METEOROLOGICAL ACCURACIES IN MISSILE TESTING

By Robert W. Lenhard, Jr., Captain, USAF

Air Weather Service, Andrews Air Force, Base, Maryland

(Manuscript received 28 October 1958)

ABSTRACT

The accuracy with which meteorological elements can be measured by currently operational radiosonde equipment is examined and representative error values calculated. A method of analyzing cross-sections for missile test ranges is described. The average of two soundings is plotted at the mid-point between them and the soundings reconstructed by use of the geostrophic extrapolation. The gain in accuracy from averaging is offset by the geostrophic approximation. These opposing effects are shown by error values calculated for a fixed range distance and by distance values over which the analysis is useful.

1. Introduction

When testing aircraft and missiles during their development, the state of the atmosphere through which they pass must be known with considerable accuracy in order to evaluate the performance of the vehicle. Similarly, accurate knowledge of existing atmospheric conditions is desired when making up ballistic tables for bombs, missiles, and artillery shells.

Radiosondes cannot follow exactly the trajectory of a bomb or missile and on-board instrumentation is not usually a very practical solution even when it is possible. Hence, some sort of range analysis of the meteorological data is necessary. A few simple methods of analysis will be examined to estimate the accuracies attainable with them using measurements obtained with conventional equipment.

2. Space and time limitations

Non-meteorological range instrumentation may be either fixed or mobile but fixed equipment is preferred and used most frequently. In order to use fixed equipment, testing must take place along a planned course. For a particular range, then, a vertical plane can be specified in which most test trajectories will be contained. Meteorological instrumentation is normally set up to provide measurements in or near this plane. The range analysis includes as a major feature, or may consist entirely of, a cross-sectional analysis of the measurements obtained.

It is not usually possible to have tests and soundings coincide exactly in space and time. Deviations may or may not introduce significant errors depending on how great they are. Persistence of meteorological events is a recognized feature that is often used in forecasting. If the persistence error is defined as the difference between two observations separated in either space or time, this error will increase from zero with increasing separation of the observations. Not until this error becomes as great as the instrument error is it possible to determine whether an observed difference is real or not. This fact provides a practical method for determining how far in space and time an observation is representative.

This study will be restricted to vertical analysis and so is applicable directly only when observations are suitably arranged in space and time. When non-representative observations are used and extrapolated, additional errors are introduced. If the magnitude of the errors due to horizontal and temporal extrapolation are known, the results of this investigation may be appropriately modified.

3. Range description and analysis

An hypothetical range will be discussed with two sounding stations in the test plane separated by 98 km (61 mi). This value is selected for arithmetic convenience but is not unrealistic. It is presumed that detailed atmospheric data are desired above and between both points—for a ballistic trajectory, for example. The parameters of interest are wind direction and speed, height of pressure surfaces, temperature, density, and dynamic pressure.

Data plotting.—A convenient chart for the analysis is one with distance along the test plane as abscissa and standard atmosphere height as ordinate. This is, approximately, also an ordinate of log p and both scales should be entered. Data are plotted at convenient intervals, such as standard pressure surfaces or 50-mb intervals, that will provide the required detail for the applications to be made of the analysis.

The entries made for each reporting surface would be the height expressed as altimeter correction $D = Z - Z_P$, the temperature, or mean virtual temperature if this differs from the temperature, and a wind measure. The wind can be entered directly as direction and velocity, or the magnitude of the wind component normal to the plane of analysis can be used. The symbols Z and Z_P represent the true height and the standard atmosphere height, respectively [3].

The linear height scale can be used as a true height scale instead of pressure height, and winds can be plotted against this if desired.

Analysis.—Winds are analyzed using isogons and isotachs and direction and velocity at successive levels along the trajectory are then extracted as desired. *D*-values are analyzed on the pressure-height scale with gradients checked for consistency by considering the plotted winds geostrophically. If the component normal to the plane has not been computed and plotted previously, it can be obtained during analysis by the use of a special wind scale. Pressure values along the trajectory can be extracted for actual heights by converting *D*-value to a pressure correction. Temperature is similarly analyzed and gradients may be checked geostrophically if wind shears are computed and plotted. The temperature profile along the trajectory can be extracted for actual height as with pressure.

