THE SHAPE OF RAINDROPS

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ABSTRACT

An investigation of the physical shape of raindrops using two cameras at right angles is described, and the results are tabulated and graphed. The data included measurements of 1783 raindrops of which 569 were classified as spherical, 496 as oblate, 331 as prolate, and 387 unclassified. The sizes measured ranged up to 6.4 mm equivalent spherical diameter. It is concluded that there is a mean shape which varies uniformly with the mass of the raindrop, but that this shape is the result of oscillation about the mean.

1. Introduction

Investigations of the physical shape of raindrops, both theoretical and observational, have been made by many investigators; however, those investigations have been based upon data gathered from measurements in two dimensions only. With the exception of Kumai and Itagaki (1954), all of the observational data have been collected in the laboratory. Consequently, there has been a need for observational data collected in three dimensions in natural rain. The present study was conducted in central Illinois during 1955 and 1956.

When water drops larger than 1.0 mm are released to fall freely in stagnant air, they are observed to flatten on the bottom and spread laterally while remaining rounded on the top surface. The forces causing this deformation of the water drops have been thoroughly discussed by McDonald (1954). As the equivalent spherical diameter of the water drop increases above 2.0 mm, harmonic distortions of the simpler oblate deformation observed in the smaller drops become increasingly important. Laboratory investigators are not agreed upon either the source of these oscillations or the region of drop size over which the three types of oscillations (rotational about the vertical axis, vibrational along the vertical axis, and vibrational along a horizontal axis) are most important. Blanchard (1949, 1950) reports observations of vibrational oscillations which he states occur only with drops smaller than 5.0 mm equivalent spherical diameter. These vibrational oscillations cause the water drop to oscillate between an oblate and prolate shape, the drop being oblate when its major axis is horizontal to the plane of the earth and the drop being prolate when its major axis is vertical. Between 4.5 and 5.5 mm, Blanchard states that deformation increases rapidly with the preferred oscillation becoming a rotational oscillation of a prolate ellipsoid about the vertical axis. He believes that the horizontal vibrational oscillation causes few drops larger than 5.5 mm equivalent spherical diameter to be observed in nature because the vibrating drops are easily shattered by slight shocks induced by sudden changes in velocity probably present in the atmosphere. Drops larger than 5.5 mm are relatively easy to form and maintain in the laboratory. In contradiction to the observations of Blanchard, Magono (1954) has observed a preferred oscillation along the vertical axes of drops of an equivalent spherical diameter of 5.9 mm, but, at the same time, he implies that the oscillation is due to the method by which he formed his water drops. The author has observed a similar vertical oscillation in early experimental work in methods of photographing water drops in the laboratory when the drops were formed by allowing tap water to drip from holes drilled in a pipe at a height of 30 feet above the point of observation. However, the size of the water drops was not determined.

Gunn (1949) reported an investigation into the mechanical resonance of freely falling water drops which he explained as having been caused by the natural frequency of water drops between 350 and 700 micrograms (0.87 and 1.10 mm diam) being excited at the right frequency by the detachment of eddies in the wake of the falling drop. At present, perhaps the best summation of the laboratory observations of the oscillations of water drops has been expressed by Best (1947) who said that the natural state may be "one of oscillation about a mean shape." It is certainly true that the gustiness and possibly even the effect of a steady wind field prevailing at the time of the rainstorm can only add to the difficulty in understanding the oscillations of a water drop falling in the atmosphere.

