COMPARATIVE STUDIES OF RELATIVE VORTICITIES COMPUTED FROM GEOSTROPHIC AND OBSERVED WINDS

By Roy E. Peterson

New York University[1,2]

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ABSTRACT

Fixed networks of rawinsonde stations were used to compute geostrophic relative vorticity from reported heights and observed relative vorticity from components of reported winds at 500 mb. The geostrophic vorticity was computed on the two different scales of 600 km and 300 km; the observed vorticity was computed on a 600 km scale. An analysis was made also of the vorticity variance produced by height errors, wind errors and small-scale wind fluctuations.

Comparison of geostrophic vorticity on different scales indicates that grid size has a very important effect upon the computed value of geostrophic vorticity. Comparison of observed and geostrophic vorticity on the same scale shows some significant differences; many of these differences, however, can be explained by the effect of height and wind errors upon the computation of each vorticity.

1. Introduction

Within recent years, as the vorticity concept has come into increasingly greater use, a common technique for computing the vertical component of relative vorticity has been to apply the geostrophic approximation to the height field of a constant pressure chart. The geostrophic technique not only provides a convenient method to compute vorticity values directly from the analysis of scalar height field, but also acts as a filter to eliminate undesirable wave solutions in the results obtained from the prediction equations. Charney (1948) established the use of geostrophic vorticity in numerical weather prediction by pointing out that the geostrophic deviation is negligible for large-scale perturbations where the mean horizontal distance between trough and wedge is of the order of 1000 km.

The considerable number of synoptic studies which ve been made of the geostrophic wind deviation ɛ.g., Houghton and Austin, 1946; Neiburger *et al*, 1948; Godson, 1950) has concluded that these deviations not only occur systematically but are at times quite large. Reed (1951), from a study of geostrophic vorticity at 4000 ft and 10,000 ft, concluded that geostrophic vorticity may be substituted for the actual vorticity whenever the spacing and curvature of the contours can be determined with a minimum of ambiguity. By pointing out the important effect of mean temperature errors in radiosonde observations, Hovmöller (1952) concluded that it was inadvisable to use unsmoothed height values for computing geostrophic vorticity. Newton (1954) also suggested that the height field be properly smoothed because the use of second-order differentials of the height field to compute geostrophic vorticity can otherwise produce large arbitrary variations.

The common practice in previous vorticity deviation studies has been to use smoothed analyses of height and wind data to compute the geostrophic- and observed-wind vorticity. The great advantage of such smoothing is that correction is applied to erroneous and unrepresentative observations; the disadvantage, however, is the large degree of subjectivity involved in the many decisions of accuracy and representativeness necessarily made in the analysis of a constant pressure chart. Since such a wide degree of discretion is afforded to the analyst, the degree of correspondence between geostrophic and observed wind vorticity can be as variable as the ability, experience and synoptic idiosyncrasy of the individual analyst. In addition, the scale of synoptic analysis is rather variable; where data are more dense, the analysis is effectively on a smaller scale than where sparser data necessitate a greater degree of interpolation. Then too, vorticity studies based upon the small sample of a few synoptic situations may lack complete generality in their results.

In the study presented here, an objective method of computing vorticity from fixed station networks was applied directly to reported height and wind data at 500 mb for the entire year 1953. The main purpose of this study was to test whether there is a significant difference between geostrophic and observed vorticity; another aim was to investigate the effect upon the size

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[2] Present affiliation: General Mills, Inc.

of the geostrophic vorticity which results from changing the scale, or the differentiation distance, used in the computation. In addition, an analysis was made of the vorticity variance arising from height errors, wind errors and small-scale wind fluctuations.

2. Methods

The objective method of vorticity measurement depended upon a choice of networks of rawinsonde stations, whose geometry provided approximate systems of perpendicular grid axes. Two such networks were selected, one twice the size of the other, and centered at a common station so that all vorticity values were computed for the same grid point. The grid axes for each network were drawn to minimize the mean distance of the outside stations from their corresponding grid positions, so that the two sets of axes do not exactly coincide. For the smaller network, the outside grid points are each 300 km from the grid center; for the larger network, 600 km.

Fig. 1 shows the smaller network of stations which was used to compute the 600-km observed vorticity from reported winds and the 300-km geostrophic vorticity from reported heights. This grid is centered at North Platte, Nebraska, and includes as outside stations Denver, Colorado; Rapid City, South Dakota; Omaha, Nebraska; and Dodge City, Kansas. The larger network, shown in fig. 2, was used to compute the 600-km geostrophic vorticity from reported heights

