weather detection.* (Tech. Rep. 12, Contract W36–039-SC-32038), Cambridge, Mass. Inst. Tech., 61 pp.
6. Marshall, J. S., R. C. Langille, and W. McK. Palmer, 1947: Measurement of rainfall by radar. *J. Meteor.*, **4**, 186–192.
7. Atlas, D., and V. G. Plank, 1953: Drop size history in a shower. *J. Meteor.*, **10**, 291–295.
8. Boucher, R. J., and D. Atlas, 1955: *Cloud and precipitation echoes on the AN/APS-34 (modified) vertical beam radar.* Paper presented at 5th weather radar Conf., Fort Monmouth, N. J.