COMPUTATION OF INSTANTANEOUS SOURCE DIFFUSION COEFFICIENTS FROM SMOKE PUFF OBSERVATIONS

By Eugene M. Wilkins

U. S. Weather Bureau[1]

(Manuscript received 16 November 1956)

ABSTRACT

A method is proposed whereby the Sutton-type diffusion coefficients appropriate for the instantaneous source may be estimated from observations of dissipation times for smoke puffs generated by standard gunpowder charges. It is logical to base the determination of these coefficients on observations of the spreading of smoke, especially if the technique is simple, inexpensive and subject to standardization.

The smoke puff experiments reported here were carried out over a level grassy surface near Convair's Nuclear Laboratory, Fort Worth, Texas, February through September, 1956. This investigation had two objectives: to obtain estimates of the Sutton-type diffusion coefficient appropriate to various meteorological conditions at the laboratory site; and to devise a simple method useable at any site for obtaining comparable data. The first objective is realized in the table summarizing the data, but the second will depend upon the acceptability of the technique in the eyes of other meteorologists working in the field of atmospheric diffusion.

The smoke puffer used has been described by Halstead [3]. The firing chamber consists of an ordinary spark plug screwed into a cast-iron reducer coupling, the larger end of the coupling serving as a receptacle for the powder charge. The spark is generated by a six-volt dry cell wired to the spark plug and to a Model "T" Ford coil through a push-button switch. Fig. 1 shows the puffer in use.

After some experimentation, it was found that three-gram charges of du Pont FFFFg black gunpowder provided a dense white cloud whose dimensions were clearly defined against a marker grid 20 m downwind. Larger charges burned inefficiently, scattering unburned particles about the puffer, and clouds were not uniform. Smaller charges were also less uniform, sometimes burning so efficiently that the cloud had a very low density. Hence, three-gram charges were used for all of the experiments. Variations in temperature and humidity had no noticeable effect.

The technique was to obtain with two stop watches the dissipation time, t_d, and the travel time, or marker time, t_m, and to estimate the vertical and horizontal dimensions of the puff as it passed through the grid. The timing was accomplished by an observer at the grid, and the dimensions were recorded by the man firing the puffs. Data were obtained in this manner for 32 runs of 20 puffs each.

It might be supposed that the dissipation time is somewhat subjective, however, it was found that different observers recording dissipation time independently had an average variation for 20 puffs of only about two tenths of a second, and this is about the limit of resolution of the stop watches used.

At the grid, markers were placed at one-meter intervals in the vertical and horizontal on a woven-

FIG. 1. The smoke puffer in action.

[1] Research carried out under the auspices of the U. S. Atomic Energy Commission.

wire fence, allowing dimensions to be estimated to about the nearest half-meter. Lighting and background could conceivably affect the dissipation time, however, light meter readings taken in the direction of travel for each run showed no significant effect. Possibly, any differences, if present, were masked by other inaccuracies in the technique. Since south winds existed for most of the experiments there was not actually an appreciable variation in background. Also, the average dissipation distance was only about 60 m (40 m from the timer) and possibly lighting and background variations were negligible at such distances.

Temperature differences between the 100-m level and shelter level were used to classify the runs according to stability conditions. These were taken from Carswell AFB radiosondes (about a half-mile away) at the sounding time nearest the time of the smoke run. Soundings are made there at 0300, 0900, 1500, and 2100 CST.

Unfortunately, very little data were obtained with a surface temperature inversion present, because these occurred very rarely during the period of experimentation. However, two of the inversion runs are believed to be representative of stable conditions.

O. G. Sutton's [5] treatment of atmospheric diffusion defines vertical and horizontal diffusion coefficients in terms of a "stability parameter" and vertical and horizontal eddy velocities. Instrumentation for measuring the eddy velocities is rather specialized, and the lack of standardization of instruments and techniques hinders not only the establishment of diffusion climatologies for various sites, but also comparisons between sites. This is particularly objectionable when one considers that Sutton's technique is widely used by meteorologists for diffusion predictions [7].

