SHORTER CONTRIBUTIONS

EMPIRICAL MODELS OF INTERLEVEL CORRELATION OF WINDS

By B. N. Charles

Sandia Corporation, Albuquerque, New Mexico

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Introduction. Studies concerned with trajectories of falling bodies, or with systematic description of large-scale turbulence mechanisms in the atmosphere, often involve statistical correlations between winds at different altitudes. Ramifications of this vertical coherence or "connectedness" also have application in statistical forecasting methods and in synoptic climatology. Accordingly, as routine observations of upper winds have accumulated for progressively higher altitudes, more effort has been applied to the study of wind correlations in time and space (Durst, 1954, 1957; Court, 1958).

In these connections, the U. S. Weather Bureau Office of Climatology, in the course of studies made for the Federal Civil Defense Administration, produced a unique collection of upper-wind data. Statistical summaries of these data by Sandia Corporation, as a Cooperator with the Weather Bureau (Anon., 1958), provide standard deviations and interlevel correlation coefficients for zonal and meridional wind components. In this paper, these results are averaged for stations located in latitudinal bands, for the summer and winter seasons, and presented for use as models of vertical wind correlations. Such models can be useful in further analysis of interlevel-correlation relationships.

Data and method. The basic data collection consists of daily 1500 GCT winds for the period 1 March 1951 through 29 February 1956, at the 950-, 850-, 700-, 500-, 400-, 300-, 250-, 200-, 150-, 100-, 80-, 50- and 30-mb levels above 51 stations. Unobserved winds were estimated as gradient winds from specially analyzed synoptic charts or as appropriate interpolations and/or extrapolations from space-time arrays of the observed data (Ratner, 1957a). The station network is identified in fig. 1 and table 1.

The summer and winter seasons were defined as June through August and December through February, respectively. All possible interlevel correlation coefficients for the zonal and meridional components were computed separately at each of the thirteen

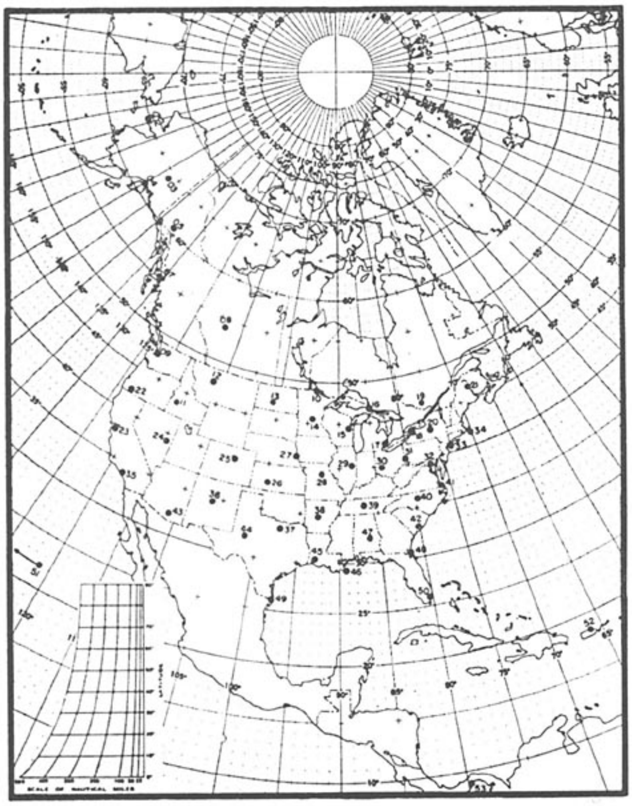

Fig. 1. Station locator.

Table 1. Station list and latitudinal grouping.

