THE COLLISION EFFICIENCY OF CLOUD DROPS OF EQUAL SIZE

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ABSTRACT

An investigation by modeling techniques has been made of the collision between cloud drops of equal size in the diameter range from 11 to 22 microns. It was found that such drops will collide with an initial vertical separation as large as 20 drop radii and an initial horizontal separation of approximately 0.3 drop radii. The approach velocity of the two drops, averaged over a vertical separation of 20 radii, is nearly constant at 0.1 of the terminal velocity of the drops.

An analysis was made of the collision rate between drops of equal size under the assumption that the drops are located randomly in space. It is shown that the collision rate is proportional to the square of the ratio of cloud liquid-water content to drop size.

1. Introduction

The problem of the collision of cloud drops and its relationship to the formation of raindrops has received considerable attention by meteorologists. However, most work has been focused on the aspect of large drops passing through a field of smaller drops. It is usually assumed that collisions do not occur between drops of equal size, since such drops have equal terminal velocities.

Recently, Telford *et al* [1] have described an experiment in which they determined the collision efficiency between equal sized drops, 154 microns in diameter, from a statistical analysis of motion-picture frames taken of a cloud composed of such drops. The collision efficiency, which is essentially the number of droplets captured by a drop as it sweeps through a volume of air divided by the actual number of droplets in that volume, was found to be 12.6. This is extremely high (usual values lie between 0 and 1) and suggests the entrainment of all droplets whose centers lie within 2.6 radii of the capturing drop. The writers indicated that the wake of the lower drop created a pressure gradient which accelerated the upper drop into collision with the lower drop.

Such a mechanism could play a significant role in the early stages of cloud-drop growth since the diffusion process tends to produce a cloud of uniform drop size which, according to the assumption given above, inhibits drop growth by coalescence.

To determine the effectiveness of the wake collection-process, an experiment was designed in which the trajectories of cloud drops of equal size were modeled.

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2. Design of the experiment

The significant parameters involved in the collisions of two drops are: the drop radius, R; the drop density, ρ_D; the relative coordinate distances, X and Y; the terminal velocities of the drops, v_1 and v_2; the medium density, ρ_M; and the medium viscosity, μ. There are eight secondary quantities and three primary quantities in this set. Application of the Buckingham theorem results in eight minus three, or five, independent dimensionless products. An equation relating these quantities may be written as follows:

$$X = Yf\left[\frac{R}{X}, \frac{\rho_M v_1 R}{\mu}, \frac{\rho_M v_2 R}{\mu}, \frac{\rho_M}{\rho_D}\right]. \quad (1)$$

The first of the quantities in the bracket represents a modeling of the geometry of the system. The second and third terms are Reynolds numbers which model the hydrodynamic forces, and the last term models the ratio of the medium density to that of the drop. The first three terms can be modeled easily [2]; however, the last term poses a problem. The density ratio of the prototype is 10^{-3}. The lightest fluid which gives a reasonably large scale factor has a density of 0.9, while the heaviest material suitable for modeling has a density of 11. The density ratio with use of such components would be too large by a factor of 100.

Since it was not possible to model the density ratio, a simplifying assumption had to be made. If the Reynolds numbers of the drops considered are limited to values of unity or smaller, the fluid inertial forces are much smaller than the fluid viscous forces. Consequently, the fluid density is no longer a significant parameter. This approximation is not too serious, since the Reynolds numbers of cloud drops smaller than 60 microns in diameter falling in air are less than unity. The significant terms remaining are X, Y, ρ_D, μ, R and g_e, where the terms have the same significance as before except that in place of v_1 and v_2 a term g_e, which is gravity minus buoyancy force, is included. This is permissible, since v_1 and v_2 are functions of R, ρ_D, μ, g_e, X and Y. An equation formed from these terms is

$$X = Yf\left[\frac{R}{X}, \frac{\rho_D g_e^{\frac{1}{2}} R^{\frac{3}{2}}}{\mu}\right]. \quad (2)$$

The first dimensionless product represents the geometry of the system; and the last term represents the ratio of the fluid viscous force to the inertial force of the drop.