Relationships of the vertical diffusion coefficient, C_z, and the horizontal C_y, to the respective puff dimensions, b and a, at a time t after release can be derived from Sutton's [5] instantaneous point source equation. These relationships are:

$$b^2 = \left(\ln \frac{100}{P}\right) C_z^2 (\bar{u}t)^{2-n} \tag{1}$$

$$a^2 = \left(\ln \frac{100}{P}\right) C_y^2 (\bar{u}t)^{2-n}, \tag{2}$$

where b and a are half of the puff height and width, respectively, and $\bar{u}$ is the wind speed during the travel of the puff. Sutton's "stability parameter," n, is defined in terms of the vertical wind-speed profile, and is thus related to the vertical gradient of air temperature. For the purpose here, values of n were taken according to the following empirical relationships quoted by Sutton [6] from investigations over a smooth surface at Porton, England:

$n = 1/5$	superadiabatic gradient	(unstable)
$n = 1/4$	gradient between dry adiabatic and isothermal	(neutral)
$n = 1/3$	mild inversion	(stable)
$n = 1/2$	strong inversion	(very stable)

The theory assumes that the concentration across the puff is Gaussian, and P is the per cent of the central concentration defining the cloud boundary. For the purpose here, P was taken arbitrarily as ten per cent. The value of the diffusion coefficient is not particularly sensitive to the choice of P.

The cross-section area of the puff viewed along the line of travel is:

$$A = \tfrac{1}{2}\pi ab \tag{3}$$

for a surface release, the lower half of the elliptical puff being flattened against the ground. It can be shown that the relationship between the area measured at the marker and the area at the time that the puff just becomes invisible is:

$$\frac{A_d}{A_m} = \left[\frac{t_d}{t_m}\right]^{2-n}, \tag{4}$$

where the subscripts d and m refer to dimensions at dissipation and at the marker respectively.

Equations (1) and (2) were used to compute C_z and C_y for the smoke puffs. Equation (3) was used to obtain A_m, and equation (4) was used to compute A_d, A summary of these calculations is given in table 1. where the runs are grouped according to the applicable n value. Run number ten was made at a time when a surface inversion was dissipating, and possibly conditions changed during the course of the run.

The averages for the first two groups in table 1 show no significant differences in diffusion coefficients, dimensions at the marker, or dissipation distances and areas as a result of the distinction between temperature profiles. In fact, if the same n-value had been used for both groups the average diffusion coefficients would have been almost identical. It was thought that the variances of the marker dimensions might be greater for the runs made in superadiabatic gradients, but they turned out to be slightly smaller on the average. It may be concluded either that thermal eddies characteristic of unstable conditions do not have an appreciable effect on instantaneous source diffusion of this scale, or else that the temperature profile in the lower 100 m is not a good indicator of the "effective n" appropriate for distinction between the two categories. However, DeMarrais [1] has also found considerable variation of n within each of these four categories of temperature profiles using data from the Brookhaven tower.

In this regard, it should be recalled that the Sutton technique assumes diffusion relative to a fixed point, which is not the same as relative diffusion of a cloud

TABLE 1. Average dissipation sizes and diffusion coefficients.