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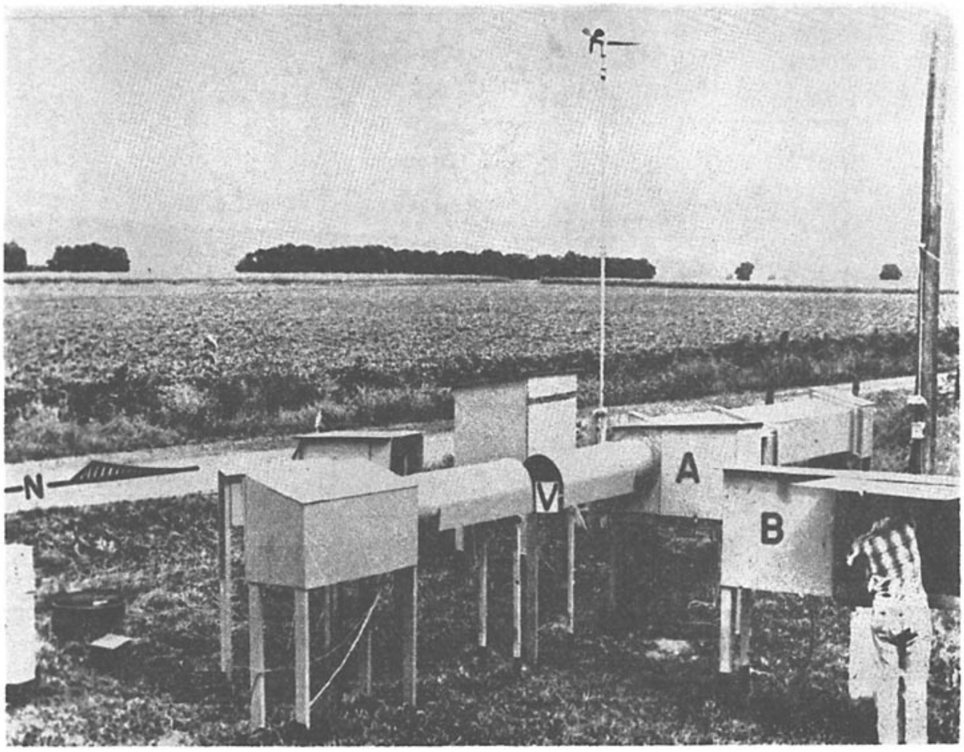

FIG. 1. Field installation of raindrop cameras.

2. Instrumentation

The data collection instruments used in this investigation are essentially two short-exposure 35-mm cameras synchronized to simultaneously photograph a volume in space from two different angles. The angle subtended from the center of the volume by the horizontal optical axes of the camera is 90 deg. Thus, the only common dimension of the raindrops measured by both cameras is the vertical. Fig. 1 is a photograph of the camera installation. The sampling volume, V, and the two cameras, A and B, are indicated.

The lighting of the raindrops and the duration of the photographic exposure to limit blur due to movement were both accomplished by using electronic flash equipment synchronized to fire with the camera shutters in the fully open position. Camera A (fig. 1) photographed the raindrops through a telecentric optical system in order to normalize the image dimensions in the object volume. Both cameras were driven by synchronous motors to obtain a constant exposure interval of 150 frames per minute. Synchronization between the two cameras was obtained by a system of cams and microswitches as shown in fig. 2. In essence, the electrical power to the synchronous motor of Camera B was intermittently interrupted until an error signal was no longer detected by the synchronizing circuit. Camera A had a magnification ratio of 1:20 obtained with a Bausch and Lomb Tessar f/4.5 Series 1C lens whose focal length was 210 mm in combination with a 4,375-mm focal-length paraboloidal mirror to form a telecentric optical system as described by Jones and Dean (1953). Camera B was fitted with a Bausch and Lomb Series f/4.5 1C Tessar lens with a focal length of 305 mm so that the magnification of the Camera B optical system was a ratio of **1:10**.

The shadowgrams of the raindrops were recorded on Eastman Fine-Grain Release Positive film. Since each camera records the projection of three-dimensional objects on a two-dimensional medium as silhouettes, any non-regular feature of the recorded objects may not be measurable even though the linear distance along each of the three coordinate axes can be accurately measured.

3. Measurements

The raindrop images from Cameras A and B were measured after magnification in two projectors placed side by side. The frame number and the positions of the drops with respect to the edges of the photographs were compared to insure the correct pairing of the drops from both photographs.