Equation (2), together with the auxiliary condition that the Reynolds number of the drops be less than unity, establishes the similarity requirements for the experiment.

3. Experimental apparatus

Apparatus (see fig. 1) was constructed for the purpose of obtaining stereoscopic photographs of pairs of simulated drops falling in a viscous medium. Only a brief discussion will be given of the equipment employed, as it has been described elsewhere [2].

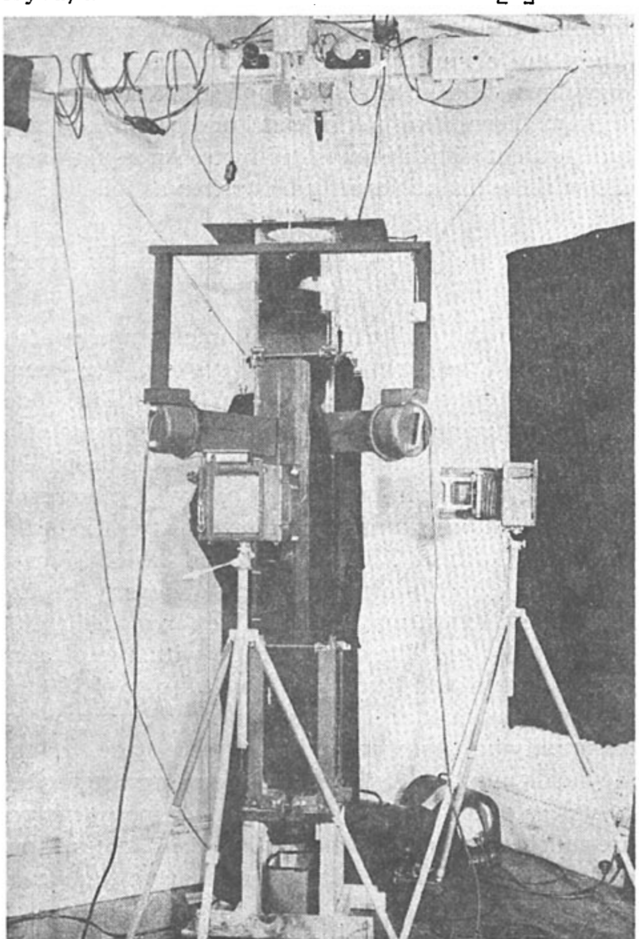

FIG. 1. Equipment used in experimental determination of collision properties of equal sized drops.

Drop model.—A drop may be characterized by its radius and density. It can be shown, by writing the viscous terms of (2) in terms of the Reynolds number, that the ratio of the drop density to medium density must be large (approaching 10^3) if the range of cloud drops from 0 to 60 microns is to be modeled with Reynolds numbers less than unity. However, if the drop range is restricted to sizes less than 25 microns, the density ratio need only be of the order of six to insure Reynolds numbers less than unity.

Polished steel ball-bearings of density 7.8 g per cm^3 and of diameter range from 1.5 mm to 1.5 cm are readily available. These model sizes allow for easy visual observation. To photograph the bearings, it was found necessary to change the characteristic of the surface so that the bearing would not act as a spherical mirror. The most satisfactory results were obtained by simply exposing the bearings to a fine spray of white enamel paint. The resulting changes in diameter were found to be less than 0.003 inch.

Fluid medium.—The fluid medium may be characterized by its viscosity and density. From the discussion in the section above, it follows that the fluid density should be as small as possible. The viscosity required of the fluid was determined from (2) by setting the viscous terms for the model (subscript M) equal to those of the prototype (subscript p):

$$\mu_M = \mu_p \frac{\rho_{DM}}{\rho_{Dp}} \frac{g_{eM}^{\frac{1}{2}}}{g_{ep}^{\frac{1}{2}}} K_1^{\frac{3}{2}}, \quad (3)$$

where $K_1 = R_M/R_p$. Substitution of the values given in the section above into (3) results in a viscosity value of approximately 18 poises. An aqueous invert sugar solution containing 76 per cent by weight of sugar was found suitable. This fluid has a viscosity of 20 poises at 21C and a density of 1.34 g per cm.^{3} Although the color of the fluid is pale yellow, it is sufficiently clear for photographic purposes.