Run no.	$\bar{u}$ (m/sec)	$\bar{d}(m)$	$\bar{A}_m$ (m^2)	$\bar{A}_d$ (m^2)	95 per cent Fiducial limits	$\bar{C}_z$	$\bar{C}_y$	$\bar{C}$
				$n = 1/5$				
1	2.36	31.3	36.9	83.9	±25.4	0.336	0.140	0.209
5	5.16	45.3	15.4	63.5	11.6	0.187	0.101	0.135
6	6.62	55.5	12.5	78.8	14.3	0.167	0.098	0.125
12	3.52	59.3	14.8	97.0	14.9	0.169	0.144	0.136
14	4.45	73.6	10.3	107.4	15.6	0.155	0.090	0.116
15	5.56	67.2	17.5	140.6	26.0	0.192	0.111	0.145
18	5.79	62.1	14.8	112.9	19.6	0.163	0.115	0.135
19	5.61	61.2	12.5	90.1	20.5	0.161	0.097	0.123
22	5.03	58.2	14.5	92.8	14.2	0.173	0.107	0.132
23	4.06	55.2	13.6	82.1	9.4	0.178	0.100	0.130
24	4.12	54.3	13.5	79.1	10.0	0.166	0.104	0.129
25	3.42	72.5	11.3	117.0	24.5	0.187	0.076	0.116
26	3.92	62.7	11.9	94.4	20.9	0.184	0.083	0.121
29	3.58	57.4	11.4	74.7	16.4	0.165	0.088	0.118
Averages	4.51	58.3	15.1	94.0	±17.4	0.185	0.104	0.134
				$n = 1/4$				
2	2.70	32.6	20.7	47.3	± 7.6	0.223	0.135	0.170
3	3.84	57.8	25.5	154.8	23.7	0.209	0.180	0.189
4	5.66	43.3	22.2	87.4	23.5	0.220	0.143	0.175
7	9.99	53.0	13.5	85.0	18.3	0.152	0.136	0.138
8*	5.88	59.1	15.7	104.5	17.0	0.152	0.156	0.150
9	5.29	63.0	17.8	131.1	20.2	0.209	0.130	0.161
11	3.65	74.3	13.5	129.9	30.0	0.204	0.087	0.132
13	1.72	47.1	18.0	76.6	11.4	0.201	0.138	0.160
16	5.29	59.1	18.0	105.7	18.6	0.212	0.121	0.157
17	5.72	74.4	11.0	98.3	14.0	0.151	0.103	0.123
20	3.55	64.0	10.8	81.3	10.3	0.198	0.079	0.124
21	3.76	71.9	11.9	105.3	20.9	0.184	0.092	0.128
27	4.60	69.2	12.6	102.3	17.6	0.191	0.094	0.133
28	5.13	70.8	9.4	81.2	16.1	0.157	0.085	0.113
30	6.93	71.4	8.1	71.3	9.0	0.139	0.085	0.107
Averages	4.84	60.8	15.2	97.0	±17.2	0.189	0.115	0.143
				$n = 1/3$				
10**	1.63	68.7	15.6	111.8	±22.0	0.185	0.159	0.167
				$n = 1/2$				
31**	2.04	81.4	11.5	93.6	±14.2	0.140	0.135	0.132
32**	2.20	84.3	12.3	104.7	±10.2	0.133	0.147	0.137

* Release made during gusts only, therefore, omitted from the averages.
** C_z computed using half of the vertical dimension, since puffs did not touch ground.

of particles. In effect, the data here have been fitted to the Sutton model in such a manner that the results are applicable only to instantaneous source diffusion and to a time scale similar to that used in the experiments.

The real measure of diffusivity for these runs is the dissipation distance, $\bar{d}$, since it is a measure of how far out a given concentration will occur. While the average $\bar{d}$ for both groups is about 60 m, it can be seen that if 60 m is taken as the division between groups there are six runs in group one that should be in group two, and *vice versa*. Analysis of the groups according as $\bar{d}$ is greater or less than 60 m gives the result below:

	$\bar{u}$	A_d	$C = \sqrt{C_z C_y}$
$\bar{d} < 60$ m (analyzed as $n = 1/5$)	4.55	88.6	0.145
$\bar{d} > 60$ m (analyzed as $n = 1/4$)	4.81	101.7	0.131

While the diffusion coefficients are revised in the right direction, there is a greater difference in $\bar{A}_d$ between the two groups. From that standpoint the distinction between n-values based on the temperature gradient is preferred to the one based on dissipation distance, since A_d depends only upon the concentration of smoke that is visible to the eye, and should, therefore, be a constant.