The drop images were measured in the vertical and horizontal and, if tilted, in the major and minor axes as well. In general, the measured vertical dimensions from the two different angles were not equal since Camera B had the optical system of an ordinary camera and the raindrop images from it would generally be either larger or smaller than the vertical

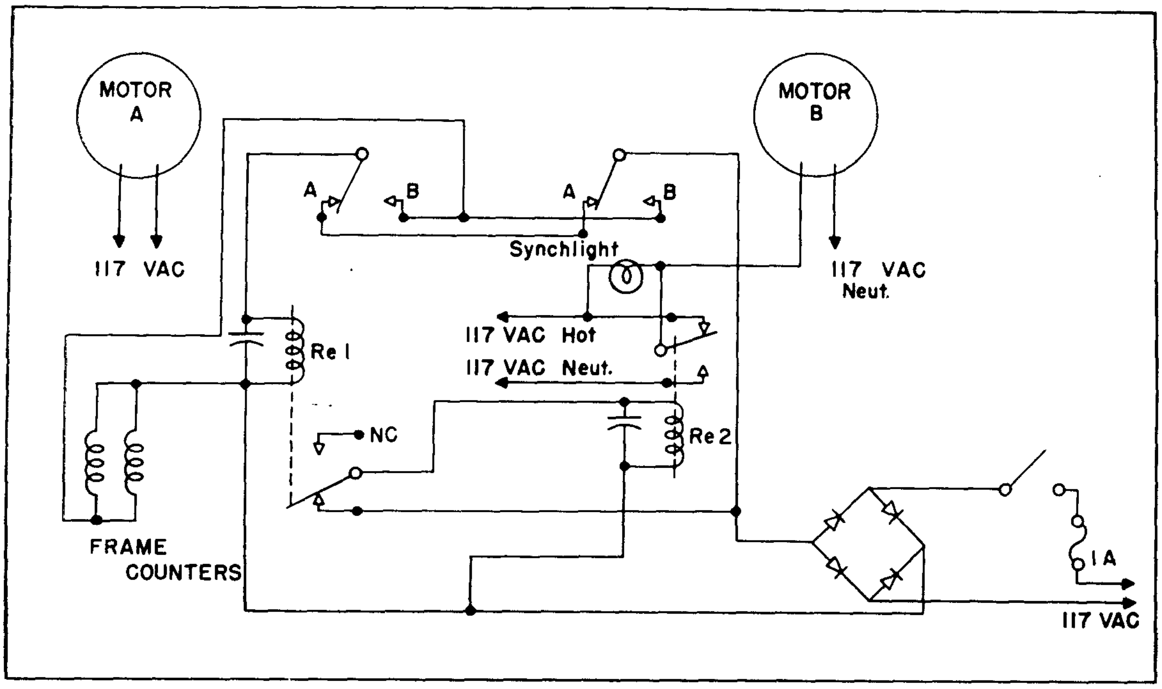

FIG. 2. Drop camera synchronizing circuit.