Averaging.—In applying the geostrophic approximation to gradients of pressure and temperature, contradictions will arise and have to be resolved. Are winds ageostrophic, or is the discrepancy due to instrument error; if the latter is true, which instrument is in error? To resolve these conflicts requires careful analysis of the individual soundings, extraneous data, and consideration of each conflict on its own merits.

A simpler method of resolving these conflicts is to average the two soundings and assume that the average sounding exists at the mid-point of the range. The geostrophic approximation can then be applied to the average soundings to reconstruct the soundings at the end of the range. The analysis can be performed graphically with the aid of appropriately constructed geostrophic scales.

This method is particularly effective when the space variability is not greater than the instrument error. With this condition, the value at one observation point cannot be said to be significantly different from that at the other and the two measurements can be considered as sample values of a single-valued parameter. The best estimate of the true value is the mean of the sample values and the standard error of this estimate is given by $\sigma_{\bar{\phi}} = \sigma_\phi/(N)^{\frac{1}{2}}$ where σ_ϕ is the standard deviation in the population and N is the sample size. In this case, there are but two items so that $\sigma_{\bar{\phi}} = 0.707\sigma_\phi$. The sample items are taken to be a set of measurements of a single-valued parameter ϕ so that σ_ϕ is the root mean square (rms) error of observation. Values of σ_ϕ for this study are obtained from previous investigations and estimates of instrument errors.

4. Accuracy

Throughout this section, the accuracy of individual soundings will be compared with that obtained over the ends of the range from averaged soundings that have been extrapolated by means of a geostrophically estimated gradient. These will be referred to as adjusted soundings. An assumption that will be made frequently is that errors are independent so that total variance is equal to the sum of the component variances: $\sigma^2 = \sigma_1^2 + \sigma_2^2$.

Wind direction and velocity.—The errors that occur in measuring winds have been investigated by Plagge and Smith [10], Rapp [11], and Singer [13], among others. There is general agreement that the error increases with altitude and that a reasonable average value is 3 to 5 kn rms vector error for current rawinsonde equipment such as the AN/GMD-1. The higher value may be considered applicable to routine field observations and the lower to carefully made, experimental observations. If it is assumed that the five-knot value is applicable at 300 mb, the spectrum of vector wind errors presented in table 1 can be constructed. The vector wind-error profile is made to compare reasonably well with those presented by Rapp [11], and the component error is computed assuming independence. Further investigation of the variation with height in the accuracy of wind measurements is desirable. The introduction of GMD-2 equipment should reduce errors. Comparison of GMD-2 with GMD-1 runs may result in the adoption of some smoothing technique to reduce GMD-1 errors such as that used by Reiter [12] in examining the reliabilitity of upper-wind data.

TABLE 1. Wind measurement errors.

Pressure (mbs)	ICAO Standard height*	Rms errors (fps)	
		Vector wind	Normal component
700	9882	6	4.2
500	18289	7	4.9
300	30065	8	5.7
200	38662	10	7.1
100	53083	12	8.5
50	67503	15	10.6
25	81926	18	12.7
10	102300**	22	15.6

* Geopotential feet with base $g = 9.80665$ msec^{-2}.
** Interpolated from tables of ARDC Model Atmosphere.

Height of pressure surfaces.—Heights of constant-pressure surfaces are determined by a solution of the hydrostatic equation to obtain the thicknesses of successive layers. The error Δh in determining the thickness of a layer is due to the error in determining the mean virtual temperature of the layer and the mean error in the measurement of pressure. If P_1 and P_2 are the lower and upper boundary pressures, $\overline{\Delta T}$ the mean error in virtual temperature, $\overline{\Delta P}$ the mean error in pressure, $\partial T/\partial P$ the baric lapse rate, and c a constant, $\Delta h = c[\overline{\Delta T} - \overline{\Delta P}(\partial T/\partial P)] \ln P_1/P_2$. These errors have been evaluated [2] and combined as independent errors to yield the height errors shown in table 2 for the individual sounding. These rms errors in height apply also to D-values since $D = Z - Z_P$ and there is no rms error in the mathematically defined parameter Z_P.