It should be borne in mind in comparing these results with coefficients recommended by Sutton that his coefficients are based on wind fluctuations rather than on the spreading of smoke. For a surface release, Sutton [6] gives (as corrected by Wanta [8]) $C = 0.181$ for "average" conditions, and presumably this corresponds to conditions where $n = \frac{1}{4}$. The agreement, though fairly good, must be regarded as pure coincidence. As for cloud shape, Sutton gives 0.6 for C_z/C_y, whereas this ratio averages about three times as great for the puffs. In order for C_z to equal C_y, the width of the puff would have to be twice its height as it passed the grid, and such was not usually

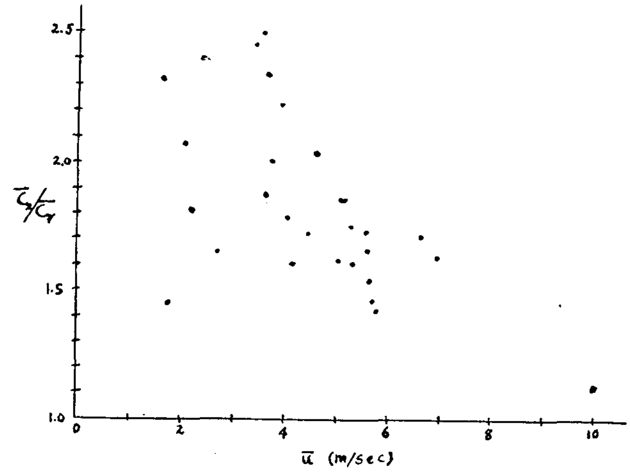

FIG. 2. Plot of $\bar{C}_z/\bar{C}_y$ *vs.* wind speed for 29 runs of 20 puffs each.

the case for these experiments. Instead most of the puffs resembled a vertical column with the lower portion flattened against the ground.

Runs 31 and 32 are believed to be typical of the inversion condition. The inversion was not very strong, and if these runs had been analyzed as $n = \frac{1}{3}$, the dissipation areas would have been somewhat larger than they are, and the diffusion coefficients smaller. It would seem better to choose in favor of the smaller dissipation size, since $n = \frac{1}{2}$ makes A_d more compatible with the rest of the data.

In run number eight, the puffs were deliberately released only during gusts in order to determine what would be the effect on the diffusion coefficients. Note that while $\bar{C}$ for this run is not much different from the mean for the group, $\bar{C}_z$ and $\bar{C}_y$ are equal, which indicates that during gusts downdrafts are present to cancel the rise of the puff due to the explosion. Puffs released during lulls often rose so high that it was impossible to estimate their dimensions as they passed the grid. However, the extreme variations of cloud shape that occurred emphasize the fact that the diffusion history of an instantaneous source depends heavily upon the eddy structure into which it is released. Despite these wide variations in cloud shape, the standard deviation of C was only about 16 per cent of the mean on the average.

$\bar{C}_z$ tends to vary inversely with wind speed, and the correlation coefficient is -0.51. C_y is unrelated, having a coefficient of -0.02. The correlation coefficient for $\bar{C}$ and wind speed is -0.33.

A slightly better correlation with wind speed is found in $\bar{C}_z/\bar{C}_y$, since this represents cloud shape and apparently is independent of n. This coefficient is -0.53, and fig. 2 is a scatter plot of $\bar{C}_z/\bar{C}_y$ *vs.* $\bar{u}$ for the runs. The relationship is not considered good enough for derivation of an empirical formula, but fig. 2 does tend to indicate an upper limit for the $\bar{C}_z/\bar{C}_y$ ratio for various wind speeds up to 10 m per sec.