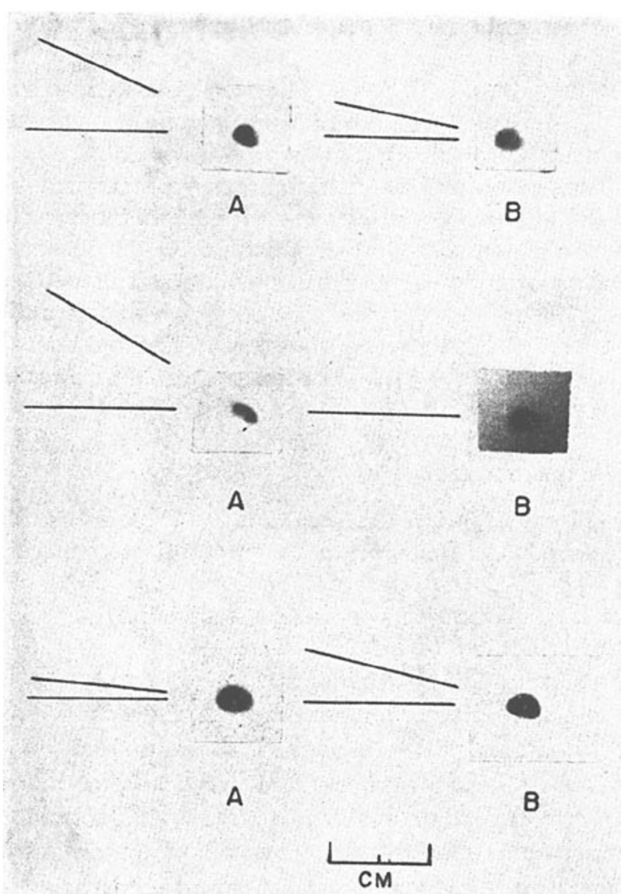

FIG. 3. Photographs of raindrops from two angles.

dimension from Camera A. Therefore, the vertical dimension from Camera A was assumed to be correct and the Camera B image was adjusted to it by the use of an appropriate multiplier for each raindrop. The multiplier required to correct the vertical dimension of Camera B was also used to correct the horizontal dimension of each raindrop. Fig. 3 illustrates pictures of raindrops from the two cameras. This figure serves to illustrate, also, one of the differences observed between raindrops and water drops photographed in the laboratory. That is, tilt of the major axis of the drop is due not only to periodic oscillations but to the wind field prevailing in the atmosphere. The tilt is indicated

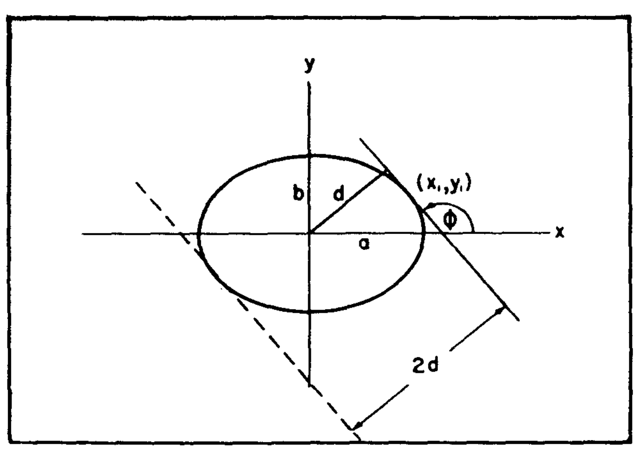

FIG. 4. Prolate ellipsoid of revolution.

by the lines drawn to the left of the drops. With only two cameras to view the drops, no attempt was made to correct the measured dimensions for errors due to angular differences between the film planes and the axes of the drops. If three cameras with their optical axes mutually perpendicular to each other had been used, all ellipsoidal drops could have been corrected no matter what their axial alignment.

Correction of the measured drop sizes to the true drop sizes entails considerable calculation and is not worth the effort for the small amount of precision gained. A simple calculation will illustrate the error possible in identifying the drops as spherical, oblate, or prolate. Assume a prolate ellipsoid where b is the minor semi-axis and a is the major semi-axis of revolution as shown in fig. 4. The angle, ϕ, is developed by the rotation of the major semi-axis, a, from the optical axis of the camera while d is the length of the line from the origin of the coordinate system to the edge of the projected image of the ellipsoid onto a plane perpendicular to the optical axis of the camera through the origin.

The equation for an ellipse may be written as

$$b^2x^2 + a^2y^2 = a^2b^2 \tag{1}$$

while

$$\tan\phi = -\frac{b^2x_1}{a^2y_1} \tag{2}$$

so that

$$2d = \frac{2ab}{\sqrt{a^2 + \dfrac{a^4\tan^2\phi}{b^2 + a^2\tan\phi}\left(\dfrac{b^2}{a^2} - 1\right)}} \tag{3}$$

expresses the length of the projection of the drop axis on the film plane when $\phi \neq 0$ deg, 90 deg.