When the averaged sounding is used, the height of a pressure surface over a station at the end of the range is given by $H = Z_P + D + x \tan\theta$ where x is one half the length of the range and θ is the slope of the constant-pressure surface in the vertical plane through the two stations. This angle is related to the geostrophic wind so that $\tan\theta = fv_g/g$. Here, f is the coriolis parameter, v_g the geostrophic wind normal to the plane, and g is the acceleration of gravity. Substitution for $\tan\theta$ yields $H = Z_P + D + fv_gx/g$.

Partial differentiation is used to examine the effect of errors;

$$\Delta H_D = (\partial H/\partial D)\Delta D = \Delta D$$

and

$$\Delta H_{vg} = (\partial H/\partial v_g)\Delta v_g = (fx/g)\Delta v_g$$

which are then combined to form $\Delta H = \Delta H_D + \Delta H_{vg}$. The errors in wind and in D-value are considered to be independent so that the rms error in height after extrapolation is given by $\sigma_H^2 = \sigma_D^2 + (fx/g)^2\sigma_{vg}^2$.

The error in D-value is the error in the averaged sounding so that $\sigma_{\bar{D}}^2 = \sigma_D^2/2$ must be used. Further, the geostrophic wind equation is an approximation and will introduce errors so that the geostrophic wind error variance σ_{vg}^2 must be considered as the sum of the component due to measurement of the wind σ_v^2 and that due to the geostrophic approximation σ_g^2. Since two soundings are averaged, $\sigma_H^2 = \sigma_{\bar{D}}^2 + (fx/g)^2\sigma_{\bar{v}g}^2$ which expands to $\sigma_H^2 = \sigma_D^2/2 + (fx/g)^2(\sigma_{\bar{v}}^2 + \sigma_g^2)$.

Murray [8] has examined the error due to the geostrophic approximation as have Giles and Peterson [5]. Their results indicate that the geostrophic departure is one to two times as large as the measurement error. Durst and Gilbert [4] obtained similar departure values. Neiburger and Angell [9] found a somewhat larger ratio, on the average. Godson [6] found that the geostrophic departure increased with wind speed with a mean vector error of about one-fourth the speed for winds over 20 kn. For convenience of computation, a ratio of $(3.5)^{\frac{1}{2}}$ will be used; that is, $\sigma_g^2 = 7\sigma_v^2/2$. This is within the range of values found by most investigators. Furthermore, doubling the ratio has very little effect on the final error evaluation. For the specified range, $x = 49$ km, $f = 1 \times 10^{-4}$, and $g = 980$ cm sec^{-2} so that $fx/g = 0.5$ sec. By use of the selected ratio for the geostrophic approximation error with the range values, $\sigma_H^2 = \sigma_D^2/2 + \sigma_v^2$ where σ_v is in length units per second. Values of wind-measurement error used in computing height errors after adjustment are taken from those for the normal component after rounding and are listed in table 2 along with the height errors.

TABLE 2. Errors in determining height of pressure surfaces.

Pressure (mb)	Rms wind error (fps)	Rms height errors (ft)		
		Single sounding	Average sounding	Adjusted sounding
700	4	34	24.0	24.4
500	5	67	47.4	47.6
300	6	119	84.2	84.4
200	7	161	113.8	114.1
100	9	229	161.9	162.2
50	11	291	206.0	206.1
25	13	355	251.0	251.4
10	16	456	322.4	322.8

Temperature.—Instrumental errors in temperature are discussed in Air Weather Service Technical Report 105–133 [2]. A reasonable value at any pressure altitude is an rms error of 1C. If the profile of temperature against pressure is desired, allowance must be made for the errors in reported pressure. Pressure sensing errors are taken as 3 mb up to and including 200 mb, 2 mb at 100 mb, and 1.5 mb at the higher levels. Table 3 lists the parametric values used for the computation and the rms errors of temperature at the listed pressures. The baric lapse rates in table 3 were determined from the mean sounding given in Air Weather Service Technical Report 105–108 [1]. The table shows that the error of pressure has a negligible effect on the error of temperature as a function of height except at very great altitudes.