Dropping mechanism.—A device was constructed for the purpose of dropping bearing pairs with a predetermined initial separation. It consisted essentially of a brass block machined to receive two brass mandrils. Each mandril was drilled to allow the passage of a given sized bearing. The block was free to slide on two horizontal brass guides. Adjustable stops were provided at each end of the guides to permit changes in the initial separation of the bearings.

Tank.—A transparent tank was constructed of quarter-inch plexiglas to contain the sugar solution. The tank was 5 ft. high, 8 by 8 in. in cross section, and held approximately 18 gal. of solution.

The dropping mechanism rested on top of the tank, and a plumb bob was hung from it to serve as a photographic reference line.

Photographic equipment.—The bearing trajectories were photographed on Kodak Contrast Process Ortho film, with two 4-by-5 Crown Graphic cameras equipped with 135-mm f4:7 Wallensak Raptar lens.

Illumination of the bearing pairs was accomplished by use of two General Radio type 1532A strobolumes. Each lamp was fitted with a sheet-metal collimator designed so that the flash could be directed at the intersection of two tank faces (see fig. 2). The flashing rate was controlled by an adjustable-speed motor-driven switch.

4. Experimental procedure

Observations of the trajectories of pairs of freely falling steel bearings were obtained using a right-angle, two-camera system. The falling bearings were photographed through two right-angle tank faces simultaneously. By an analysis of the two resulting photographs it was possible to fix, at all times, the position of one bearing relative to the other in space. The two cameras (noted as A and B in fig. 2, according to the tank face through which they photographed) were set and focused at such a distance from their respective faces that the resulting photographed images were the same size on both negatives. Care was taken to keep both cameras at the same height and oriented perpendicular to the relative tank faces.

Visual observations were made with silver test bearings to determine the initial settings for the dropping mechanism. Repeated visual test runs were made for the given-sized bearing pair, adjusting each time the position of the bearings, until an initial separation was found whereby collision was definitely observed to occur. When an approximate collision course had thus been determined for given-sized bearing pairs, a series of photograph runs was taken using white coated bearings. The time delay between the release of the bearings was varied so that data were obtained concerning the effect of increasing the initial vertical separation for each fixed horizontal separation.

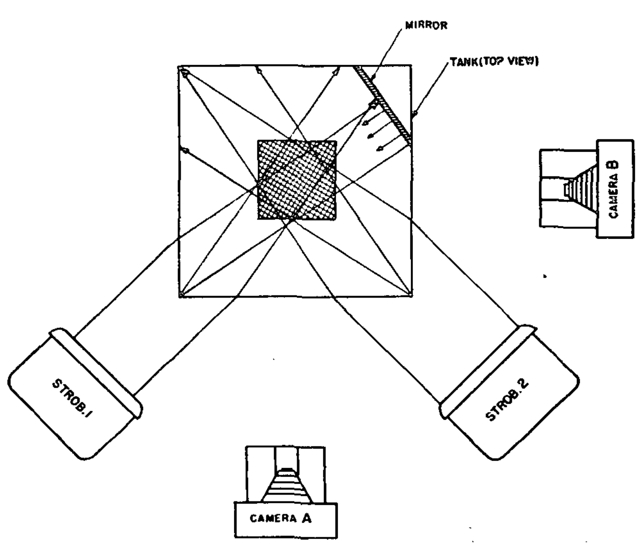

FIG. 2. Lighting arrangement used in experiment. Hatched area represents bearing fall-zone.

5. Computation

The negatives obtained from either camera, as a result of the setup, represented the bearings' trajectory projected to a plane perpendicular to the optical axis of each camera. Since the negatives obtained from camera A represent a projection of the bearing pairs in an XZ-plane and the negatives from camera B represent components in the YZ-plane, the actual position of the bearings in space can be determined from the resultants of the XZ- and YZ- components.