Assuming a prolate ellipsoid with $2a = 5$, $2b = 4$, and $\phi = 45$ deg, then $2d = 4.53$. Equation (3) can be used to show that all prolate ellipsoids with $b/a = 0.8$ will be interpreted correctly as prolate ellipsoids unless the tilt of the major axis is between 35 deg and 55 deg from one of the camera axes. On the average, 22 per cent of the prolate ellipsoids with horizontal major axes will fall in this critical orientation and will be interpreted as oblate ellipsoids. Other axial ratios will have similar interpretation errors. Oblate ellipsoids cannot be misinterpreted when viewed by three 90-deg cameras.

A total of 1783 raindrops larger than 1.9-mm diam were measured to obtain the results given in this report. Table 1 gives the average axial ratios of all the raindrops measured, including drops determined to be oblate or prolate ellipsoids. The equivalent spherical diameter is D. When all raindrops are considered together, b/a is the ratio of the vertical dimensions to the horizontal dimensions, and is also the ratio of the minor to the major axes for the oblates. The ratio of the major to the minor axes for the prolates is a/b while n is the number of raindrops in each classification.

TABLE 1
Average axial ratios of raindrops.

	All drops		Oblates		Prolates	
D	b/a	n	b/a	n	a/b	n
2.0	0.98	103	0.90	7	1.16	14
2.1	0.99	87	0.91	33	1.15	32
2.2	0.97	160	0.87	46	1.14	16
2.3	0.96	96	0.88	19	1.14	16
2.4	0.95	127	0.84	22	1.14	24
2.5	0.97	151	0.92	21	1.12	22
2.6	0.95	130	0.89	35	1.12	51
2.7	0.96	121	0.89	38	1.15	16
2.8	0.95	127	0.90	25	1.11	22
2.9	0.92	96	0.87	24	1.14	18
3.0	0.92	100	0.90	26	1.10	23
3.1	0.90	66	0.85	29	1.06	14
3.2	0.89	54	0.85	18	1.16	5
3.3	0.90	49	0.83	17	1.12	5
3.4	0.90	35	0.83	10	1.12	6
3.5	0.85	51	0.84	24	1.15	12
3.6	0.88	49	0.84	23	1.18	6
3.7	0.85	25	0.85	12	1.10	1
3.8	0.87	26	0.85	12	1.28	2
3.9	0.90	20	0.83	4	1.11	6
4.0	0.88	22	0.81	11	1.14	6
4.1	0.83	16	0.78	7	1.18	2
4.2	0.79	8	0.74	4	1.05	1
4.3	0.80	7	0.80	2	1.14	1
4.4	0.82	13	0.73	7	1.10	2
4.5	0.92	5	0.78	2	1.14	2
4.6	0.81	6	0.78	3	1.19	2
4.7	0.81	9	0.76	3	1.09	2
4.8	0.65	2	0.61	1		
4.9	0.75	4	0.73	2		
5.0	0.74	3	0.73	3		
5.1	0.84	5	0.75	2	1.14	2
5.2	0.76	2	0.71	1		
5.3	0.61	1	0.60	1		
5.4	0.83	3				
5.5	0.76	1	0.75	1		
5.6	0.65	1				
6.0	0.75	1				
6.4	0.79	1	0.79	1		
		1783		496		331

The determination of the type of ellipsoid in which the raindrops should be grouped was based upon consideration of the ratios between the three axes measured. An axial ratio of 0.95 was assumed as a criterion for separating the measured raindrops into spherical, oblate ellipsoid, prolate ellipsoid, and irregular ellipsoid classifications. Thus, a raindrop was classified as a spheroid if the ratio of its shortest to the longest axis was 0.95 or greater. If two of the drop axes resulted in a ratio of the shortest to the longest of 0.95 to 1.00 while the third axis resulted in a ratio with either of the other two axes less than 0.95 or greater than 1.05, the drop was classified as an oblate or prolate ellipsoid, respectively. All raindrops that did not meet these criteria were classified as irregular ellipsoids. Of the 1783 raindrops measured, 569 were classified as spherical, 496 as oblate, 331 as prolate, and 387 as irregular ellipsoids.