Latitude		
> 50N	02 Nome, Alaska 03 Fairbanks, Alaska 04 Anchorage, Alaska	05 Whitehorse, Yukon T. 07 Annette, Alaska 08 Edmonton, Alberta
40N–50N	09 Seattle, Wash. 10 International Falls, Minn. 11 Boise, Idaho 12 Great Falls, Mont. 13 Bismarck, N. D. 14 St. Cloud, Minn. 15 Green Bay, Wisc. 16Sault Ste Marie, Mich. 17 Mt. Clemens, Mich.	18 Buffalo, N. Y. 19 Maniwaki, Que. 20 Rome, N. Y. 21 Caribou, Me. 22 Medford, Ore. 27 Omaha, Nebr. 29 Rantoul, Ill. 31 Pittsburgh, Pa. 33 Hempstead, N. Y. 34 Nantucket, R. I.
30N–40N	23 Oakland, Calif. 24 Ely, Nev. 25 Denver, Colo. 26 Dodge City, Kans. 28 Columbia, Mo. 30 Dayton, O. 32 Washington, D. C. 35 Long Beach, Calif. 36 Albuquerque, N. M. 37 Ft. Worth, Texas 38 Little Rock, Ark.	39 Nashville, Tenn. 40 Greensboro, N. C. 41 Norfolk, Va. 42 Charleston, S. C. 43 Tucson, Ariz. 44 Midland and Big Springs, Tex. 45 Lake Charles, La. 47 Montgomery, Ala. 48 Jacksonville, Fla.
< 30N	46 Burrwood, La. 49 Brownsville, Texas 50 Miami, Fla.	51 Lihue and Hilo, T. H. 52 San Juan, P. R. 53 Albrook, C. Z.