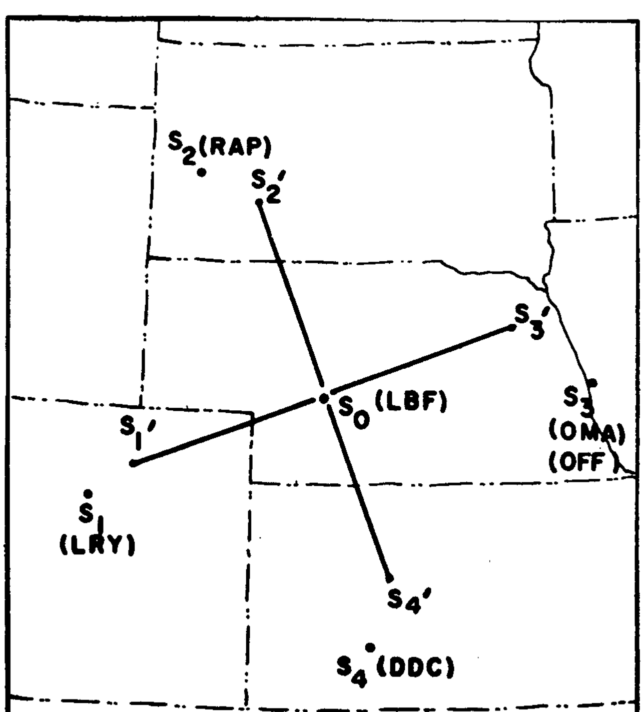

FIG. 1. Rawinsonde network used to compute 600-km observed vorticity from reported winds and 300-km geostrophic vorticity from reported heights at 500 mb.

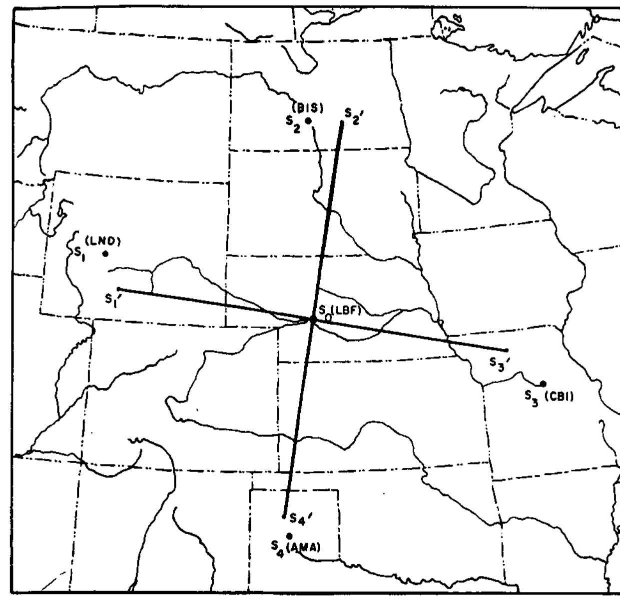

FIG. 2. Radiosonde network used to compute 600-km geostrophic vorticity from reported heights at 500 mb.

of a grid whose outside stations are Lander, Wyoming; Bismarck, North Dakota; Columbia, Missouri; and Amarillo, Texas.

Since the data-reporting stations do not exactly coincide with their respective grid positions, a linear process of interpolation and extrapolation was used to apply small corrections. In brief, lines were drawn joining two station values, and by interpolation the value was obtained at the intersection of this line with the grid axis. Then, depending upon whether this intersection was outside or within the proper grid distance, interpolation or extrapolation was used to obtain the value at the grid point.

The observed relative vorticity can be computed from the basic equation

$$\zeta = \partial v/\partial x - \partial u/\partial y \qquad (1)$$

by first considering the finite-difference form

$$\zeta = (v_3 - v_1)/\Delta x - (u_2 - u_4)/\Delta y, \qquad (2)$$

where u and v are wind components with respect to a Cartesian system. Since both Δx and Δy are equal to 600 km and the wind components are expressed in knots,

$$\zeta = 0.086\,(v_3 - v_1 - u_2 + u_4) \times 10^{-5}\ \text{sec}^{-1}.$$

This equation is written in terms of the wind components at the grid points; if the grid values are expressed in terms of station values, one obtains

$$\zeta = 0.086\,(0.69\,v_3 - 1.11\,v_1 + 0.14\,v_2 + 0.28\,v_0 - 0.86\,u_2 + 0.96\,u_4 - 0.10\,u_0) \times 10^{-5}\ \text{sec}^{-1}, \qquad (3)$$

so that the observed relative vorticity can be computed directly from the reported values. It may be noted that interpolation introduces wind components from the central station into the calculations.

The equation for computation of the geostrophic relative vorticity can be derived from the basic equation (1) by considering the geostrophic wind equations for the components, u and v;

$$u = (-g/f)\,\partial h/\partial y \quad (4)$$

and

$$v = (g/f)\,\partial h/\partial x, \quad (5)$$

where g is the standard acceleration of gravity, f is the Coriolis parameter, and h is the geopotential height of the isobaric surface. If we substitute (4) and (5) into equation (1) and neglect the term $\partial f/\partial y$, we obtain as the geostrophic vorticity

$$\zeta_g = (g/f)(\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2).$$

The term $(\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2)$ can be approximated, however, by the expression $L^{-2}(h_1+h_2+h_3+h_4-4h_0)$, where $L = \Delta x/2 = \Delta y/2$. By substitution, we obtain

$$\zeta_g = (g/fL^2)(h_1 + h_2 + h_3 + h_4 - 4h_0). \quad (6)$$

In this study, L is defined as the scale of the geostrophic vorticity, and Δx (or Δy), as the scale of the observed vorticity. Thus, the smaller network of stations could be used to compute both the 300-km geostrophic vorticity and the 600-km observed vorticity.