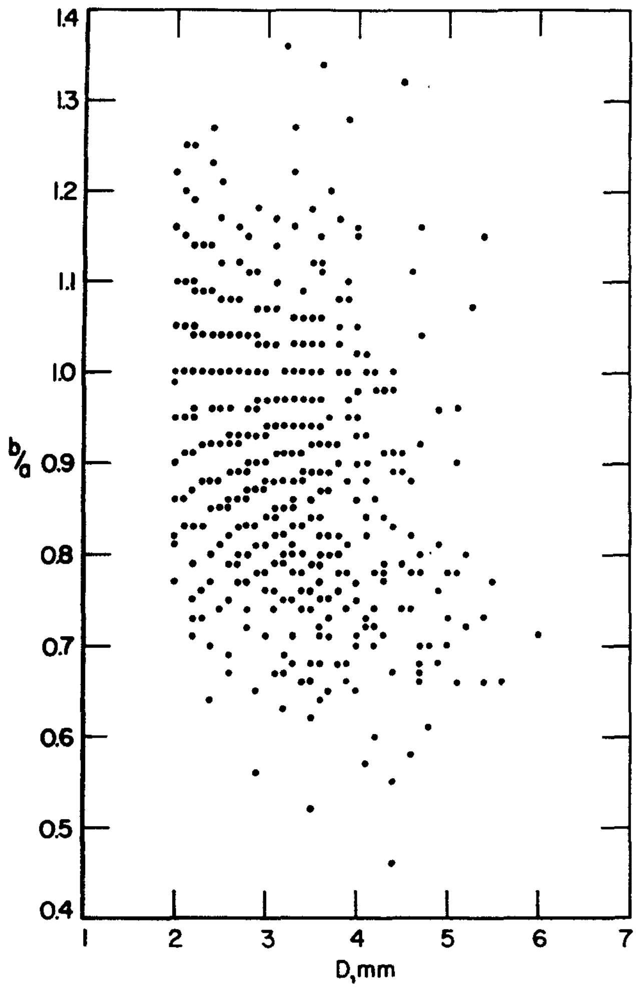

FIG. 5. Axial ratios of raindrops.

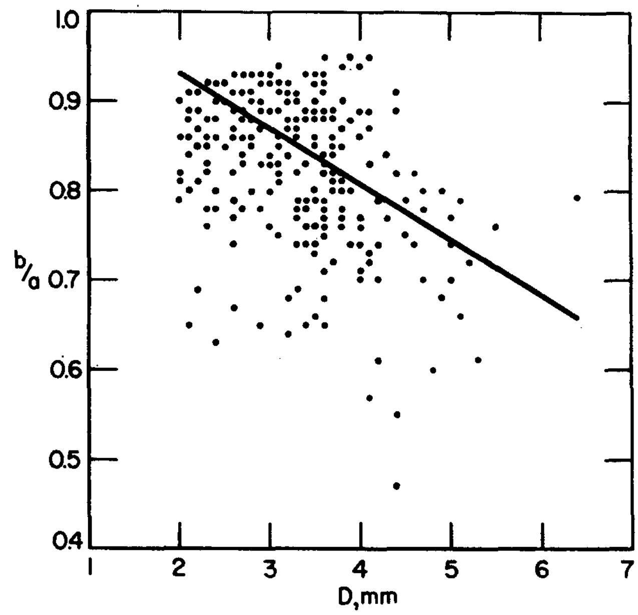

FIG. 6. Axial ratios of oblate raindrops.

