# SHORTER CONTRIBUTION

## A POSSIBLE SUNSPOT INFLUENCE ON THE GENERAL CIRCULATION

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*Introduction.*—In the course of an investigation into the utility of hemispheric weather types in long-range forecasting [1], a detailed study was made of the behavior of the hemispheric wave number in middle latitudes. The hemispheric wave number was defined as the number of major troughs in the mid-troposphere westerlies that could be discerned about the northern hemisphere between latitudes 30 to 45 deg. Each trough counted had a tightened gradient of contour lines accompanying it, indicating that it was a part of a major jet-stream circulation. No systems were included that obviously resulted from local surface thermal influences.

Waves were counted each day for the years 1946 and 1947 at 500 mb, and for the years 1952 to 1955 at 700 mb. No differences could be found in wave-number distributions that could be ascribed to enumeration at differing heights in the atmosphere (see below).

*Year-to-year homogeneity of wave-number distribution.*—Table 1 shows the distribution of wave numbers

Table 1. Frequency distributions of wave numbers at middle latitudes.

| Year | Wave number 4 | 5 | 6 | 7–8 | Total* |
|---|---|---|---|---|---|
| 1946 | 2 | 184 | 167 | 6 | 359 |
| 1947 | 0 | 86 | 212 | 67 | 365 |
| 1952 | 18 | 175 | 143 | 3 | 339 |
| 1953 | 37 | 210 | 104 | 14 | 365 |
| 1954 | 1 | 153 | 129 | 82 | 365 |
| 1955 | 14 | 80 | 160 | 77 | 331 |
| Total | 72 | 888 | 915 | 249 | 2124 |

* Some years incomplete.

for the six years. Extreme wave numbers of infrequent occurrence have been combined with adjacent, more frequent wave numbers.

The standard $\chi^2$ test for homogeneity [2] was applied to these data. As applied here, this test asks whether the probability of occurrence of a given wave number is the same from year to year or, alternatively, whether the distributions of wave numbers found for the various years could have come from independent sampling from a single population. The null hypothesis is made that all distributions are random samples from a universal population.

The following tests were made: (1) Were the individual years homogeneous among themselves? $\chi^2 = 393$ with 15 degrees of freedom; (2) Were the wave numbers of the years 1952–1955, counted at 700 mb, homogeneous with those for the years 1946–1947, counted at 500 mb? $\chi^2 = 63.7$ with 3 degrees of freedom. The probability that the samples as described were independently drawn from a homogeneous population is much less than 1 per cent in both cases, so that the null hypothesis of homogeneity must be rejected.

The lack of homogeneity results, in part, from the day-to-day persistence of the wave number which makes successive "samples" dependent upon each other, thus violating the hypothesis of independent sampling. Methods are not immediately available to show the effects of day-to-day persistence on the expectation of homogeneity, but a simple estimate may be made for the present case. It may be shown [3] that the wave number for a given day has insignificant effect on the wave number seven or more days later. It may then be argued that rather than 365 independent samples per year, only $365 \div 7$ independent samples have been taken. This is likely an underestimate, but it suffices for the present purposes. The effect of this reduction of the number of samples is to divide each value of $\chi^2$ by 7, the number of degrees of freedom remaining unchanged. The new values of $\chi^2$ thus obtained still indicate lack of homogeneity, significant at the 1-per cent level among years and at the 5-per cent level between 700 and 500 mb. It is of interest that much of the significance of the inhomogeneity between 700- and 500-mb wave distributions has been lost, suggesting that the difference in level of observation did not of itself produce a material difference in wave-number distribution.

The lack of homogeneity among years thus appears to be greater then can be produced by day-to-day persistence, and must be ascribed to some factor outside of wave number itself, something that materially

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changes the odds on the wave numbers from year to year.

*Wave-number transition probabilities.*—It follows that the probabilities of transition from one wave number to another must similarly be inhomogeneous from year to year. The transition probability is defined as the conditional probability that, if today's wave number is $\nu$, tomorrow's will *not* be $\nu$. An estimate of the transition probability is

$$p_{\nu\bar{\nu}} = n_{\nu\bar{\nu}}/n_{\nu},$$

where $n_{\nu\bar{\nu}}$ is the number of days on which wave number $\nu$ is not followed the next day by $\nu$, and $n_{\nu}$ is the number of days on which wave number $\nu$ occurrs.

A tabulation of transitions in the years 1946–1947 and 1952–1955 is shown in table 2. Here wave numbers of infrequent occurrence have been omitted. The number of occurrences of wave number seven was usually too small to form stable estimates of transition probability. Application of the test for homogeneity shows inhomogeneity, significant below the 1-per cent level, indicating that the transition probabilities are not constant from year to year.