TABLE 3. Errors in determining temperature at specific pressure surfaces from individual soundings.

Pressure (mb)	$\partial T/\partial P$ (°C/mb)	Rms errors (°C)		
		Sensing	Pressure	Total
700	0.06	1.00	0.18	1.015
500	0.09	1.00	0.27	1.036
300	0.14	1.00	0.42	1.085
200	0.13	1.00	0.39	1.073
100	−0.02	1.00	−0.04	1.008
50	−0.16	1.00	−0.24	1.028
25	−0.32	1.00	−0.48	1.109
10	−1.12	1.00	−1.68	1.955

If an averaged sounding is used for range analysis, it is necessary to determine the error in estimating the temperature T' over a sounding station. If the averaged temperature $\bar{T}$ is assumed to be the mean for a layer across which the geostrophic wind shear component ψ normal to the plane of analysis is computed, the horizontal temperature gradient can be obtained. Let $G = T' - \bar{T}$ be the temperature difference as computed using the shear. Then the error in T' is obtained from $\sigma_{T'}^2 = \sigma_{\bar{T}}^2 + \sigma_G^2$.

The approximate thermal-wind equation can be writen as $\psi f/g = G/Tx$ where ψ is in units of sec^{-1}. Errors in G can be examined by differentiation,

$$\Delta G_T = (\partial G/\partial T)\Delta T = (fx\psi/g)\Delta T$$
$$\Delta G_\psi = (\partial G/\partial \psi)\Delta\psi = (fxT/g)\Delta\psi,$$

and combined into a statement of rms errors by assuming independence between shear and temperature errors so that

$$\sigma_G^2 = (fxT\psi/g)^2[(\sigma_T/T)^2 + (\sigma_\psi/\psi)^2].$$

Tolefson [14] has examined shears and their errors and the results indicate that, in general, $\sigma_\psi/\psi > 0.2$ which is several orders of magnitude larger than σ_T/T. Neglecting the latter yields, $\sigma_G^2 = (fxT\sigma_\psi/g)^2$. For the range selected, $fx/g = 0.5$ sec and the error in estimated temperature becomes $\sigma_{T'}^2 = \sigma_{\bar{T}}^2 + T^2\sigma_\psi^2/4$ where σ_ψ is in units of sec^{-1}.

The errors in shear can be related to the errors in wind. For two related variables, the standard deviation of their difference is given by $\sigma_{1-2}^2 = \sigma_1^2 + \sigma_2^2 - 2r_{12}\sigma_1\sigma_2$. The difference may be regarded as an error in shear and the standard deviation taken as the rms value of that error. Then σ_1 and σ_2 are the rms errors in winds at two successive levels, and r_{12} is the correlation coefficient between these errors. For layers not over a few thousand feet in depth, the errors in boundary level winds will be of about the same magnitude so that $\sigma_1 = \sigma_2 = \sigma$ and $\sigma_{1-2}^2 = 2\sigma^2(1 - r_{12})$.

In the case of averaged soundings, geostrophic winds and shear are being considered so that both measurement and approximation errors must be included. Previous analysis used

$$\sigma_{\bar{v}g}^2 \doteq \sigma_{\bar{v}}^2 + \sigma_g^2 = \sigma_v^2/2 + 7\sigma_v^2/2 = 4\sigma_v^2.$$

This value applied to the shear yields

$$\sigma_\psi^2 = 8\sigma_v^2(1 - r_{12})/\Delta H^2.$$

The error in estimated temperature now becomes

$$\sigma_{T'}^2 = \sigma_{\bar{T}}^2 + 2T^2\sigma_v^2(1 - r_{12})/\Delta H^2.$$

Here, ΔH is the depth of the layer across which the shear is determined.

