AN EXPERIMENT IN FORECASTING THE DISPLACEMENT OF 500-MILLIBAR TROUGHS AND RIDGES

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ABSTRACT

A number of simple objective methods of computing the motion of troughs and ridges at the 500-mb level were tested for a fifty-day period, with use of data over and near North America. The most useful methods, which are shown to be significantly better than extrapolation, are Petterssen's jet-stream wave formula, Rossby's long-wave formula, the grid or "box" method, and pressure-kinematic extrapolation.

1. Introduction

As part of an extended project to develop methods of forecasting the formation and intensification of circulatory systems, Petterssen [8] has examined the dynamic and thermodynamic processes involved in development at sea level. A product of this investigation is the hypothesis that "cyclogenesis will occur when and where a well-developed high-level (e.g., 300 mb) positive-vorticity advection region becomes superimposed over a low-level barocline (frontal) zone along which the thermal advection is discontinuous." Tests of this rule [9] indicate that it has definite prognostic value.

Since practical application of the hypothesis involves the relative motions of upper- and lower-level systems, it is important, for the proper timing of the onset of cyclogenesis, to establish the best means of forecasting the rate of movement of these systems. Experience suggests that, in conditions where cyclogenesis is imminent, the movement of surface frontal systems can often be forecast with greater confidence than those of upper-level troughs and ridges.

It was therefore decided to undertake a test of various objective methods of forecasting motion of upper-level systems. The purpose of such a test is mainly to remove some of the uncertainty concerning which methods can be considered useful, rather than to establish hard and fast statistics on the efficacy of an individual method.

The experiment reported here was conducted during the fifty-day period 24 January to 14 March 1953. Computations (from data at 500 mb) were made on a current basis. After the forecast period ended, all computations were carefully reviewed with regard to adequacy of initial data. Recomputation or rejection of some cases on grounds of data availability were not significantly influenced by results of the initial computations, rejections being made primarily on the basis of deficiencies noted when the computation was made.

The participants in the computations reported herein were Miss Dorothy L. Bradbury and Messrs. John O. Ellis, Carl O. Erickson, Mariano A. Estoque, Keith D. Hage, John M. Mihaljan, Chester W. Newton, and William L. Schallert.

2. Statistical presentation of data

Verification data are presented pictorially and, in those cases where it is deemed useful, in the form of tables. In the case of Rossby's formula (section 7, below), it is felt that tabular presentation of data is not justified because of an inadequate basis for comparison with other tests. It is left for the reader to judge the usefulness from the diagrams.

Since some methods require more data or place different requirements upon the character of the flow pattern than others, the sampling of cases was not the same for all methods. As an aid in comparing results, in addition to the more commonly used measures, the coefficient of variation $V = \sigma/\bar{x}$, where $\sigma$ is the standard deviation of errors and $\bar{x}$ the mean observed movement for the appropriate sample, is used. The coefficient of variation serves to describe the broadness of the wings of an error distribution curve and is strongly influenced by extreme errors. A low coefficient of variation, together with a large percentage of errors within relatively small limits (e.g., $\pm 3$ deg lat/day), indicates a good computation method. Methods which were no better than linear extrapolation will not be discussed in detail; a breakdown of the results for those cases can be found elsewhere [17].

It is not intended to imply that the systematic errors given in the tables would be strictly applicable to other samples chosen, for instance, in summer months or in different geographical locations. Subse-
Table 1. Results of forecasts using linear extrapolation of past 24-hr movement (LE) and movement with climatological mean speed (CM). Movements are in deg lat.

<table>
<thead>
<tr>
<th></th>
<th>Troughs (24 hr)</th>
<th>Troughs (48 hr)</th>
<th>Ridges (24 hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long wave</td>
<td>Short wave</td>
<td>All cases</td>
</tr>
<tr>
<td>Number of cases</td>
<td>LE 25</td>
<td>CM 27</td>
<td>LE 78</td>
</tr>
<tr>
<td>Mean motion</td>
<td>7.4</td>
<td>7.4</td>
<td>11.3</td>
</tr>
<tr>
<td>*Mean error</td>
<td>3.6</td>
<td>2.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>4.8</td>
<td>3.1</td>
<td>4.6</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td>66</td>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td>Systematic error</td>
<td>+1.3</td>
<td>0.0</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