For the 600-km geostrophic vorticity, where $L = 6 \times 10^5$ m, $g = 9.81$ m sec^{-2}, and $f = .96 \times 10^{-4}$ sec^{-1}, h is expressed in units of ten feet, and the equation is written in terms of reported station values:

$$\zeta_g = 0.087\,(0.86\,h_1 + 0.80\,h_2 + 1.08\,h_3 + 1.13\,h_4 - 3.87\,h_0) \times 10^{-5}\,\text{sec}^{-1}. \quad (7)$$

And similarly for the 300-km geostrophic vorticity; the following equation can be derived in terms of station values:

$$\zeta_g' = 0.347\,(1.11\,h_1 + 1.12\,h_2 + 0.97\,h_3 + 0.96\,h_4 - 4.16\,h_0) \times 10^{-3}\,\text{sec}^{-1}. \quad (8)$$

3. Data

A total of 385 cases, for which all three vorticity values could be computed, were selected by examination of all 500-mb data for the year 1953, reported by the nine stations comprising the two networks. These data were obtained from original teletype data, Daily Upper Air Bulletins, and from microfilmed copies of WBAN Form 33, Summary of Constant Pressure Data. The wind data used were those reported for the 500-mb surface; with the exception of Omaha where only sparse pibals were available, so that RAWIN data from Offutt AFB were used by interpolating the wind there for the RAOB height reported by Omaha.

WBAN 33 would have been used exclusively but for the coding of wind directions on this form to only 16 points, so that its usefulness in this respect was limited to a check for gross errors in the transmitted values of wind direction and to occasionally filling in gaps where the data were otherwise complete. The height values from WBAN 33, listed on this form in terms of meters, were converted to units of ten feet and used to check the transmitted height data and to complete occasional gaps. This rigorous checking system eliminated many errors of coding and transmission, so that the data used in the vorticity calculations were the best available.

The following is a list of the monthly totals of cases for the successive months January–December: 18, 14, 15, 36, 27, 39, 47, 47, 44, 40, 31, 27. The summer months have the larger numbers of cases; the winter months, the smaller number. To eliminate some of the seasonal bias possibly resulting from this uneven distribution, and also to attempt to find any noticeable seasonal effects, the cases were divided into two groups: a "summer" group comprising the months May–October and a "winter" group of the remaining months. The summer cases total 244, the winter cases 141.

4. Analysis of geostrophic and observed vorticity

The relationship between the 600-km vorticities, ζ_g and ζ, is shown for the separate winter and summer groups of cases by scatter diagrams (figs. 3 and 4). Table 1 summarizes the following statistics: n—the

TABLE 1. Statistical analysis of ζ_g and ζ (vorticity in units 10^{-5} sec^{-1}).

Group	n	$\bar{\zeta}_g$	$\bar{\zeta}$	σ_g	σ	r	S_g	S	RMS
Winter	141	0.97	−0.33	3.55	2.98	.764	2.28	1.92	2.65
Summer	244	1.16	−1.25	2.69	2.60	.653	2.04	1.97	3.26
Total	385	1.09	−0.91	3.03	2.78	.689	2.20	2.02	3.05

number of cases in each group; $\bar{\zeta}_g$ and $\bar{\zeta}$—the arithmetic means; σ_g and σ—the standard deviations of the respective vorticities; r—the linear correlation coefficients; S_g—the standard error of estimating ζ_g from ζ; S—the standard error of estimating ζ from ζ_g; RMS—the root mean square of the absolute differences between the two vorticities.

There are some general seasonal differences in the distributions of each vorticity and in the relationships between them. For the winter cases, the correlation coefficient is considerably larger; the difference between the vorticity means is less; the standard deviations of both vorticities, especially the geostrophic, are larger; the RMS value is 20 per cent less than in the summer group. There would thus appear to be a small but significant difference in the seasonal distributions of each vorticity. In the summer, for example, the

500-mb circulation is less well-developed, as shown by the smaller standard deviations of each vorticity. In addition, the geostrophic and observed vorticities are not as well correlated in summer as in winter, where the correlation is significant.