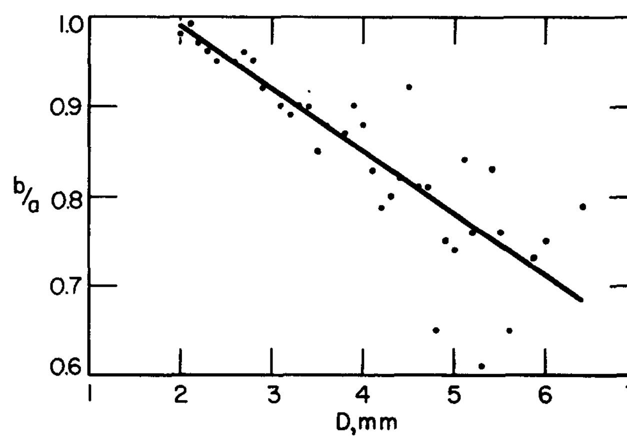

FIG. 7. Average raindrop axial ratio.

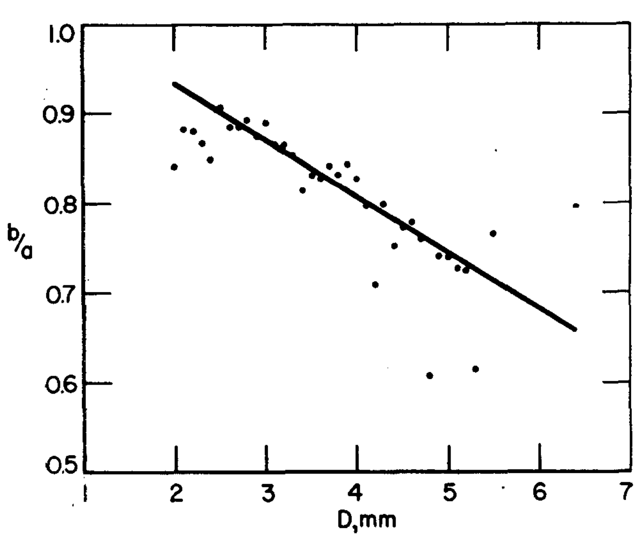

FIG. 8. Average oblate raindrop axial ratio.

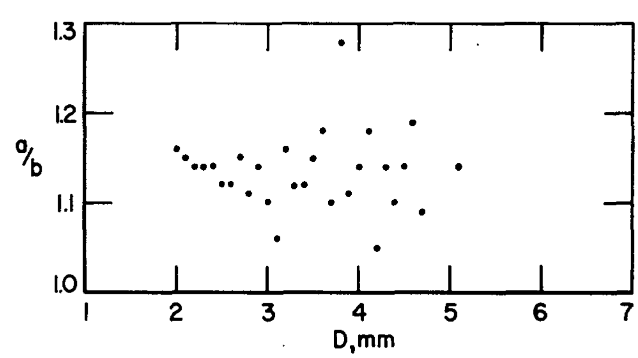

FIG. 9. Average prolate raindrop axial ratio.

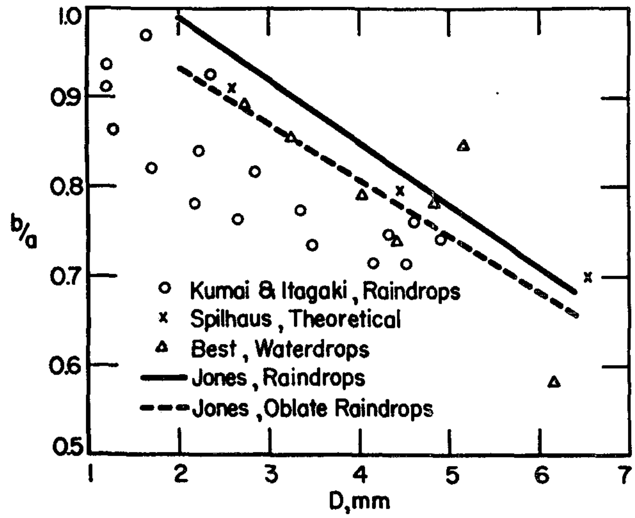

Fig. 10. Comparison of water drop axial ratios.