* After subtraction of systematic error.

sequent routine use of the systematic errors presented suggests, however, that these are reasonably adequate. In lieu of testing an independent sample, systematic errors were computed independently for the first and second halves of the total number of cases in a given category. The differences were not regarded as significant in comparison with the total errors involved. For example, for 24-hr short-wave trough forecasts by the grid method (table 3), the systematic errors for the first and second halves were 1.1 and 1.6 deg lat/day; a variation of this magnitude has an insignificant effect on the other statistical measures shown. Complete omission of the correction for systematic error in this case (typical of results for other methods) would increase the values of the mean absolute error and the standard error deviation by 10 per cent or less, not affecting the conclusion that the method is superior to extrapolation.

3. Linear extrapolation and climatology

Two methods of forecasting requiring no skill are linear extrapolation of past movement, and movement with a climatological mean speed. Fig. 1 indicates the method of linear extrapolation of trough or ridge motion. Direction lines AB and BC were drawn from the chosen initial point A, making equal angles with the successive past and present positions of the trough or ridge line. The angle of turn of this line was extrapolated by inspection to determine the 24- and 48-hr forecast directions. The angle of turn was not based only on the past rate of turning, but in general was influenced by the character of the large-scale flow pattern. For example, in the case of minor troughs moving through a major wave system, the direction of motion was varied to conform with the large-scale flow pattern expected during the forecast period. For 24-hr linear extrapolation, the forecast movement AE = AB was laid out along direction line AD, and for 48-hr forecasts the movement EF = AB was added. In almost all cases, two points near the extremes of a given trough or ridge line were extrapolated, the mean of their movements being taken as the motion for verification purposes.

After the experiment was completed, “climatological mean” speeds were computed by averaging movements of all cases of a given type, and hindcasts were made by prognosticating all troughs or ridges to move with the appropriate mean speed. The results of one-day linear extrapolation forecasts are presented in fig. 2, and are compared with climatological mean speed forecasts in table 1.

The points in fig. 2 show a high degree of scatter, indicating (particularly in the case of troughs) that appreciable accelerations are common between successive 24-hr periods; thus, extrapolation of past motion is unreliable as a forecast method. Table 1 suggests that, particularly for long-wave troughs, movement with a climatologically-determined average speed may be more reliable. It should, however, be recognized that the “climatological means” used here were determined solely from data within the fifty-day test period.

4. Extrapolation with acceleration

In this method, lines along which forecasts were made were determined as described above, and the forecast movement AE (fig. 1) was computed for 24 hr only, on the basis of past acceleration as indicated by previous 24- and 24-to-48-hr movements AB and BC. In computation of the forecast movements, the acceleration graphs given by Wasko [18] were used. The routine of extrapolation was not the same as followed in the control-line method, for which these graphs were devised, and the results are not to be construed as relating to the efficienacy of the control-line method, which has met with considerable success in practice [10; 18].

The results in this test were clearly inferior to those
TABLE 2. Factors for converting sum of height differences \([Z_5 - Z_t] + (Z_4 - Z_t) + (Z_6 - Z_t)\) across grid, in hundreds of feet, to geostrophic movement in deg lat/day. Grid size is 20 deg lat, true at latitude 45 deg.

<table>
<thead>
<tr>
<th>Latitude</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambert conformal projection</td>
<td>.57</td>
<td>.48</td>
<td>.42</td>
<td>.37</td>
<td>.34</td>
<td>.31</td>
<td>.30</td>
<td>.28</td>
<td>.27</td>
<td>.26</td>
<td>.25</td>
<td>.24</td>
</tr>
<tr>
<td>Polar stereographic</td>
<td>.67</td>
<td>.54</td>
<td>.45</td>
<td>.38</td>
<td>.34</td>
<td>.30</td>
<td>.27</td>
<td>.25</td>
<td>.24</td>
<td>.23</td>
<td>.22</td>
<td>.21</td>
</tr>
</tbody>
</table>

Fig. 2. Computed vs. observed 24-hr displacements of troughs (left) and ridges (right) by linear extrapolation. Dash-dotted lines enclose errors less than 3 deg lat/day.

obtained by linear extrapolation. For 24-hr trough forecasts, the coefficient of variation was about 10 per cent higher than that for linear extrapolation, while only 2/3 as many forecasts fell within a ±3 deg lat/day error range. For 24-hr ridge forecasts, the coefficient of variation was 70 percent, also inferior to linear extrapolation.