If both groups of cases are considered together, the mean geostrophic vorticity is positive and the mean observed vorticity is negative. The separation of the means is large for this sample size. Also, the root-mean-square of the absolute differences between the vorticities is quite large, approximately equal to the standard deviation of each vorticity distribution. The standard error of estimating one vorticity from the other is equal to about 70 per cent of the vorticity standard deviations. The correlation coefficient indicates that, considering the entire year, the variance of one vorticity accounts for just about one half the variance of the other vorticity. There is, therefore,

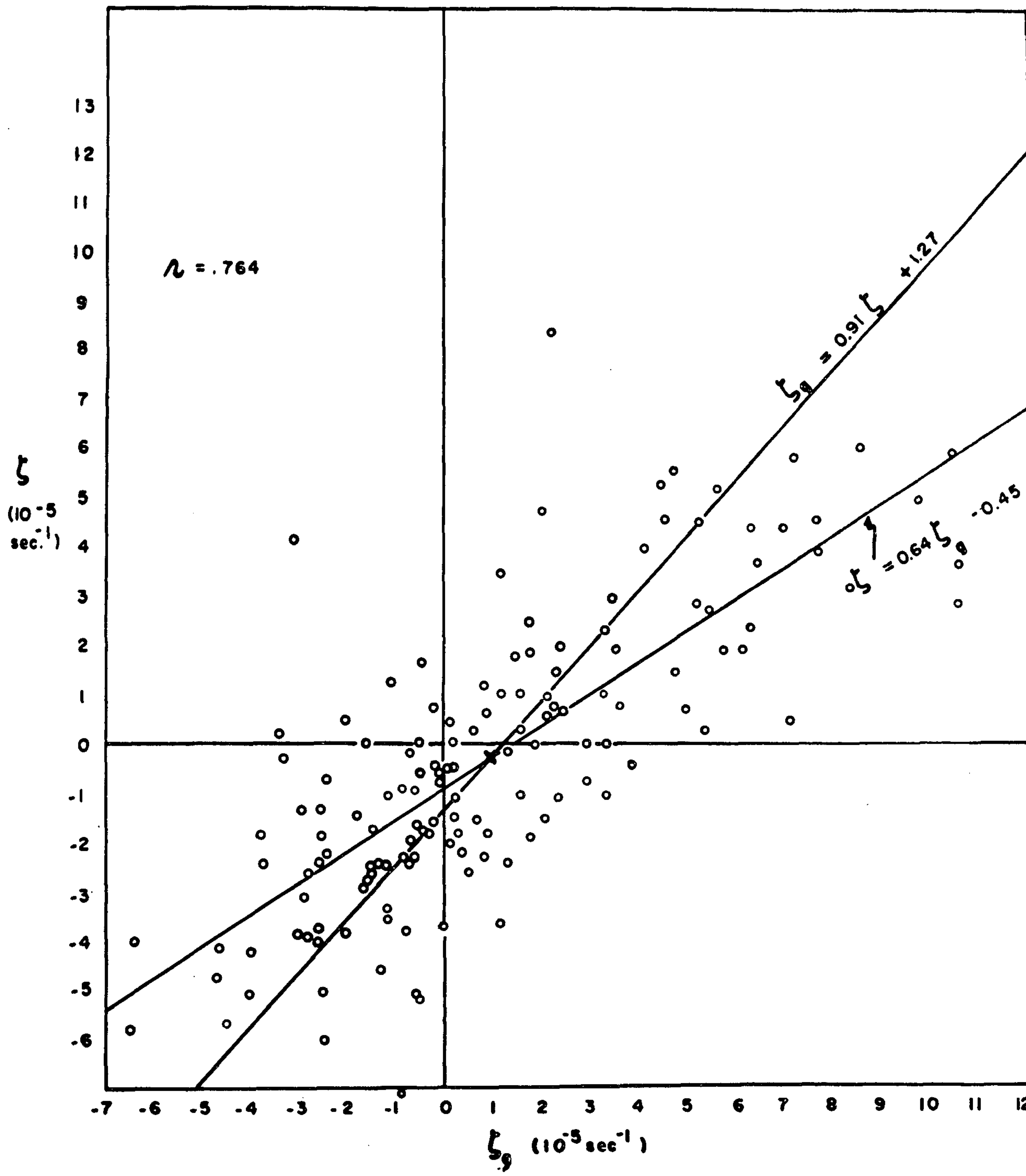

FIG. 3. Scatter diagram. ζ *vs.* ζ_g. Winter cases.

a large residual variance which can be assumed to result from real vorticity differences, from the effects of height errors, wind errors and small-scale fluctuations, and to a lesser extent from the non-coincident and different-sized computational networks used in this study and from the small interpolative corrections which were applied.