From fig. 3 it is seen that Y_{12} is the distance, measured on the negative from camera B, from the center of the upper bearing to the center of the lower bearing. X_{12} is a corresponding distance measured on the negative from camera A. Z_{12} is the vertical distance separating the bearings. The resultant of these distances represents the actual separation of the bearings in space, D_{12}, or

$$D_{12} = [X_{12}^2 + Y_{12}^2 - Z_{12}^2]^{\frac{1}{2}}. \quad (4)$$

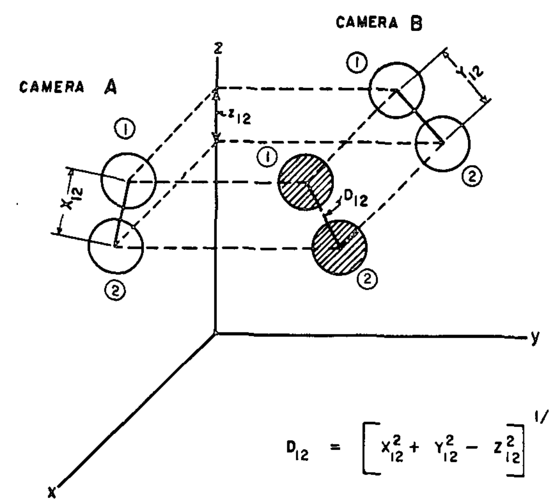

FIG. 3. Component distances measured for determination of separation of ball bearings in space.

To facilitate the measurements of the components, the negatives were placed in a photographic enlarger and the images were projected on millimeter graph paper, enlarged by a ratio of 1 in. (true) equals 64 mm. (projected). The centers of the bearing images were transferred to the graph paper with the aid of circular templates having center points and diameters equal to those of the projected images. The template circles were superimposed on the projected bearing images; and when they coincided exactly, a sharp point was inserted through the center of the circle on the template, thus locating the bearing center on the graph paper. After all the image centers had been transferred to the graph paper, a smooth curve was drawn connecting the center points of each bearing. These curves represented the trajectories of the bearings.

The separation of the bearing trajectory-lines was measured to ± 0.1 mm. The values for this separation obtained from each negative were substituted into (4), and the actual separation was determined. When $D_{12} = 2R$, the bearings were said to be in contact and a collision trajectory had been obtained.

Component measurements were also made, in each case, of the initial separation, R_i, of the bearings' centers. These measurements were made at a point, early in the bearings' trajectory, before they began to influence each other's motion:

$$R_i = [(X_{12})^2 + (Y_{12})^2]^{\frac{1}{2}}. \qquad (5)$$

In denoting the largest initial separation where the upper bearing still collides with the lower bearing, R_i becomes R_c. The collision efficiency, E, was computed from

$$E = [(R + R_c)/R]^2. \qquad (6)$$

It should be stated at this point that all measurements were made directly from uncorrected, projected bearing trajectories. To be accurate, three corrections would have to be made. The first inaccuracy would arise from the fact that the bearings do not always fall in the planes of focus of the cameras. A maximum displacement from the plane of focus of 3 cm. being assumed, the error involved is less than 5 per cent. A second inaccuracy results from the fact that, when the bearings lie in a plane not parallel to the focal plane of the camera, true projected distances will not be recorded. In the procedure used in this experiment, the camera was located at a distance which was at least 100 times the separation of the bearings. The error resulting from such an effect is less than 1 per cent and may be neglected. The third inaccuracy would result from the fact that the photograph had been made into a fluid with an index of refraction higher than that of air. Computations show that measurements of distances could be off by as much as 8 per cent. The factor, however, would be a constant for all measurements at a fixed point on the trajectories and would, therefore, be automatically taken into account in the projection scale-factor and would not contribute to measurement errors. In most cases, the errors that might be expected to result from neglecting the above inaccuracies would be less than those which would result from the inability to measure distances to better than ±0.1 mm. The majority of initial separation distances were greater than 2.5 mm., so that the uncertainty of the computed efficiency may be written approximately as $E(1 \pm 0.2)$.