5. The grid or box method

This method was originally developed by Riehl [11] for forecasting tropical hurricane displacements, and its use for high-latitude closed circulations has been described by Wilson [19]. For the present purpose, the grid size adopted by Wilson, 20 deg lat square, was used. With the grid center O (fig. 3) on the initial point, the trough displacement OC was assumed to be given by the mean geostrophic wind over the area covered by the grid,

\[
C = \frac{g}{f \Delta n} \sum Z \cdot K(\phi) \left[ (Z_5 - Z_t) + (Z_4 - Z_t) + (Z_6 - Z_t) \right],
\]

where \(Z_t\), etc., represents 500-mb heights in hundreds of feet at corresponding points on the grid. Table 2 gives the conversion factors \(K(\phi)\).

The trough speed determined by use of the grid was taken to be applicable throughout a 24- or 48-hr period. In almost all cases, the speed shown in the statistics is the mean of two computations at points near the extremes of the well-defined portion of a trough or ridge line.\(^3\)

The results are presented in fig. 4 and table 3. The results indicate (compare table 1 and fig. 2) that the grid method is clearly superior to linear extrapolation of past motion, when applied to short-wave troughs, and when the systematic error is subtracted

\(^3\) Separate statistics for “north” and “south” points indicated the same degree of efficiency in both cases. Because of the large size of the grid, the speed obtained is usually not sensitive to the exact choice of the point of computation. The different speeds obtained at north and south points are often useful in indicating changes in orientation of troughs or ridges, though the indicated change may be exaggerated.

Fig. 3. Schematic 500-mb contours with grid superimposed on short-wave trough. See text.
TABLE 3. Statistics on forecasts of trough movement by grid method. Units are deg lat.

<table>
<thead>
<tr>
<th></th>
<th>24 Hours</th>
<th></th>
<th>48 Hours*</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short wave</td>
<td>Long wave</td>
<td>Short wave</td>
<td>Long wave</td>
</tr>
<tr>
<td>Number of cases</td>
<td>100</td>
<td>27</td>
<td>73</td>
<td>114</td>
</tr>
<tr>
<td>Mean motion</td>
<td>11.6</td>
<td>7.4</td>
<td>23.2</td>
<td>19.8</td>
</tr>
<tr>
<td><strong>Mean error</strong></td>
<td>2.8</td>
<td>3.7</td>
<td>5.2</td>
<td>6.2</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.4</td>
<td>4.6</td>
<td>6.4</td>
<td>7.9</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td>30</td>
<td>64</td>
<td>29</td>
<td>40</td>
</tr>
<tr>
<td>Systematic error</td>
<td>+1.4</td>
<td>+7.1</td>
<td>+4.7</td>
<td>+6.8</td>
</tr>
</tbody>
</table>

* Statistics for two-day forecasts are favorably biased, with respect to those for one day, by omission of a number of troughs which, at the end of the two-day forecast period, had either filled or moved over the Atlantic where their locations could not be determined with confidence.

** Systematic error subtracted from forecast motion in each case.

from the computed movement. In Table 3, to make the systematic errors meaningful, cases of transition from long- to short-wave troughs, or vice versa, have been eliminated, except in the last column. In Fig. 4, such transitional cases (21 percent), mostly short- becoming long-wave troughs, were included among the cases represented by dots. These were predominantly forecast to move too rapidly and contribute strongly to the scatter shown.

It was frequently observed that the grid method gave remarkably consistent results when applied to individual troughs; e.g., a large positive forecast error on one day was likely to be followed by an overforecast by about the same amount, for the same trough, on the following days. Relatively large positive errors generally occurred when the method was applied to troughs which deepened rapidly, while good results were obtained for troughs of small amplitude, when the amplitude decreased or remained constant with time. As a rough guide, this method should not be applied when the contours in the neighborhood of the initial point greatly exceed an amplitude of 15 to 20 deg lat.