5. Analysis of different-scale geostrophic vorticity

The relationship between the different-scale geostrophic vorticities, ζ_g (600 km) and ζ_g' (300 km), is shown for the separate winter and summer categories by scatter diagrams (figs. 5 and 6). In table 2 is a statistical summary similar to that of the previous table; here S_g^* is the standard error of estimating ζ_g

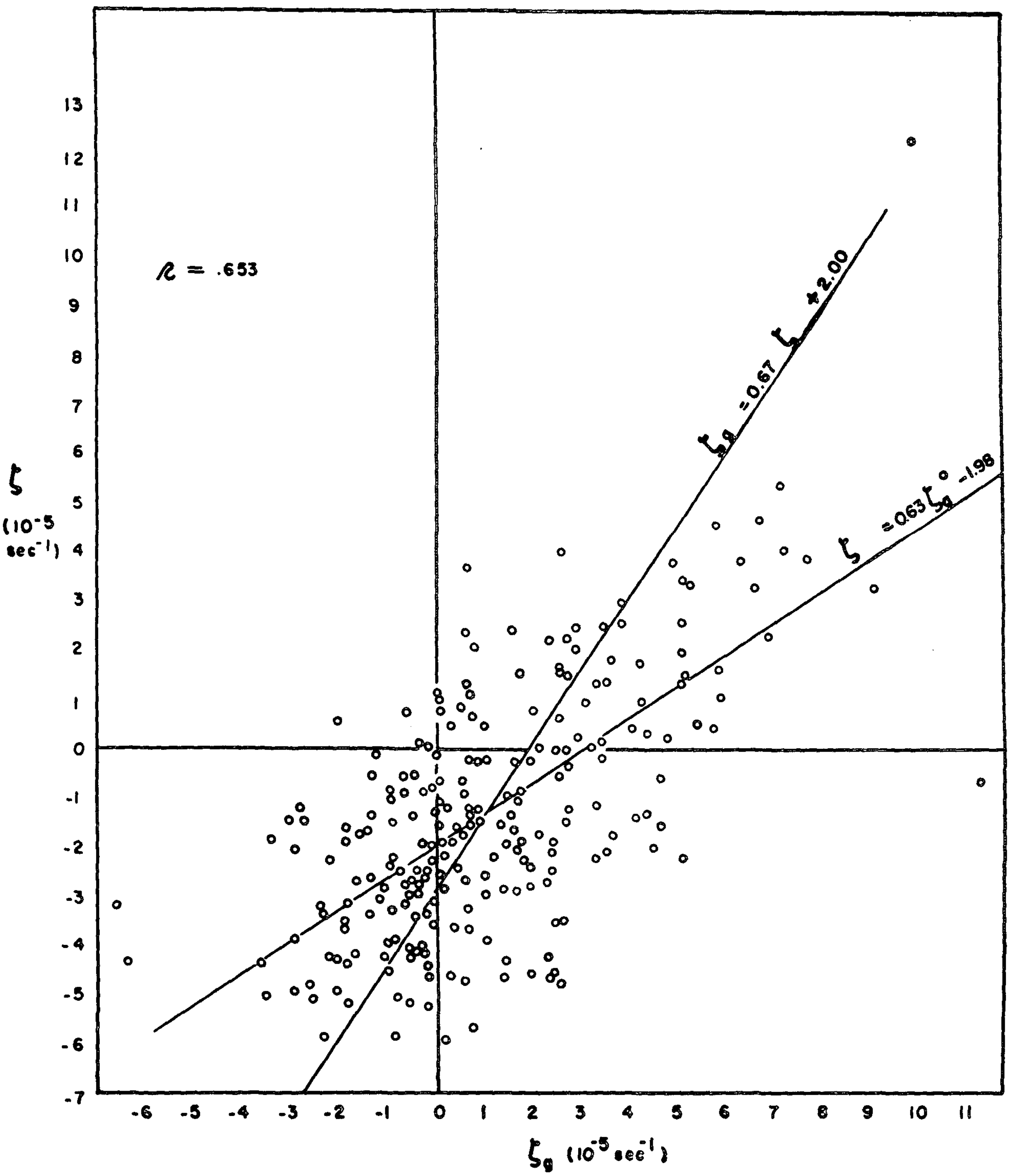

FIG. 4. Scatter diagram. ζ *vs.* ζ_g. Summer cases.

