AN EXAMINATION OF THE VERTICAL WIND PROFILE IN THE
LOWEST LAYERS OF THE ATMOSPHERE

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ABSTRACT

Wind-profile measurements from 1/4 to 16 m above ground over prairie grass and over a snow surface at
the Suffield Experimental Station and from 2 to 16 m over short grass at O'Neill, Nebraska, have been
analysed in order to determine the relative accuracy of the logarithmic and power-law representation of
the variation of wind speed with height. It was found that over prairie grass and over a snow surface the
wind speed increased more rapidly than predicted by the logarithmic law while, except under strong in-
verson conditions, the data are well represented by a simple power law. The O'Neill data were represented
equally well by the two laws under adiabatic and lapse conditions, while the power law represented the
data better under inversion conditions.

1. Introduction

It appears to be generally accepted that, under
conditions of neutral equilibrium, the vertical profile
of wind speed in the lower atmosphere is most ac-
curately represented by a logarithmic function of the
height above the surface. In theoretical studies of
eddy diffusion of matter from artificial sources, it has
been found necessary to use the so-called power law
of variation of wind speed with height in order to
solve the equations of diffusion. This law states that
the wind speed is proportional to a power of the height
above the surface. Sutton [1] has used the power law
with the assumption that the gradient of wind speed
is indicative of the diffusivity of the air close to the
ground.

While the literature suggests that a “log law”
represents the vertical wind profile more accurately
than the “power law,” there appears to be little
published comparison of the relative exactness of the
two. From the practical point of view, if the difference
between the two is small, then the diffusion equations
of Sutton [1] and others can be used with greater
confidence although the “log law” may be a little
more exact generally.

Studies of the vertical wind profile in the lowest
layers of the atmosphere have been made by a number
of workers in the field of micrometeorology. Best [2]
found that over grass 1 cm high under neutral stability
conditions the wind speed was proportional to the
logarithm of the height in the height range of 2.5 cm
to 500 cm. Thornthwaite and Kaser [3] obtained

\[
\frac{d\bar{u}}{dz} = \alpha \varepsilon^{-\beta},
\]

where \(\bar{u}\) is the mean wind speed, \(\varepsilon\) the height, \(\alpha\) a
constant independent of \(\varepsilon\), and \(\beta\) a constant for a
given temperature gradient or stability. Here \(\beta = 1\)
for adiabatic conditions, \(\beta > 1\) for superadiabatic lapse rates, and \(\beta < 1\) for inversions. \(\beta = 1\) leads to
the logarithmic profile while other values of \(\beta\) lead
to a power law.

This paper presents a comparison of the accuracy
of the logarithmic law and the simple power law under
different conditions of vertical temperature gradients.
The data used for this comparison consist of wind-
profile measurements made at the Suffield Experimen-
tal Station over a snow surface as well as over
prairie grass. In addition, wind-profile data obtained
by the Massachusetts Institute of Technology Round
Hill Field Station at the Great Plains Turbulence
Project at O'Neill, Nebraska [6], were analysed.
2. Experimental

Wind-speed measurements at the Suffield Experimental Station were made with Sheppard-Casella 3-cup contact anemometers connected electrically to recording counters which were photographed at 1-min intervals. Anemometers were located on an open steel tower at heights of \( \frac{1}{2}, 1, 2, 4, 8, \) and 16 m above ground level, the 16-m anemometer on top of the tower and the remainder on rods 30 cm long. The tower was placed so that the anemometers were upwind of the tower.

The site was a level section on an area of gently rolling prairie with no large obstructions within two miles. The snow-free surface consisted of sparse grass 6 to 10 cm high with very sparse stalks up to 30 cm. Wind measurements were made on this surface for a continuous 25-hr period from 1700 26 November to 1800 27 November, 1955. The measurements over snow were made for two periods of one hour each: 1420 to 1530 under lapse conditions, and 1705 to 1805 under inversion conditions, both on 25 February 1957. The snow was moderately smooth, 5 in deep with very sparse stalks of grass protruding 3 to 4 in.

3. Results

Wind profile over prairie grass.—Fig. 1 shows a plot of the mean wind speed for one-hour periods against the logarithm of the height for zero-temperature
gradient and for a 2.0C inversion between 4 and ½ m. The curves obtained are typical of those obtained under these stability conditions, both being curved toward the wind-speed axis, the curvature for inversions being greater than for zero gradient.

In fig. 2, the same data are plotted on log-log paper. It is apparent that the points lie reasonably close to a straight line for zero-gradient conditions, while a very slight curvature towards the wind-speed axis is indicated for the inversion. This shows that the data can be represented much more closely by a power law than by the logarithmic law.

Wind profile over a snow surface.—Fig. 3 shows a plot of wind speed against the logarithm of the height over a snow surface using the mean wind speeds for 40-, 20-, and 50-min periods under lapse, near neutral, and inversion conditions, respectively. The plots show that the best fitting curves are all concave to the wind-speed axis.