The grid method was also used for prognosticating the movement of closed low centers, by placing the grid center O (Fig. 3) on the low center with one arm of the grid along a meridian, and computing west-east and north-south components of motion by the expression given earlier and an analogous expression for the mean gradients in the east-west direction. The results are given in the vector scatter diagrams of Fig. 5, where, for comparison, the actual vectorial movements are given along with the error distribution for linear extrapolation of past 24-hr movement. The results by the grid method are much better than those obtained by extrapolation, the mean errors and standard deviations for the meridional and zonal components from the grid being roughly 2/3 of the errors given by extrapolation. There was a tendency to forecast too rapid eastward motion of low centers, the systematic error in zonal displacement being 1.2 deg lat/day.

A study of individual cases confirmed, in general, the results found by Wilson [19] for high-latitude lows. The largest errors were associated with the following synoptic situations:

1. Lows centered in troughs flanked by an intense blocking ridge.
2. Small lows which moved southeastward around a well-developed ridge. These moved more rapidly than forecast in every case.

![GRID METHOD THROUGHS 24 HR](image1)

**GRID METHOD THROUGHS 24 HR (x Long waves) (• All others)**

![GRID METHOD THROUGHS 48 HR](image2)

**GRID METHOD THROUGHS 48 HR**

Fig. 4. Results of trough displacement computations by grid method; left: 24 hr; right: 48 hr. In left part, crosses indicate troughs of long-wave character at computation time; in both parts, dots include short-wave troughs and troughs in transition between short- and long-wave character. Dashed line is "perfect forecast" line displaced by amount of systematic error for short-wave troughs, and dash-dotted lines enclose points whose forecast error, after correction for systematic error, was less than 3 deg lat/day.
3. Lows centered in long-wave troughs between strong ridges.
4. Cases in which rapid and intense sea-level development occurred just ahead of the upper low.

Demands on data being small, the grid method is applicable to practically all short-wave troughs. Grid forecasts are particularly useful in the case of "northwest troughs," for which neither past history nor the details of the wind field may be known with confidence.

6. Petterssen's wave formula

The displacement given by Petterssen's wave formula [7],

\[ C = \frac{L}{1 + \left( \frac{L}{2\pi B} \right)^2} \]

was determined from a nomogram devised by Fletcher [16]. In the formula, \( C \) is the speed of the trough or ridge line, \( u \) the maximum wind speed, \( \beta \) the northward variation of the Coriolis parameter at the corresponding latitude on the trough or ridge line, \( L \) the wavelength, \( \gamma \) the angular deviation of the trough or ridge line from the meridian, and \( B \) the half-width of the current. All quantities were evaluated from 500-mb charts.

Fig. 6 illustrates the determination of the parameters involved. In all cases, isotachs were drawn, although this is not necessary when wind observations are plentiful. Whenever possible, the half-width was taken as the mean of the half-widths (fig. 6a) on the two sides of the wind maximum. In many cases, as in fig. 6b, two or more wind maxima were present, and the half-width in such a case was measured on one side only.

Figs. 6c to 6f illustrate some of the types of wave patterns encountered which are not sinusoidal throughout. Since the formula was derived on the basis of a sinusoidal wave shape, which is supposed to describe the character of the flow in the neighborhood of a trough or ridge, it is necessary in such cases to determine an "effective wavelength" by sketching a continuation of the portion of the wave pattern which is sinusoidal, as indicated by the dashed lines in the figure. In the cases represented by figs. 6c and 6e, for example, it is clear that the curvatures of the upstream ridges are too weak or too strong with respect to those of the troughs, indicating that the observed half-wavelength upstream is too large in the first and too small in the second case. Although a certain degree of subjectivity is involved in "completing" the sine wave, the results obtained show that this procedure is well justified. The wavelength was, in the majority of cases, obtained by doubling the half-wavelength upstream from the trough or ridge being forecast.