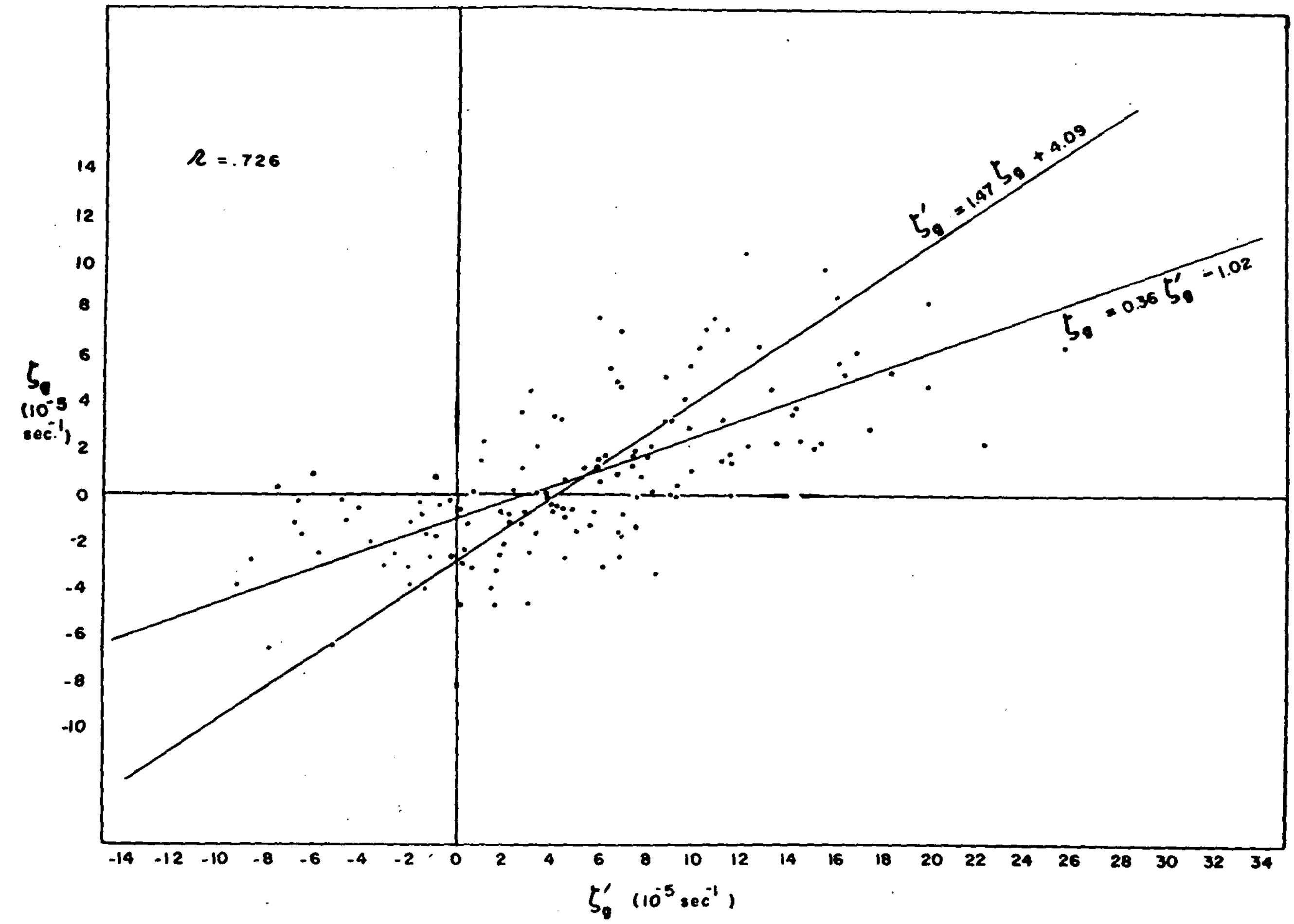

FIG. 5. Scatter diagram. ζ_g *vs.* ζ_g'. Winter cases.

from ζ_g', and S_g' is the standard error of estimating ζ_g' from ζ_g.

In this instance, the correlation coefficients for the separate winter and summer groups are in close agreement.

TABLE 2. Statistical analysis of ζ_g and ζ_g' (vorticity in units 10^{-5} sec^{-1}).

Group	r	$\bar{\zeta}_g$	$\bar{\zeta}_g'$	σ_g	σ_g'	r	S_g*	S_g'
Winter	141	0.97	5.52	3.55	7.16	.726	2.43	4.93
Summer	244	1.16	7.44	2.69	7.44	.740	1.81	5.01
Total	385	1.09	6.74	3.03	7.39	.723	2.09	5.11

The spread of the two vorticity means, however, is considerably larger for the summer group, because of the larger summer mean of the smaller-scale vorticity. Another slight seasonal effect is found in the lower summer value of the standard error of estimating the larger-scale vorticity from that on the smaller scale.

The most interesting feature of this table is found in the result that both the mean and standard deviation of ζ_g' are so very much larger than the corresponding values of ζ_g for both the summer and winter groups. For the combined cases, the standard deviation of ζ_g' is about two and one-half times as large as the standard deviation of ζ_g. Yet, in spite of this large disparity in their individual distributions, the two geostrophic vorticities are rather well correlated, with standard errors of estimation in each instance equal to about 70 per cent of the respective standard deviations.

The large difference in the individual vorticity distributions mainly arises from the use of second order height differentials in the computation of geostrophic vorticity. As shown in (6), the geostrophic vorticity is inversely dependent upon the square of the grid-differentiation distance L, and directly dependent upon the sum of the height differences between the central station and each of the outside stations. If one were to assume, for example, that the height gradient is linearly distributed along each grid axis, the effect of halving the grid size would be to double the geostrophic vorticity computed at the central point. In addition to this effect arising from the mathematics, as the scale of geostrophic vorticity is reduced, the effect of more pronounced smaller-scale features in the height field is increased. In this way, the large disparity in the individual distributions of the two different-scale vorticities can be reasonably explained.