Fig. 4 shows the same data plotted on log-log paper. It can be seen that the points lie very close to a straight line for the lapse and inversion. For the slight lapse case, the four lowest points also lie very close to a straight line while the top two are off about 0.3 mph. This could be mainly due to error in measuring the wind speed. At any rate, two of the profiles are well represented by the simple power law.

Wind profiles at O'Neil, Nebraska.—About 40 wind profiles measured by the Round Hill Field Station at O'Neil, Nebraska, were plotted both on semi-log and log-log paper. These were hourly mean values of wind speeds measured at 2, 4, 8, and 16 m above ground under stability conditions which varied from a lapse of 1.6C to an inversion of 3.2C between 2 and 16 m. Fig. 5 shows four typical plots of wind speeds against the logarithm of the height in the above stability range. For adiabatic ($\Delta T = -0.08^\circ C$) and lapse ($\Delta T = -1.45^\circ C$) conditions, the points lie close to a straight line, while for moderate ($\Delta T = 1.5^\circ C$) and strong ($\Delta T = 3.2^\circ C$) inversions, they lie on lines curved towards the wind speed axis, the curvature being greater in the case of the strong inversion.

In fig. 6, the same data are plotted on log-log paper, and it is apparent that the points lie on straight lines except in the case of the strong inversion. In the latter case, a curve can be drawn through the points which
been determined for the stability conditions investigated. Table 1 gives the values of $n$ obtained from

<table>
<thead>
<tr>
<th>Surface</th>
<th>Values of $\Delta T$ and $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prairie</td>
<td></td>
</tr>
<tr>
<td>$\Delta T(4m - 1m) \degree C$</td>
<td>$n$</td>
</tr>
<tr>
<td>Snow</td>
<td>$\Delta T(4m - 1m) \degree C$</td>
</tr>
<tr>
<td>Grass at O'Neill</td>
<td>$\Delta T(16m - 2m) \degree C$</td>
</tr>
<tr>
<td>Grass</td>
<td></td>
</tr>
<tr>
<td>$\Delta T(4m - 1m) \degree C$</td>
<td>$n$</td>
</tr>
<tr>
<td>Snow</td>
<td>$\Delta T(4m - 1m) \degree C$</td>
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</tbody>
</table>

The graphs in figs. 2, 4, and 6 for prairie grass, a snow surface, and over grass at O'Neill, respectively. It is interesting to compare the values of $n$ obtained from the profiles over the different surfaces and its variation with stability. For example, $n$ is somewhat greater over prairie grass than over snow under similar stability conditions, indicating that it increases with increasing roughness of the surface. It is also greater over prairie grass than over the terrain at O'Neill.

The value of $n$ (0.165) obtained over the snow surface under near adiabatic conditions is slightly greater than the value of 0.149 reported by Frost [4] from observations from 4 to 1000 ft made at Cardington, also under adiabatic conditions. However, it is difficult to compare these results since $n$ probably decreases somewhat with height.

The increase in the value of $n$ with increasing stability is evident from the table. However, this increase is not as great as was found by Frost [4] for the profile up to 400 ft at Cardington where $n$ increased from 0.145 to 0.77 as the temperature gradient between 400 and 4 ft varied from 3F lapse to 12F inversion.

Variation of gustiness with height.—The wind-profile measurements over prairie grass and over a snow surface afforded an opportunity to determine the variation of gustiness with height. Table 2 gives the values of the gustiness $\sigma/\bar{u}$ as determined from the standard deviation of one-minute wind speeds $\sigma$, and the mean one-hour wind speeds $\bar{u}$, for heights of $\frac{1}{4}$, 1,
2, 4, 8, and 16 m, over a snow surface for lapse and for inversion conditions.

Table 2 shows that the standard deviation increases with height in both cases and is somewhat greater for lapse conditions. However, \( \sigma/\bar{u} \) varies little with height for lapse conditions while it decreases considerably with height for the inversion period. These results may be interpreted as showing that, for the inversion case, the contribution of mechanical turbulence is greater in the low levels than for lapse conditions.

4. Discussion

The results of the wind-profile measurements over prairie grass and over a snow surface show that the wind speed increases more rapidly with height than predicted by the logarithmic law, even under adiabatic and superadiabatic conditions. There is some indication, however, that there is closer agreement with this law at the low levels than at the higher levels. It may be pointed out that most of the data which have given good agreement with the logarithmic law have extended only up to 5 or 8 m while Frost [4] reported good agreement with the power law from 4 ft to 1000 ft. This suggests the possibility that in general the logarithmic law may be valid at low levels while the power law may give better agreement at levels above 8 m, for instance. However, the data analysed in this paper are represented as well or better by the power law, even at low levels.

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REFERENCES

6. Massachusetts Institute of Technology Round Hill Field Station, 1953: Summary of observations from slow-response instrumentation at O’Neill, Nebraska, during August and September, 1953.