Additional information is needed to evaluate r_{12}, the correlation between errors in measuring winds at successive levels. Reiter [12] has analyzed the fluctuations about a smoothed profile of winds obtained by GMD-1 equipment and found evidence of quasi-periodic oscillations. Preliminary comparisons between GMD-1 and GMD-2 soundings have shown similar oscillations. The inference is that these oscillations represent errors arising from measuring and data-processing limitations. Reiter has examined the periods of these oscillations and found the most frequent ones to be in the range of two to six minutes, nominally corresponding to layers 2000 to 6000 ft deep. If a shear is measured across a full oscillation, the errors in wind should be almost perfectly correlated in a positive sense; if measured across a half oscillation, the correlation will be almost perfectly negative. The frequency of various periods obtained by Reiter [12] can be used to weight probable correlation coefficients to make an estimate of the correlation for a layer of specified depth. Periods that yield correlation coefficients of +1 or −1 are considered individually; all others are grouped into an assumed random residual with zero correlation. The results are presented in table 4 for layers 2000 and 4000 ft thick.

TABLE 4. Correlation between boundary level wind errors for shear layers of 2000- and 4000-ft depth.

Period (min)	Frequency (per cent)	Layer depth (ft)	Correlation coefficient	
			For period	For layer
2	33		+1	
4	35	2000	−1	−0.02
Other	32		0	
2	33		+1	
4	35	4000	+1	+0.56
8	12		−1	
Other	20		0	

The equations are

$$2000 \text{ ft layer} - \sigma_{T'}^2 = \sigma_T^2/2 + 0.51 \times 10^{-6}(T\sigma_v)^2$$

and

$$4000 \text{ ft layer} - \sigma_{T'}^2 = \sigma_T^2/2 + 0.055 \times 10^{-6}(T\sigma_v)^2.$$

An alternative hypothesis is that wind-observation errors are random when compared over intervals of a minute or more, and that is essentially zero for layers several thousand feet thick in which case the equations are

$$2000 \text{ ft layer} - \sigma_{T'}^2 = \sigma_T^2/2 + 0.5 \times 10^{-6}(T\sigma_v)^2$$

and

$$4000 \text{ ft layer} - \sigma_{T'}^2 = \sigma_T^2/2 + 0.125 \times 10^{-6}(T\sigma_v)^2.$$

Since the latter two equations will usually provide the larger value for estimated temperature errors, only these have been computed. The parametric values and resulting total errors are presented in table 5. The rms-error values are not likely to be too small, and it is at least possible that they are slightly too large. The values of temperature are from Air Weather Service Technical Report 105–108 [1]. It can be seen that the adjusted values are at best only slightly more accurate than the original readings from individual soundings for the most part. Improvement comes only

TABLE 5. Residual errors in determing temperature at specific pressure levels after adjustment of soundings.

Pressure (mb)	Temp (°C)	σ_v (fps)	Rms temperature errors (°C)			
			σ_T	$\sigma_{\bar{T}}$	$\sigma_{T'}$ for layer of 2000 ft	$\sigma_{T'}$ for layer of 4000 ft
700	+ 1	4.2	1.02	0.72	1.08	0.83
500	−14	4.9	1.04	0.74	1.16	0.87
300	−40	5.7	1.09	0.77	1.21	0.90
200	−54	7.1	1.07	0.76	1.34	0.94
100	−61	8.5	1.01	0.71	1.46	0.95
50	−57	10.6	1.03	0.73	1.78	1.09
25	−51	12.7	1.11	0.78	2.14	1.27
10	−45	15.6	1.96	1.39	2.87	1.87

with shears taken over a fairly deep layer, in which case the assumption that the point temperature is the mean for the layer may be invalid.