6. Analysis of height and wind errors

Estimates of the vorticity variance produced by height errors and by wind errors and small-scale fluctuations will now be considered. The error standard deviation of the geostrophic vorticity can be shown (Scarborough, 1930) to depend upon the individual height errors in the following way:

$$\hat{\sigma}_g = \left[\sum_{i=0}^{i=4} (\partial\zeta_g/\partial h_i)^2 \sigma_{h_i}{}^2 \right]^{\frac{1}{2}}, \qquad (9)$$

where the partial derivatives, $(\partial\zeta_g/\partial h_i)$, are obtained by differentiation of (6), and the height error variances, $\sigma_{h_i}{}^2$, are estimated experimentally. By assuming that the height-error variances are equal for the five stations in the network, we obtain after simplifying

$$\hat{\sigma}_g = (20)^{\frac{1}{2}}(g/fL^2)\,\sigma_h. \qquad (10)$$

The error in the height of the central station is seen to be most important, since if there were no error in this height, the numerical factor would be 2 instead of $(20)^{\frac{1}{2}}$. If Rapp's estimate (1952) of the error variance of the 500-mb height is now used, $\sigma_h{}^2 = 104.5$ m^2, we obtain as the error standard deviation of ζ_g, $\hat{\sigma}_g = 1.3 \times 10^{-5}$ sec^{-1}; and similarly for ζ_g', $\hat{\sigma}_g' = 5.2 \times 10^{-5}$ sec^{-1}. These results indicate that the size of the geostrophic vorticity error is strongly dependent upon the grid scale for given height errors, since the error standard deviation of the geostrophic vorticity is inversely dependent upon the square of the grid differentiation distance.

The error standard deviation of the observed vorticity can be similarly shown to be

$$\hat{\sigma} = \left[\left(\frac{\partial\zeta}{\partial v_3}\right)^2 \sigma_{v_3}{}^2 + \left(\frac{\partial\zeta}{\partial v_1}\right)^2 \sigma_{v_1}{}^2 + \left(\frac{\partial\zeta}{\partial u_2}\right)^2 \sigma_{u_2}{}^2 + \left(\frac{\partial\zeta}{\partial u_4}\right)^2 \sigma_{u_4}{}^2 \right]^{\frac{1}{2}}, \qquad (11)$$

where the partial derivatives are obtained by differentiation of (2), and the wind error variances are estimated experimentally. If we assume that $\sigma_{v_3}{}^2 = \sigma_{v_1}{}^2$ and $\sigma_{u_2}{}^2 = \sigma_{u_4}{}^2$, we obtain, after simplifying,