In verification, observed displacements were measured normal to trough or ridge lines except in those cases where there was rotation, the measurements then being made along an arc normal to initial and final positions. Table 4 and fig. 7 show that, when the systematic error is subtracted from the forecast movement, Petterssen's wave formula gives much better results than linear extrapolation in the case.
Table 4. Verification of Petterssen's wave formula. Numbers in parentheses refer to results for linear extrapolation (in about 10 per cent of cases for troughs, linear extrapolations could not be made with confidence). Units are deg lat.

<table>
<thead>
<tr>
<th></th>
<th>24 hr</th>
<th>Troughs 36 hr</th>
<th>48 hr</th>
<th>24 hr</th>
<th>Ridges 36 hr</th>
<th>48 hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Number of cases</td>
<td>86</td>
<td>72</td>
<td>64</td>
<td>31</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>Mean motion</td>
<td>11.2</td>
<td>16.7</td>
<td>21.6</td>
<td>6.9</td>
<td>10.5</td>
<td>12.9</td>
</tr>
<tr>
<td>**Mean error</td>
<td>1.9(3.5)</td>
<td>3.0(5.4)</td>
<td>5.8(8.2)</td>
<td>2.0(2.5)</td>
<td>2.8(3.9)</td>
<td>3.3(4.6)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.4</td>
<td>4.0</td>
<td>7.1</td>
<td>2.5</td>
<td>3.9</td>
<td>4.5</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td>22(41)</td>
<td>24(41)</td>
<td>33(48)</td>
<td>36(47)</td>
<td>38(45)</td>
<td>35(47)</td>
</tr>
<tr>
<td>Systematic error</td>
<td>+0.9</td>
<td>+1.0</td>
<td>+1.5</td>
<td>+2.5</td>
<td>+3.2</td>
<td>+4.6</td>
</tr>
</tbody>
</table>

* Decrease in number of cases with time results from inability accurately to verify locations at end of longer forecast periods, generally due to flattening out of the most rapidly moving systems.
** Systematic error corrected for.

of troughs, while somewhat less superiority is evident in the case of ridges. The results here are comparable with those obtained by Johannessen and Cressman [5]. In the present case, however, less restriction was placed upon the character of the flow, considerable departure from sinusoidal shape being allowed.

Troughs (15 cases) located west of 110°W and south of 45°N had a systematic error of +2.2 deg lat/day as compared with +0.9 for all troughs, while 13 ridges west of 120°W (mostly over Alaska and British Columbia) showed a systematic error of +3.5 deg lat/day compared with +2.5 for all ridges. These figures agree well with the systematic positive errors determined by Johannessen and Cressman, 1 deg long/day for troughs, 3 deg long/day for ridges east of the Rockies, and 5 deg long/day for ridges over the Rockies.

The largest errors tended to be associated with short wavelengths and low wind speeds; with short wavelengths, the formula is quite sensitive to errors in determining L. No relationship was found between errors and wave amplitude, some of the best forecasts being made for very flat troughs. In most cases involving large errors, it was found that significant changes of u, L or B occurred during the forecast period, the changes being of the right sign to account for the forecast error. No reliable synoptic indications were found for foreseeing the changes in the parameters.

7. Rossby's long-wave formula

In computations by Rossby's long-wave formula [12],

$$C = u - \frac{\beta L^2}{4\pi^2} = \frac{\beta}{4\pi^2} (L_0^2 - L^2),$$

where $L_0$ is the stationary and L the actual wavelength, the diagram devised by Byers [1] was used. Troughs were classified as long-wave troughs according to the criteria described by Cressman [2]. In the present test, no restriction was made regarding acceleration, as in Cressman's test.

The 600-mb zonal wind $u$ was taken from the

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Fig. 6. Determination of parameters in Petterssen's wave formula. In (a) and (b), isoclinas (thin lines) labeled in knots. Heavy curves represent contour pattern, and dashed curves continuation of sine waves representative of curvature in troughs.
ZWNH (Zonal winds, northern hemisphere) teletype transmission of data computed at Andrews Air Force Base, the value corresponding to maximum $L_S$ (or maximum $u$ where maximum $L_S$ was ambiguous) being used. Three-day means, giving slightly better results than one-day wind data, were used in the verifications. In cases where parts of the wave in question were located in two of the 120-deg longitude zones transmitted, the zonal wind was weighted according to the portion of the wave lying in each zone. The wavelength $L$ was measured from the trough computed to the next major trough upstream. On some occasions, because of large amplitudes, $L_S$ was computed from constant absolute-vorticity trajectory data by the procedure outlined by Cressman [2].