6. Discussion of results

The collision radii determined by this experiment increase approximately in a linear manner with drop size. A graph of critical collision radius *versus* drop size is given in fig. 4. The collision efficiency computed with (6) is constant with drop size, within the precision of the experiment, and is equal to $1.7(1 \pm 0.2)$.

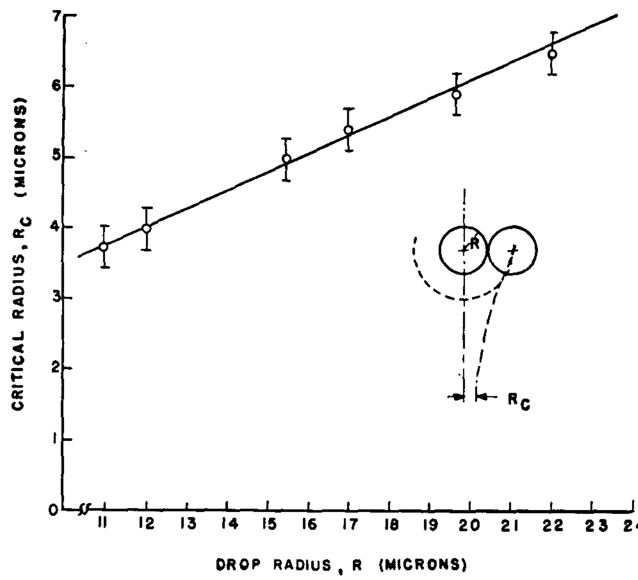

FIG. 4. Plot of critical collision radius as function of drop radius.

The approach velocity between two drops on a collision course, averaged over a vertical separation of 20 radii, is given in fig. 5. This velocity is roughly 0.1 of the drop terminal velocity.

Fig. 6 shows the trajectories of pairs of ball bearings which are 17 microns in model size. A_{122} and B_{120} are examples of collision trajectories, and A_{123} is an example of a miss. Only one of the negative pairs is shown for each trajectory, since in these examples there was little horizontal motion in the orthogonal plane. The largest observed initial vertical separation resulting in collision was 12 drop diameters.

The relative trajectories of equal- and unequal-sized colliding pairs of drops differ. In case of equal-sized drops, the approach velocity of the upper drop increases with time until collision occurs with the lower drop. The reverse is true for drops of unequal size.

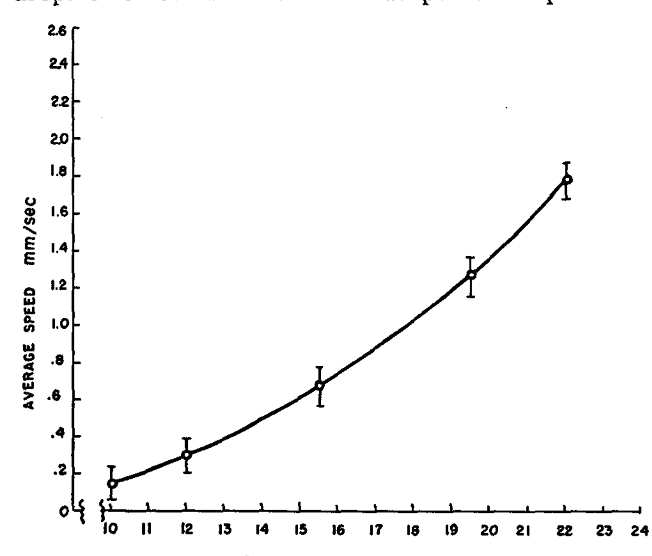

FIG. 5. Plot of approach velocity between two equal sized bearings averaged over vertical distance of 20 drop radii as function of drop size.