As mentioned previously, the cross section area of the puff at time of dissipation, A_d, should be a constant, and selection of the n-value according to temperature gradient gave better agreement than selection according to dissipation distance. The tendency for A_d to be constant is also seen by the way that exceptionally large A_m values are offset by small t_d/t_m values and *vice-versa*. Variations in A_d between individual runs show no relationship to $\bar{u}$, A_m, brightness, temperature gradient or any of the other items recorded along with the smoke runs. Variations between individual determinations thus reflect on the accuracy of the technique, and limits about the means for a 95 per cent confidence factor are given in table 1. This limit is about 17 per cent of the mean on the average. In statistical terminology, one can be 95 per cent confident that the true $\bar{A}_d$ lies within ± 17 per cent of the computed mean. Furthermore, if each is allowed a value anywhere within those limits, it is possible to make most of them equal to the mean for the group. By comparison, the standard deviation of A_d was about one third of the mean on the average.

The true dissipation size is then estimated by averaging the 32 determinations of $\bar{A}_d$, and this is 96.5 m². The standard error is 22 m², but one can be 95 per cent confident that the true A_d lies within ± 8 m² of the computed mean. By fixing A_d, a formula can be derived from which the diffusion coefficient can be calculated using only the dissipation time (for puffs generated by three-gram charges), the wind speed and n. The formula is

$$C^2 = \frac{2A_d}{\left(\ln \frac{100}{P}\right)(\bar{u}t_d)^{2-n}}. \qquad (5)$$

The factor 2 is eliminated from the numerator if the puffs are not flattened against the ground. Fig. 3 is a graph of this equation, and C computed therefrom is a sort of virtual coefficient, making no distinction

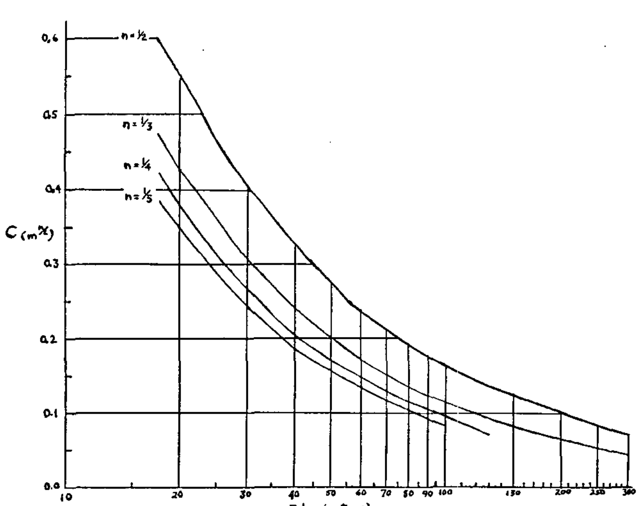

FIG. 3. Diffusion coefficient, C, as a function of stability parameter, n, and dissipation distance, $\bar{u}t_d$, for 3-g powder charges.

between the vertical and horizontal. By estimating the ratio of the vertical dimension of the puff to the horizontal dimension during its visible travel one can obtain C_z and C_y separately using the relationship

$$C^2 = C_z C_y. \tag{6}$$

Use of this method to calculate Sutton-type diffusion coefficients appropriate for instantaneous sources requires only one observer if the wind speed is obtained from an anemometer. It is convenient to use two observers if wind speed is obtained from the travel of the puff, since the latter method requires operation of two stop watches in addition to the puffer.

The appropriate value of n apparently is not an important consideration for any lapse rate of temperature, and this situation can be expected for the most part in the daytime. Even if no instrumentation is available to measure the temperature profile the presence of an inversion can probably be ascertained from the dissipation distance of the puffs.