The result of a computation was considered as the movement of the downstream trough, rather than the mean motion of the upstream and downstream troughs. Attempts to take into account the motion of the upstream trough gave worse results. Where forecast motion was negative, retrogression was indicated; no attempt was made to verify such cases numerically.

During the test period, only 54 troughs met the customary restrictions of relatively long wavelength, large amplitude, and relative symmetry, and could be adequately verified. The results are shown in fig. 8. Of the 54 24-hr forecasts, 34 verified within ±3 deg lat/day. For troughs having closed lows in middle latitudes, represented by the circles in fig. 8, there was no evident relationship between computed and observed motion. In the seven cases wherein there was a minor wave between sinusoidal major troughs, the downstream troughs were forecast to move too slowly. During much of the period there were blocking situations or the wave pattern departed markedly from sinusoidal, restricting the use of Rossby's formula to the relatively small number of cases re-

**Fig. 7**. Results of computations using Pettersen's wave formula; top left: 24-hr troughs; top right: 48-hr troughs; bottom left: 24-hr ridges; bottom right: 48-hr ridges. Dash-dotted lines enclose points whose forecast error (after correction for systematic error) was less than 3 deg lat/day.
ported above. An attempt to extend the use of the formula to troughs of moderately large amplitude, which were clearly not long-wave troughs, did not give useful results. Fig. 8 includes a number of marginal or hybrid long-wave situations which contribute to the scatter about the “correct forecast” line.

The results shown are inferior to those reported by Cressman [2]. However, the cases examined by Cressman were, by choice, those in which little acceleration occurred. In the present group of cases, some acceleration, often pronounced, occurred in most instances and, as shown later, was generally of the sign indicated by the forecast.

8. Constant absolute-vorticity trajectories

Constant absolute-vorticity trajectories (CAVT) [13; 4] were determined from the wind at points of inflection, by use of tables [15]. Computations were attempted only in those cases where the current was broad and approximately sinusoidal at least one-quarter wavelength up- and down-stream from the inflection point. The average wind over an 800-ft contour interval was reduced by 20 per cent to approximate the 600-mb wind speed; results were, on the whole, about the same when a 400-ft contour interval was used.

Because of difficulties in interpreting the results in terms of individual wave elements, the procedure followed in verification was to determine the time and location of a CAVT crest or trough, and to compare this with the interpolated observed crest or trough location at the appropriate time and latitude, at 1/4 and 3/4 wavelength downstream from the initial point. The results are indicated in Table 5 and Fig. 9.

It should be noted that the statistics in Table 5 concern only those cases which could be verified, and particularly for the ridges at 3/4 wavelength a significant number dropped out because of the effects of minor perturbations or because the large-scale wave pattern was completely out of phase with the portion of the trajectory concerned. Thus, the apparent good results (considering the long time involved) for (3/4)-wavelength forecasts cannot be taken at face value. Some of the large errors at 3/4 wavelength

<table>
<thead>
<tr>
<th>Cases verified</th>
<th>31</th>
<th>20</th>
<th>13</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean time of</td>
<td>24</td>
<td>66</td>
<td>19</td>
<td>68</td>
</tr>
<tr>
<td>trajectory (hr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>3.7</td>
<td>8.8</td>
<td>3.2</td>
<td>6.5</td>
</tr>
<tr>
<td>Systematic error</td>
<td>0.9</td>
<td>6.0</td>
<td>-0.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Fig. 9. Error frequency diagrams for (a) trough and (b) ridge locations at 1/4 wavelength downstream from inflection point, for constant-vorticity trajectories.
involved either cases of discontinuous retrogression or blocking situations.

In the case of both troughs and ridges, it was noted that all (1/4)-wavelength trajectories ending over the Rocky Mountains (12 cases) gave forecast positions too far east, the systematic error in location being +3 deg lat; for the remaining cases terminating elsewhere, the systematic error was —2.5 deg lat.