Density.—From the equation of state, the density ρ of a gas is given by $\rho = P/RT$ where R is the gas constant. Partial differentiation can be used to examine the errors in computing density;

$$\Delta p_T = (\partial P/\partial T)\Delta T = (-P/RT^2)\Delta T,$$

$$\Delta \rho_P = (\partial \rho/\partial P)\Delta P = \Delta P/RT,$$

$$\Delta \rho = (P/RT)(-\Delta T/T + \Delta P/P).$$

If density error is expressed as a percentage, the rms error in density at the instrument can be obtained from $\sigma_\rho^2/\rho^2 = (\sigma_T/T)^2 + (\sigma_P/P)^2$ where σ_T and σ_P are the rms errors of measurement of temperature and pressure, respectively.

When density soundings are adjusted by use of the geostrophic approximation, the equation for the final error involves the wind component normal to the plane of analysis. This is a climatological quantity. In order to avoid localization of this study, ρ_P/ρ can be expressed in terms of height by use of the equation of state and the hydrostatic equation can be expressed as $\sigma_\rho^2/\rho^2 = (\sigma_T/T)^2 + (g\sigma_z/RT)^2$.

When used in this form, care must be taken in selecting appropriate values for σ_T and σ_z. The equation as derived expresses the rms error in density computed from the reported readings. This is the density error at the point in space where the instrument is located. The appropriate value for σ_T is 1C, the sensing error of the instrument. The appropriate value for σ_z is the error in determining the height of the instrument. These errors have been estimated [2] and are much larger than those that occur in determining the height of a pressure surface. Only the latter will be examined in detail.

For a constant-pressure surface, $\sigma_P = 0$, so that the error in density is $\sigma_\rho = \sigma_T/T$ where the error in temperature has been adjusted for the errors in reading pressure. This is the error in temperature for a specific pressure surface as given in the total column of table 3 and the $\sigma_{T'}$ columns of table 5.

When the height of the constant-pressure surface is computed, there is an error involved that can be converted into an error in density at the computed height. This conversion produces the same equation involving height as before but requires the use of different height errors. For the error in density at heights computed for pressure surfaces, the appropriate values for σ_z are the height errors given in table 2. These have been combined with the temperature errors for individual soundings and for adjusted soundings using 2000- and 4000-ft-thick shear layers. The component errors and resulting density errors are presented in table 6.

TABLE 6a. Percentage errors in density and dynamic pressure as functions of height for individual soundings.

Pressure (mb)	Rms errors			
	Temp (°C)	Height (ft)	Density (%)	Dynamic press. (%)
700	1.02	34	0.39	0.64
500	1.04	67	0.48	0.70
300	1.09	119	0.71	0.87
200	1.07	161	0.91	1.04
100	1.01	229	1.22	1.32
50	1.03	291	1.48	1.56
25	1.11	355	1.74	1.81
10	1.96	456	2.25	2.31

TABLE 6b. Percentage errors in density and dynamic pressure as functions of height for adjusted soundings.

Pressure (mb)	Rms errors						
	Height (ft)	2000-ft shear layer			4000-ft shear layer		
		Temp (°C)	Density (%)	Dynamic pressure (%)	Temp (°C)	Density (%)	Dynamic pressure (%)
700	24.4	1.08	0.41	0.64	0.84	0.32	0.59
500	47.6	1.16	0.49	0.70	0.87	0.39	0.63
300	84.4	1.21	0.64	0.81	0.90	0.54	0.74
200	114.1	1.34	0.82	0.96	0.94	0.69	0.85
100	162.2	1.46	1.05	1.17	0.95	0.91	1.04
50	206.1	1.78	1.29	1.38	1.09	1.11	1.22
25	251.4	2.14	1.52	1.60	1.27	1.31	1.40
10	322.8	2.87	1.94	2.00	1.87	1.69	1.76

Dynamic pressure.—The dynamic pressure is defined as $q = K\rho W^2$ where K is a constant and W is the missile velocity. The equation for the rms error in dynamic pressure is that for density with an additional term to include errors in estimating the velocity of the missile—

$$\sigma_q^2/q^2 = (\sigma_T/T)^2 + (g\sigma_z/RT)^2 + (3\sigma_W/W^2.$$

A similar analysis has been done by Gordon [7] and he used values of $W = 2000$ fps and $\sigma_W = 5$ fps. These values, when combined with the various density errors, provide the rms errors in dynamic pressure given in table 6.