$$\hat{\sigma} = (2\sigma_u{}^2 + 2\sigma_v{}^2)^{\frac{1}{2}}/L. \qquad (12)$$

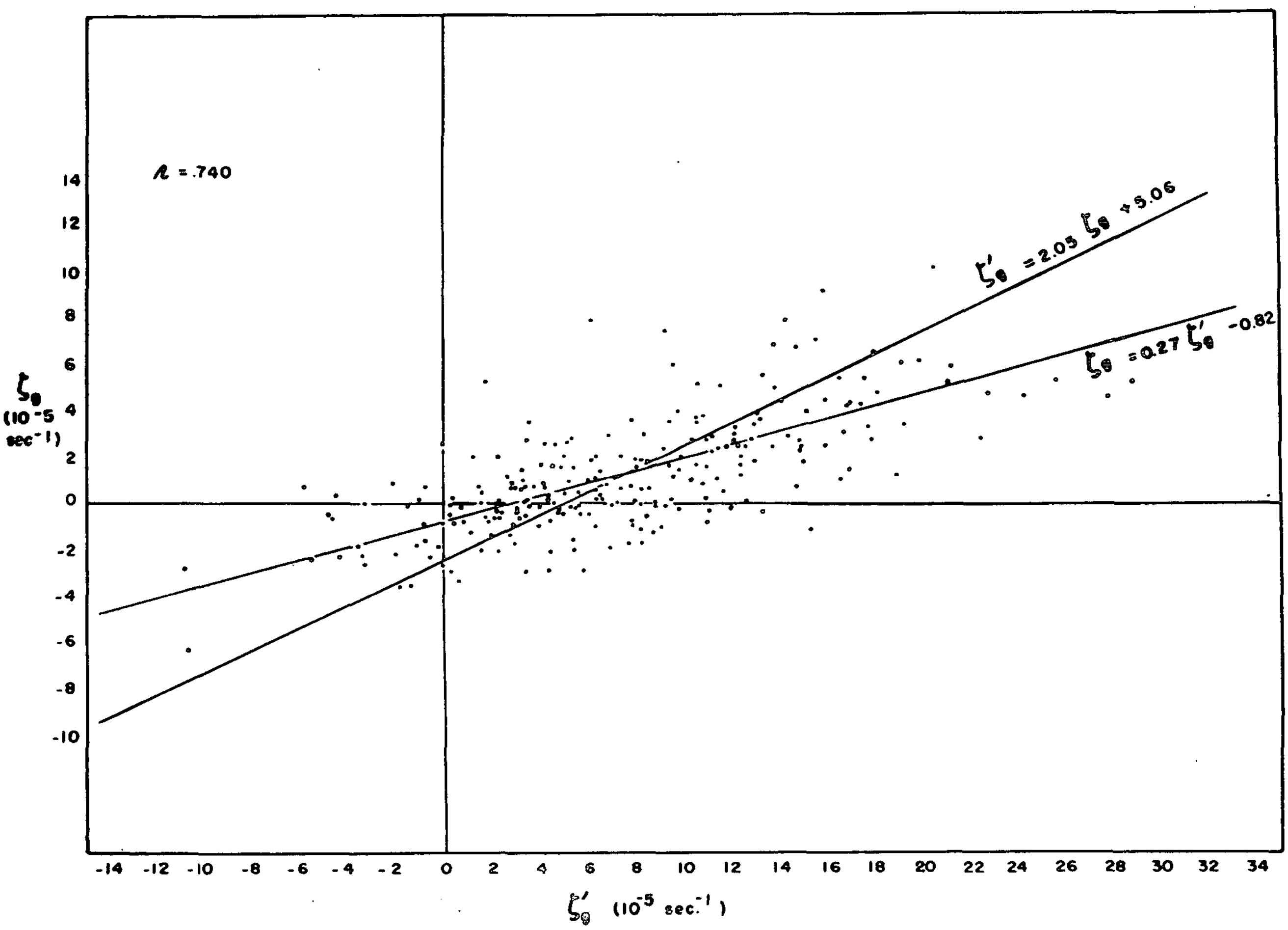

FIG. 6. Scatter diagram. ζ_g *vs.* ζ_g'. Summer cases.

From Rapp's results (1952), the average error variances of the zonal and meridional wind components between 5.5 and 6.0 km (the approximate range of the 500-mb height) are $\sigma_u^2 = 0.65$ m² sec⁻² and $\sigma_v^2 = 0.79$ m² sec⁻². Thus, by substitution in (12) the error standard deviation of ζ is $\hat{\sigma} = 0.24 \times 10^{-5}$ sec⁻¹. This value is considerably lower than the value obtained for the geostrophic vorticity on the same scale. The variance of small-scale wind fluctuations should also be considered, however, since wind observations are effectively non-simultaneous, and small-scale fluctuations of periods ranging from 5 min to 3 hrs may seriously affect the vorticity computations based upon the reported winds of five stations.

Rapp (1952) has presented some tentative approximations to the variances of small-scale wind fluctuations. These were determined for various heights in four separate experiments. For our purposes, the variance values which he obtained for 5–6 km were averaged separately for the zonal and meridional components. The mean of seven zonal variances was 3.3 m^2 sec^{-2}; the mean of five meridional variances was 10.1 m^2 sec^{-2}. A representative value of the fluctuation variance for each component was assumed to be 5.0 m^2 sec^{-2}. Upon substitution in (12), the fluctuation standard deviation of ζ is $\hat{\sigma}_f = 1.7 \times 10^{-5}$ sec^{-1}.

The above estimates of error and fluctuation standard deviations are proposed as qualitative estimates. The error standard deviations are probably on the low side, since Rapp's results were based upon data more accurate than ordinarily obtained in the routine observational program. It would appear, however, that the variance of observed vorticity due to wind fluctuations considerably exceeds that due to wind measurement errors, and that this combined variance in observed vorticity is comparable to the error variance in the geostrophic vorticity on the same scale of 600 km. It would also seem, from the nature of (10) and (12), where $\hat{\sigma}_g \sim L^{-2}$ and $\hat{\sigma} \sim L^{-1}$, that for given height errors, wind errors and fluctuations, the smaller the scale chosen, the more inaccurate the geostrophic vorticity becomes relative to the accuracy of the observed vorticity.

7. Conclusions

It has been shown that the selection of grid size has a pronounced effect upon the size of the geostrophic vorticity. The results of the study on two scales of 300 km and 600 km indicated that the standard deviation of the smaller scale geostrophic vorticity was more than twice that of the larger scale vorticity.

The study of comparable geostrophic and observed vorticities showed that they are better related in the winter months and that on the average there is a considerable difference between the vorticities. Our examination of the variance arising in the computation of each vorticity, however, indicated that the effect of height errors, wind errors and small-scale fluctuations can explain most of the difference between the computed vorticities. On the scale of 600 km, the over-all errors in the computation of each vorticity seem comparable; as the scale of computation becomes smaller, the error in the geostrophic vorticity begins to exceed considerably that of the observed vorticity.

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