9. Streamline-isotherm amplitude relationship

A test of the relationship [14]

\[ \frac{A_T}{A_S} = \frac{u}{u - C}, \]

where \( A_T \) and \( A_S \) are amplitudes of trajectories and streamlines, respectively, \( u \) is the wind speed and \( C \) the wave speed, was made under the customary assumption that \( A_T \) may be represented by the isotherm amplitude. Three methods were used: the amplitudes of contours and isotherms at 500 mb, contours at 500 mb and mean isotherms 500–1000 mb, and contours at 700 mb with mean isotherms 500–1000 mb. No numerically useful relationship between forecast and observed movements was evident. Phase relationships, not being numerically verifiable, were not tested.

10. Pressure-kinematic extrapolation

In use of Petterssen's formula for pressure-kinematic extrapolation [6],

\[ C = -\frac{\partial Z}{\partial x} \frac{\partial^2 Z}{\partial x^2} \]

where \( C \) is trough or ridge speed, \( Z \) the 500-mb height, and \( x \) the direction of displacement, the time derivatives were replaced by the 12-hr height changes prior to forecast time. The space derivatives were usually measured from a graphically constructed mean of the current and 12-hr previous maps. In cases where the averaging process suppressed the amplitude of the wave pattern (i.e., when the waves at 12-hr intervals were nearly out of phase), space derivatives were measured from the latest chart and displaced so as to coincide with trough or ridge positions 6 hr earlier. The displacement thus computed was applied to the position at forecast time, forecast and observed displacements being computed along a normal to the appropriate trough or ridge line.

Computations were made at one to three points on a given trough line, and averaged. Results are shown in fig. 10 and table 6. The data for troughs in table 6 and fig. 10 include only those cases with large amplitudes, most of these being long-wave troughs. The pressure-kinematic technique was found to be of no value for small-amplitude troughs, due to difficulties in obtaining accurate values for either space or time derivatives. For ridges, table 6 includes both large- and small-amplitude cases; the accuracy for large-amplitude ridges was roughly the same as for large-amplitude troughs.

This method should clearly be applied only to those cases where troughs and ridges are well defined and where the time changes are large enough not to be seriously affected by observational or analytical errors. Due allowance should be made for changes in structure which affect the tendency field, and very large differences between structures on individual and mean charts should be treated with caution.

11. Fjörtoft's barotropic graphical technique

Twenty-four hour forecasts of the 500-mb flow pattern were made by the graphical technique of numerical prognosis, based on conservation of vorticity, developed by Fjörtoft [3]. The simplification was made that the effect of variation of the Coriolis parameter was taken into account by advecting the relative vorticity in the mean-flow field, then uniformly shifting the vorticity pattern 4 deg long west-

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**Table 6.** Statistics on 24-hr trough and ridge forecasts by pressure-kinematic formula. Units are deg lat.

<table>
<thead>
<tr>
<th></th>
<th>Troughs</th>
<th>Ridges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cases</td>
<td>85</td>
<td>41</td>
</tr>
<tr>
<td>Mean motion</td>
<td>9.4</td>
<td>7.5</td>
</tr>
<tr>
<td>Mean error</td>
<td>2.9</td>
<td>4.3</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.7</td>
<td>6.8</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td>39</td>
<td>91</td>
</tr>
<tr>
<td>Systematic error</td>
<td>—0.9</td>
<td>+2.6</td>
</tr>
</tbody>
</table>

**Fig. 10.** Computed vs. observed 24-hr displacements of troughs by pressure-kinematic formula. Dashed-dotted line enclose errors (after subtraction of systematic error) less than 3 deg lat/day.
ward (for a 24-hr forecast). The "grid interval" used was 600 km.

The results, as regards trough and ridge motions, proved to be inferior to those obtained by linear extrapolation. Short-wave troughs were, in general, forecast to move too rapidly. Both low and high centers were, on the average, forecast to move to the left of the actual path (15 deg for lows). The poor results may in part be due to difficulty in interpreting the proper location of trough or ridge lines in the forecast contour pattern; in some situations the procedure resulted either in an apparent flattening of troughs or in the introduction of fictitious troughs in the final pattern. At any rate, the attempted abstraction of positions of troughs and ridges does not necessarily reflect the success of the method in predicting the broad-scale flow pattern.