5. Conclusion

When two soundings are sampling an environment, a decrease in the effects of instrument errors is achieved by averaging the soundings and using the geostrophic wind to establish horizontal variations in the quantities. A gain of 30 per cent achieved by averaging is partially offset by the use of geostrophic approximation. This loss is not great if not carried over too great a distance.

Accuracy also depends on the depth of the layer across which shear is computed. A depth of 4000 ft reduces the gain in temperature accuracy considerably. Greater accuracy would be obtained with deeper shear layers but these seem unrealistic when it is actually the temperature at a level and not the mean layer temperature that is being extrapolated. Smaller shear layers, such as 2000 ft, reduce the accuracy gained by averaging so drastically that it is better to use individual soundings even over short ranges.

The effect of distance on the accuracy resulting from this type of analysis can be examined, and this effect can be used to determine the appropriate spacings of soundings along the range. For the latter purpose, there are two criteria:

(1) There is no point in extending the analysis to distances where its error would be greater than for individual soundings.

(2) There is no point in placing soundings so close together that errors after averaging and extrapolating are larger than the spatial variability of the parameter.

These criteria may be applied to the error equations which may then be solved for critical distance. The coriolis parameter is taken as 1×10^{-4} sec^{-1} and zero correlation is assumed for errors in winds bounding a shear layer.

(1) By setting the residual error after averaging and extrapolating equal to the error in individual soundings, the following equations are obtained:

$$x_T = 8 \times 10^4 \Delta H \sigma_T / T \sigma_v,$$
$$x_D = 11.3 \times 10^4 \sigma_D / \sigma_v,$$

and

$$x_\rho = 11.3 \times 10^4 \Delta H (\sigma_T^2 + k^2 \sigma_D^2)^{\frac{1}{2}} / \sigma_v (2T^2 + k^2 \Delta H^2)^{\frac{1}{2}},$$

where $k = g/R$ and x is in feet. The appropriate values for the specified range have been computed and are presented in table 7.

TABLE 7. Distance in miles from mid-range to point at which errors in adjusted soundings are equal to those in individual soundings.

Pressure (mb)	Distance in miles to equivalence of error for		
	Temperature	Height	Density
700	54	174	57
500	50	293	59
300	50	447	75
200	42	486	77
100	34	577	86
50	27	588	84
25	24	599	82
10	33	626	87

(2) The spatial variability S_T of an element such as temperature is defined as $S_T^2 = \sum (T_0 - T_x)^2/N$ where T_0 is the temperature at the mid point or origin and T_x the temperature at a distance x away from the origin. It is not illogical to expect that S is a function of x. Equating S to the errors after adjustment is equivalent to equating the errors to the function of x. Thus, if $S_T = f_T(x)$ for temperature and similarly for other elements, the equations for temperature, height and density are:

$$[f_T(x)]^2 - \sigma_T^2/2 = (xT\sigma_v)^2/(2^7 \times 10^8 \Delta H^2),$$
$$[f_D(x)]^2 - \sigma_D^2/2 = x^2\sigma_v^2/(2 \times 10)^8,$$

and

$$[f_\rho(x)]^2 - (\sigma_T^2 + k^2\sigma_D^2)/2T^2 = x^2\sigma_v^2(2T^2 + k^2\Delta H^2)/(T\Delta H)^2(2 \times 10^8).$$

The values of S as a function of x can be obtained climatologically.