After this paper was completed, Dr. Frank Gifford pointed out in an informal communication that definition of the visible concentration in terms of an integration of Sutton's equation along the line of sight provides an alternate means for determining C. Such a technique has been employed in a more elaborate treatment of smoke puff data by Kellogg [4] and by Frenkiel and Katz [2], however, using a different technique. In the present case, Gifford shows that C can be determined without recourse to an arbitrary assumption about P. This is possible only if the puff is measured twice, *i.e.*, marker grid dimensions and dissipation time. If r is defined as the visible radius of the puff, then at $t = t_m$, $r = r_m$, and at $t = t_d$, $r = 0$. Integration of Sutton's instantaneous point source equation along the line of sight yields

$$\frac{\chi_m}{Q} = \frac{2}{C^2(\bar{u}t_m)^{2-n}} \exp \frac{-r_m^2}{C^2(\bar{u}t_m)^{2-n}}, \tag{7}$$

where χ_m is the integrated concentration along the line of sight defining the visible edge of the puff at the marker, and Q is the quantity of material emitted. The corresponding equation for dissipation time is

$$\frac{\chi_d}{Q} = \frac{2}{C^2(\bar{u}t_d)^{2-n}}, \tag{8}$$

where χ_d is the integrated concentration through the center of the puff at the time that the puff becomes just invisible. Since $\chi_m = \chi_d$, equations (7) and (8) yield

$$C^2 = \frac{r_m^2}{(2-n)(\bar{u}t_m)^{2-n} \ln (td/t_m)}. \tag{9}$$

Equation (9) does not replace (5), but it could have been used in place of (1) and (2). Of course, if this concept had been used, $A_d \equiv 0$ in equation (4), and equation (5) would be useless, having lost the constant represented by the critical cross-section area of the standard puff at dissipation time. Computing C from equation (9) gives values that are about 15 per cent greater on the average than those listed in table 1, and the range for ten samples picked at random was from zero to 30 per cent greater. While these differences might seem insignificant compared with other imponderables attending diffusion predictions, equation (9) is certainly preferred for computing C where a marker grid and two observers can be used.

An equation analogous to (9) for use with a continuous source is

$$C_{cps}^2 = \frac{r_m^2}{\frac{2-n}{2} x_m^{2-n} \ln \frac{x_d}{x_m}}. \tag{10}$$

Here, r_m is the visible radius of the plume from a cross-wind view at a distance x_m downwind from the source, and x_d is the visible length of the plume.

Diffusion parameters determined in this manner must be superior to those obtained from wind fluctuations for the purpose of making diffusion predictions. In the interest of promoting greater accuracy, the experimenter is again cautioned against the use of such parameters for diffusion on a scale appreciably different from that for which the parameters were determined. Parameters determined for an instantaneous source may not be applicable to a continuous source and *vice-versa*.

Acknowledgments.—The author is most grateful to Murray J. Young, Convair meteorologist, for assistance in planning, taking and analyzing the smoke puff experiments. Colleagues in the Office of Meteorological Research, U. S. Weather Bureau, especially Fred D. White, Donald H. Pack, and Frank Gifford, are thanked for their suggestions and encouragement.

REFERENCES

1. DeMarrais, G. A., 1956: *Wind velocity profiles at Brookhaven National Laboratory during various temperature lapse rate conditions.* (To be published.)
2. Frenkiel, F. N., and I. Katz, 1956: Studies of small-scale diffusion in the atmosphere. *J. Meteor.*, **13**, 388–394.
3. Halstead, M. H., 1954: The fluxes of momentum, heat and water vapor in micrometeorology. *Publications in Climatology*, **VII** (2), Johns Hopkins University.
4. Kellogg, W. W., 1956: Diffusion of smoke in the stratosphere. *J. Meteor.*, **13**, 241–250.
5. Sutton, O. G., 1932: A theory of eddy diffusion in the atmosphere. *Proc. r. Soc.* (London), **135A**, 143.
6. Sutton, O. G., 1947: The theoretical distribution of airborne pollution from factory chimneys. *Quart. J. r. meteor. Soc.*, **73**, 426.
7. U. S. Weather Bureau, 1955: *Meteorology and atomic energy.* Superintendent of Documents, Washington 25, D. C.
8. Wanta, R. C., 1952: *On the mean diffusion coefficients in Sutton's theoretical distribution of airborne pollution from factory chimneys.* (Unpublished.)