These results should not be taken to indicate what ultimately can be obtained by graphical integrations. Later experiments have shown conclusively that the accuracy of the graphical integrations is considerably increased by use of a smoothing interval of approximately 1000 km rather than 600 km.

12. Summary

Four of the above methods are significantly better for trough forecasts than is linear extrapolation: the

![Graphs showing computed vs. observed acceleration for 24-hr forecasts for grid method (top left), Petterssen's wave formula (top right), Rossby's wave formula (bottom left), and pressure-kinematic formula (bottom right). Dashed lines enclose errors less than 3 deg lat/day after subtraction of systematic error. In top left, dots indicate short-wave troughs and crosses troughs in transition between short- and long-wave. In top right, dots are short- and crosses long-wave troughs. In bottom left, troughs having closed centers have been eliminated.](image_url)
grid method, Petterssen's and Rossby's wave formulae, and pressure-kinematic extrapolation. Constant-vorticity trajectories, when used with care, give useful results; however, their interpretation is often subjective and their value is difficult to assess statistically.

For ridge movements, only Petterssen's wave formula (after correction for systematic error) appeared to give results significantly superior to linear extrapolation.

Linear extrapolation of past movement being used as a basis for comparison, the usefulness of a forecasting method may be judged by whether it gives a satisfactory indication of acceleration or deceleration. Fig. 11 shows diagrams of computed versus observed accelerations, for 24-hr periods, for the four best methods of trough-speed computation enumerated above. Acceleration, as such, is not computed by any of the methods, but may be inferred from the difference between the past speed and the forecast speed. In fig. 11, "computed acceleration" refers to the forecast 24-hr movement minus the past 24-hr movement, and "observed acceleration" to the observed movement during the forecast period minus the past observed movement. In all cases, fairly good correlations are evident.

For short-wave troughs (fig. 11, top left), the grid method indicated the sign of acceleration correctly in 98 cases and incorrectly in 54, while no acceleration was indicated for eight cases in which acceleration occurred. Large accelerations were well indicated.

Petterssen's wave formula (fig. 11, top right) indicated the correct sign of acceleration in practically all cases, both for long- and short-wave troughs. The scatter about a 1:1 line is smaller than for any other method, indicating that Petterssen's equation is more reliable than other methods for those cases with sufficient wind data for its application.

Out of a total of 77 cases (Fig. 11, bottom left),° Rossby's formula indicated the sign of acceleration of long-wave troughs correctly in 53 cases and incorrectly in 16, 8 cases having either no observed or no computed acceleration.

The lower right portion of fig. 11 shows that the pressure-kinematic extrapolation formula gives useful indications of the acceleration of troughs. Use of this method must be restricted to large-amplitude systems and to those areas where changes of the height field are known accurately.

Although in a statistical sense the above computation methods give good results, in some individual instances situations can be recognized in which computations (or extrapolation) are likely to give misleading results. Some of these are mentioned above in regard to the grid method, which assumes essentially that the relative vorticity field is advected with the mean geostrophic wind over a fairly large area surrounding a trough or low center. It was expected beforehand that this method would not be useful for long-wave troughs, since the variation of the Coriolis parameter is important in that case.

Although Petterssen's formula, which involves few assumptions and utilizes more information than the other methods, gives best results in the long run, there are certain situations which must be treated with caution. A particular example arises in the case of deep troughs with strong cold advection to the rear and weak thermal advection ahead. Such troughs (on rare occasions) may remain nearly stationary, although appreciable movement may be computed from the formula. This suggests that the thermal advection pattern is connected with an asymmetrical divergence field contributing to retardation of the trough.

The general results indicate that the utilization of ordinary extrapolation procedures in prognostication of trough movement is rarely justifiable, since in practically all cases one of the four methods above may be applied, and all are statistically superior to extrapolation of past movement. The selection of the method most likely to give best results is largely a function of the character of the flow pattern and availability of data.

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REFERENCES

° This diagram includes data for the period 24 January to 1 May 1955. For only the 50 days of the experiment, the correlation was essentially the same.