4. Results

The ratio of the major and minor axes was chosen as the means of presenting the experimental results because the results would then be directly comparable to the results of other investigators. Graphs of the individually plotted points determined from the measurements of each raindrop larger than 1.9 mm were plotted. Fig. 5 is the graph for all of the raindrops as classified into 0.1-mm intervals of equivalent spherical diameter, D, and fig. 6 is the graph of those raindrops selected as oblate ellipsoids as defined above. These graphs illustrate the scatter of the data used in determining a mean curve of deformation with increasing size of the raindrops. The usual procedure for reducing experimental results in meteorological experiments to useful equations or graphs has been to employ the method of least squares. However, the method of least squares involves the assumption that the experimental points are normally distributed about the mean. Comparison of fig. 5 with fig. 7, which is a graph of the mean raindrop ratio (b/a) in each 0.1-mm size classification, shows that the experimental points are far from being normally distributed. Accordingly, the data have been reduced by fitting free-hand curves (Ezekiel, 1956) to the average points to obtain fig. 7 and 8. Reference to fig. 9 will show that a curve fitted to the average axial ratios of the prolate drops would be of no significance. For the oblate and average raindrops, a straight line appears to be the curve of best fit. Values picked from the hand-fitted curves are tabulated in table 2.

A comparison of the results of different investigators into the shape of rain and water drops is shown in fig. 10. Kumai and Itagaki (1954) photographed raindrops in two dimensions to obtain results which indicate that all of the drops which they measured were assumed to be oblate ellipsoids, a natural assumption from a two-dimensional picture. Generally, their points indicate a greater deformation than the mean oblate raindrop in three dimensions which could be explained by the fact that they photographed raindrops whose trajectory had been collimated in falling the length of a short tunnel.

Table 2. Average deformation of raindrops.

D (mm)	Average raindrop b/a	Oblate raindrop b/a
2	0.99	0.93
3	0.92	0.87
4	0.85	0.81
5	0.78	0.74
6	0.71	0.68

Within the range of D usually observed in rain, the three points given by Spilhaus (1948) as fitting his theoretical determination of water-drop shape lie between the mean shape for all raindrops and the mean oblate raindrop shape. Best's waterdrops are also plotted in fig. 10.

The curve delineating the mean axial ratio of all raindrops should not be expected to be comparable to any of the measurements of the referenced papers as these papers do not include drops other than oblate. The usefulness of this mean axial ratio with increasing equivalent spherical diameter lies in the fact that it includes all of the measured drops; thus, it represents, for example, the shape of the mean raindrop in a radar scanning volume.

The frequency of occurrence of the different shapes of the raindrops is of some significance, particularly in the smaller size intervals where larger numbers of drops were measured. As shown by the number of occurrences, n, for each type of ellipsoid in table 1, there is no increase or decrease in the frequency of occurrence of prolates (20 per cent) with increasing D, but the frequency of oblates increases with increasing D with a tendency for the frequency to stabilize near 45 per cent of all drops at $D = 4.0$ mm. Wind speed and direction, which were recorded continuously during the data-collection period, qualitatively indicated that the tilt of the major axes of the raindrops with respect to the horizontal plane was dependent upon the gustiness and direction of the wind.

5. Conclusions

It may be said that there is no static shape of a freely falling raindrop but that all large raindrops are oscillating about a preferred shape. It is doubtful that the forces causing the observed instantaneous shape of a raindrop can be traced back to some prior event. However, even though the different investigators do not agree upon the size intervals through which the different oscillations are most important, the oscillations which have been observed in the laboratory can explain the shape of raindrops observed in nature. For radar-rainfall studies, it is probable that the average curve derived in this study will be sufficient for any studies made of the radar reflectivity where average axial ratios are assumed.

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