TABLE 2. Wave-number transitions.

| Year | $n_5$ | $n_{5\bar{5}}$ | $n_6$ | $n_{6\bar{6}}$ | $n_7$ | $n_{7\bar{7}}$ |
|---|---|---|---|---|---|---|
| 1946 | 184 | 29 | 165 | 30 | 6 | 5 |
| 1947 | 85 | 23 | 212 | 41 | 64 | 21 |
| 1952 | 171 | 40 | 140 | 33 | 3 | 3 |
| 1953 | 209 | 39 | 104 | 33 | 14 | 6 |
| 1954 | 152 | 26 | 129 | 48 | 79 | 28 |
| 1955 | | | 174 | 49 | | |

Examination of the tabulation of transition probabilities, table 3, reveals a definite pattern. Transition probabilities from wave numbers five and seven cannot be demonstrated to be inhomogeneous from year to year. The transition probability from wave number six, on the other hand, varies widely and contributes heavily to the inhomogeneity shown.

TABLE 3. Transition probabilities of wave numbers, and sunspot numbers.

| Year | $p_{5\bar{5}}$ | $p_{6\bar{6}}$ | $p_{7\bar{7}}$ | Mean annual sunspot number: Number | Mean annual sunspot number: Logarithm |
|---|---|---|---|---|---|
| 1946 | 0.158 | 0.182 | (0.833) | 92.6 | 1.9666 |
| 1947 | 0.267 | 0.193 | 0.328 | 151.6 | 2.1804 |
| 1952 | 0.234 | 0.236 | (1.000) | 31.5 | 1.4983 |
| 1953 | 0.187 | 0.317 | (0.429) | 13.9 | 1.1430 |
| 1954 | 0.171 | 0.372 | 0.354 | 4.4 | 0.6435 |
| 1955 | | 0.282 | | 38.0 | 1.5798 |
| Correlation with logarithm of sunspot number: | +0.49 | −0.955<br>(−0.973)* | — | | |

* First 5 years.

Tentatively, then, the inhomogeneity of the wave-number distribution may be ascribed to the wide year-to-year variation in the transition probability of the wave number six at middle latitudes. To the extent that large-scale weather behavior is related to the wave-number distribution, the differences in weather from one year to another would appear to be related to variations of stability of wave number six.

*Correlation with sunspot numbers.*—In an attempt to "explain" the variation of the transition probability of wave number six, it was compared with sunspot activity. The logarithm of the mean annual sunspot number[3] was taken, approximately to normalize its distribution. The linear correlation between transition probability of wave number six and the logarithm of the mean annual sunspot number is −0.973 for the five pairs, significant by the $t$-test at less than the 1-per cent level.

The only independent data immediately available were the wave counts for the year 1955. A "forecast" of the transition probability of wave number six from the mean sunspot number of the year was not as successful as the correlation would indicate, but inclusion of the 1955 data merely reduced the correlation to −0.955. A partial explanation of the relatively poor fit of the 1955 data may be found in the great variation in sunspot numbers observed that year. Since the logarithm of the sunspot number is used, the mean logarithm may differ materially from the logarithm of the mean; the departure of the transition probability $p_{6\bar{6}}$ in 1955 from the "predicted" can be materially reduced by using the mean of the logarithm of the mean monthly sunspot numbers for the year. Since this practice was not followed in the other years, it cannot legitimately be used here; the correlation remains very significant without such refinement.

The transition probability from wave number five shows a positive correlation with the logarithm of the mean sunspot number, but not of demonstrable significance. Transitions from other wave numbers did not occur with sufficient frequency to permit stable estimates of transition probabilty.

*Discussion.*—The customary $t$-test of significance of correlation appears valid here, as the use of the test for homogeneity to select a general-circulation parameter of considerable variability cannot have affected correlation. Although but six data pairs were used, each transition probability from wave number six was determined from a number of transitions and may be regarded as a fairly stable estimate. The transitions represent subjective judgments, but the number of transitions encountered reduces all but bias errors from this source.

[3] From *J. geophys. Res.*

It cannot be claimed that the "true" correlation between the estimated transition probability from wave number six and the logarithm of the mean annual sunspot number is as high as −0.955. All that may be said is that it is improbable that the two quantities are uncorrelated. The high numerical value of the coefficient must be regarded as a fortunate accident.

The sign of the correlation is not entirely expected. It is usually taken that the influence of solar disturbance is to perturb the atmosphere, but the present study appears to indicate that a spotted sun favors longer persistence of wave number six, possibly at the expense of stability of wave number five. To the extent that wave number six represents a winter type pattern and five a summer type, some solar influence on local weather is indicated. However, the significance of any correlation such as found here must be somewhat doubtful until a physical explanation is at hand.

## REFERENCES

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