The actual values of parameters quoted in this study are meant to be indicative, not definitive. The accuracies stated are probably representative but, although quoted as rms errors, should not have the probabilities attached to them that are normally associated with the rms statistic. Greater precision could be obtained by a more intensive examination of available data (*e.g.*, incorporation of the dispersion of the baric lapse rates rather than the mean). It is doubtful if this is worth while unless additional experimental investigation is also accomplished (*e.g.*, on the variation of wind measurement error with height). When additional information on inherent instrument accuracies becomes available, errors of analysis can be determined more precisely and appropriately for specific ranges, incorporating the local climatology.

6. Acknowledgment

The concept and details of accomplishing the cross-sectional analysis were proposed by Dr. K. R. Johannessen of Air Weather Service during a consultant visit in November 1955 to the detachment providing weather support at White Sands Proving Ground, New Mexico. The concept of reducing errors by averaging was proposed by Captain Rodney S. Wheeler of the same weather detachment and its particular applicability verified by Lieutenant John M. Pierrard through a preliminary examination of the time and space variability on the White Sands Proving Ground. Further investigation of space and time variability between El Paso, Texas and Albuquerque, New

Mexico was accomplished by the Directorate of Climatology, Air Weather Service. Dr. Johannessen also contributed many valuable suggestions during the preparation of this report.

REFERENCES

1. Air Weather Service, 1953: *Temperatures at the 10-mb level.* AWS Tech. Rep. No. 105–108, 26 pp. & appendix.
2. ——, 1955: *Accuracies of radiosonde data.* AWS Tech. Rep. No. 105–133, 12 pp.
3. Bellamy, J. C., 1945: The use of pressure altitude and altimeter corrections in meteorology. *J. Meteor.*, **2**, 1–79.
4. Durst, C. S., and G. H. Gilbert, 1950: Constant height balloons—calculation of geostrophic departures. *Quart. J. r. meteor. Soc.*, **76**, 75–88.
5. Giles, K. C., and R. E. Peterson, 1957: A study of wind velocities from constant pressure balloons. *J. Meteor.*, **14**, 569–572.
6. Godson, W. L., 1950: A study of the deviations of wind speeds and directions from the geostrophic values. *Quart. J. r. meteor. Soc.*, **76**, 3–15.
7. Gordon, J. J., 1954: *Effect of the accuracy of Air Weather data on dynamic pressure measurements.* Unpubl. memo. Douglas Aircraft Co., 9 pp.
8. Murray, R., 1951: Practical value of the contour chart as a method of representing upper winds. *Meteor. Res. Pap. No. 663*, Meteor. Res. Comm., 6 pp.
9. Neiburger, M., and J. K. Angell, 1956: Meteorological applications of constant pressure balloon trajectories. *J. Meteor.*, **13**, 166–194.
10. Plagge, H. J., and L. B. Smith, 1956: *Project Rawijet—A study of the wind variability in space and time at the Salton Sea test base.* Sandia Corp. Res. Rep. SC-3880(TR).
11. Rapp, R. R., 1952: The effect of variability and instrument error on measurements in the free atmosphere. *Meteor. Pap., N. Y. Univ.*, **2**, No. 1, 41 pp.
12. Reiter, E. A., 1958: The layer of maximum wind. *J. Meteor.*, **15**, 27–43.
13. Singer, B. M., 1955: Wind variability as a function of time. *Air Force Surv. in Geophys. No. 72*, Air Research and Development Command.
14. Tolefson, H. B., 1956: *An investigation of vertical wind shear intensities from balloon soundings for application to airplane and missile response problems.* Natl. Advis. Comm. Aero., Tech. Note, No. 3732, 33 pp.

Author's Postscipt

Recent experience and investigations that are either unreported or incompletely analyzed indicate that the overall system error in temperature for the GMD-1 is in the neighborhood of 0.5C rather than a sensing error alone of 1C, at least up to about 80,000 ft. This will have a great effect on the accuracy of density computations. The temperature errors in tables 3 and 6 and the density and dynamic-pressure errors in table 6 are probably an overestimate of the rms values. This indication of higher temperature accuracy refers to standard equipment and not to special instruments equipped with coated-bead thermistors and hypsometers. Such instruments would, of course, provide even greater accuracy.