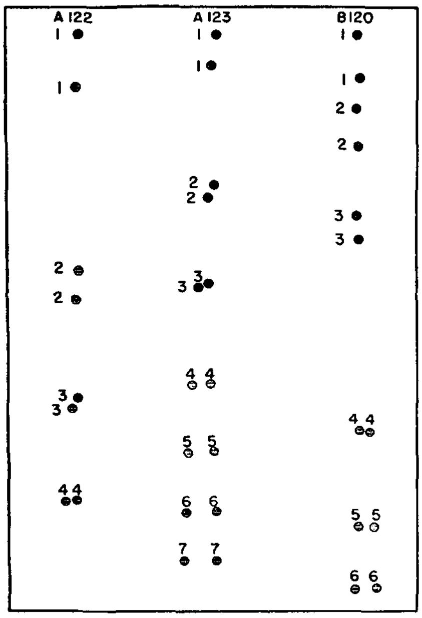

FIG. 6. Trajectories of pairs of ball bearings which are 17-micron diameter in model size.

After collision or after the point of closest approach has been reached and the lower drop is moving about the upper, a second difference becomes evident. With equal sized drops, the process of the lower drop moving about the upper drop continues until the line of centers becomes horizontal. At this point, the drops begin to separate horizontally while maintaining identical terminal velocities. The horizontal separation was observed to depend upon the initial horizontal separation. With zero initial horizontal separation, a horizontal center separation of roughly 1.5 diameters was observed following collision. An initial horizontal separation corresponding to a near miss would be followed by a 4- or 5-diameter horizontal spread. Similar horizontal separation was observed with drops of nearly the same size. However, after the upper drop passed out of the region of mutual influence, both drops regained their characteristic terminal velocities.

7. An application of the wake collection-process to a cloud

A measure of the effectiveness of the wake collection-process in producing collisions between cloud drops of equal size can be determined from the data obtained in this experiment. Consider a cloud composed of water drops which are equal in size and randomly positioned in space.

Let the following quantities be characteristic of the cloud: W = cloud liquid-water content (g per m^3), R = drop radius (m), R_c = critical radius = $0.3\,R$ (m), N = drop concentration (m^{-3}), Z_c = largest vertical separation over which wake collection occurs = $20\,R$, and V_z = vertical approach velocity (averaged over Z_c) = $0.1\,V_t$.

The largest volume V_c which two drops of equal size can occupy and still collide by virtue of the wake process is a cylinder Z_c in height and R_c in radius. This volume can be expressed as

$$V_c = 1.8\,\pi R^3. \qquad (7)$$

The average number of drops contained in this volume is

$$M = NV_c = 1.8\,\pi R^3 N. \qquad (8)$$

The probability that n drops will occupy volume V_c, as obtained from Poisson statistics [3], is

$$P(n) = M^n e^{-M}/n!. \qquad (9)$$

The number of volumes V_c contained in a unit volume is $1/V_c$. Therefore, the average number of pairs of drops contained in collision volumes is

$$P(2)\frac{1}{V_c} = \frac{1}{V_c}\,\frac{M^2 e^{-M}}{2}.$$

The maximum time, t_c, required for these pairs of drops to collide is

$$t_c = Z_c/V_z. \qquad (10)$$

Consequently, the minimum number of collisions occurring each second in a unit volume, B, is given by

$$B = \frac{1}{V_c t_c}\,\frac{M^2 e^{-M}}{2}, \qquad (11)$$

or, substituting for W, V_c and t_c in metric units,

$$B = 9.9 \times 10^{-8}\, W^2/R^2$$
collisions per second per cubic meter. (12)

As an example, let $R = 10^{-5}$ m (10 microns), and $W = 1$ g per m^3. The number of collisions per second in a volume of 1 m^3 from (12) is 990 per second, and the resulting number of 12.6-micron drops formed, it being assumed that each collision results in coalescence, is 450 per second. For a cloud of similar liquid-water content and a drop radius of 5 microns, the collision rate is 2960 per second, and 1980 drops of 6.3-micron radius will be formed per second per cubic meter.

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