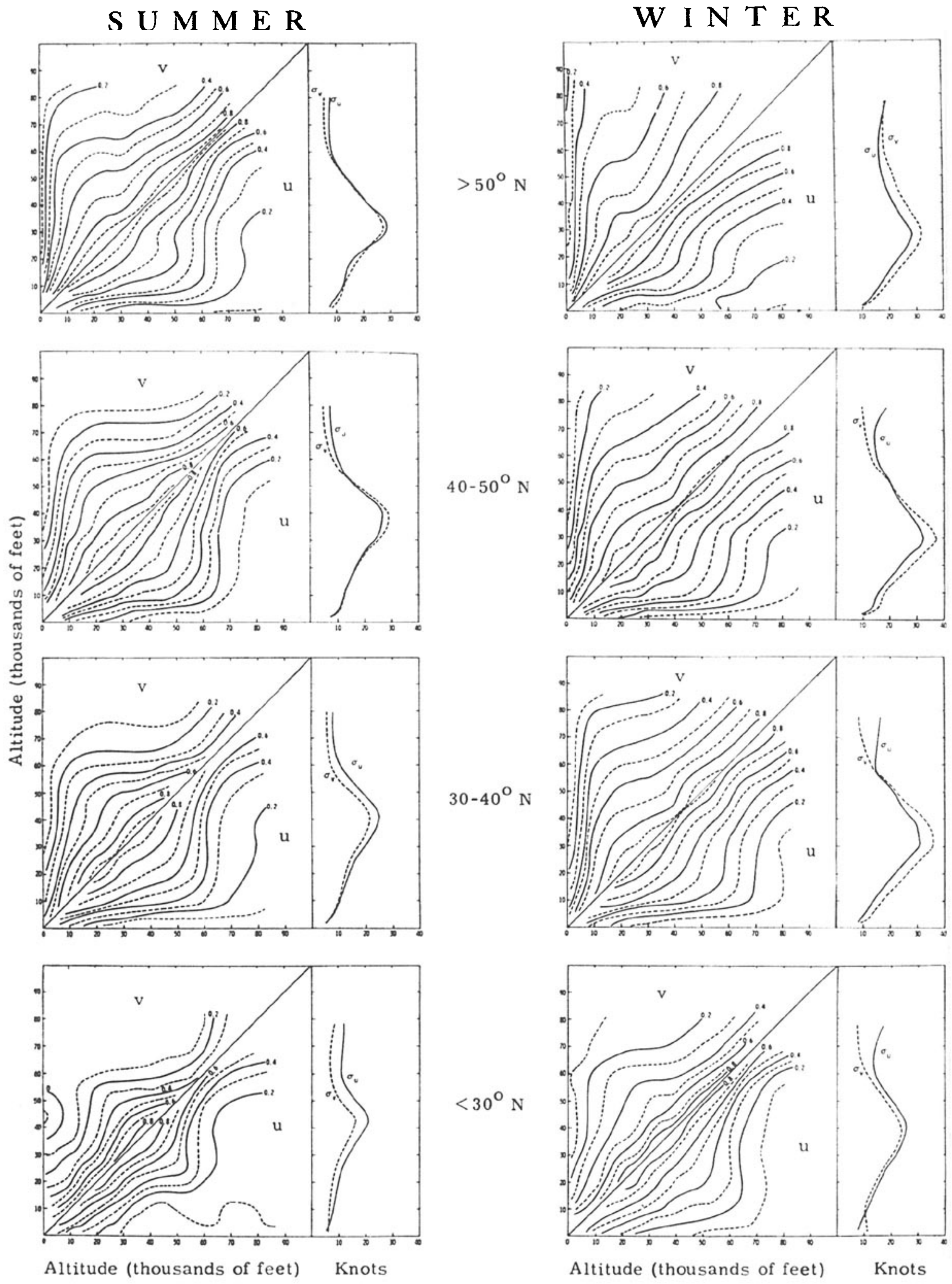

FIG. 2. Seasonal interlevel correlation coefficients and wind-component standard deviations, averaged over latitudinal bands. Correlation isopleths above diagonals apply to meridional components; those below diagonals, to zonal components.

pressure levels from

$$r = \frac{\sum u_1 u_2}{N\sigma_1\sigma_2}, \tag{1}$$

where u represents the paired departures from mean values of the wind components at pressure-altitudes 1 and 2, N is the sample size, and σ represents the standard deviations of the wind components at the appropriate altitudes.

The values so obtained were averaged for the latitudinal groups of stations indicated in table 1 and plotted as field values on appropriate charts, using approximate mid-latitude-band pressure heights from recent normal maps (Ratner, 1957b) to relate pressure-altitude and geometric height. Isopleths of correlation were drawn with a minimum of subjective smoothing, and the results are shown in fig. 2, together with vertical profiles of the mean standard deviations of the zonal (σ_u) and meridional (σ_v) wind components.

Discussion. The calculation of each interlevel correlation coefficient involved 460 and 452 data pairs in the summer and winter seasons, respectively. Whether these coefficients are significantly different from zero cannot be determined on the basis of these sample sizes, however, because of the serial correlations present in the time series. The magnitudes of these serial correlations appear to be greatest in winter at the higher altitudes, judging from 16 widely separated stations considered previously (Charles, 1959). For most of these stations, the lag correlation decayed to 0.3 or less after four days, but at Albrook AFB, Canal Zone, the four-day lag correlations in winter near 70,000 ft appeared to be in excess of 0.5, as shown in table 2.

Bartlett's formula for approximating the effective degrees of freedom in testing correlation between series is used to estimate significance of the interlevel-correlation coefficients (Quenouille, 1952). If the serial correlation coefficients in two series of N terms are $r_1, r_2, r_3 \ldots$ and $r_1', r_2', r_3' \ldots$, the effective number of independent observations to use in testing the correlation between the series is roughly

$$N_e = \frac{N}{1 + 2r_1r_1' + 2r_2r_2' + \cdots}. \tag{2}$$

From the data of table 2, the denominator of equation 2 is 9.3, so N_e is about 49. The standard error of the computed correlation coefficients, obtained from

$$\sigma_r = \frac{1 - r^2}{\sqrt{N_e}} \tag{3}$$

is then about 0.05 for $r = 0.80$, and 0.14 for $r = 0.3$. For a station exhibiting more rapid time decay, the standard errors are less. At Omaha, Nebraska, for example, N_e is about 87, so the standard errors corresponding to $r = 0.80$ and $r = 0.30$ are 0.04 and 0.10, respectively. The correlations pertinent to each individual station used in averaging the latitudinal bands of fig. 2 are accordingly highly significant statistically when they exceed 0.3 (McDonald, 1957).

The arithmetic averages over latitudinal bands are not associated herein with statistical confidence bands. They are simply mean correlation quantities pertinent to locations characterized by arbitrary geographical homogeneity. The individual correlations entering into each arithmetic mean may indeed be statistically nonhomogeneous; the vertical wind coherence of western stations might differ sufficiently from that of eastern stations to justify additional meridional stratification. But this was not suggested by visual examination of the data, as were the latitudinal and seasonal groupings. The basic data collection is believed to be suitable for more stringent statistical analysis, and it is hoped that future work can include additional study of the homogeneity of the correlation coefficients, using more rigorous statistical criteria. Table 3 is presented, however, for the benefit of those concerned with the statistical homogeneity of the

Table 2. Zonal lag correlations, Albrook AFB, in winter.

Level	Lag, days 1	2	3	4	5	6	7	8	9	10
30 mb	0.88	0.84	0.80	0.77	0.77	0.76	0.75	0.73	0.72	0.69
50 mb	0.68	0.60	0.56	0.55	0.52	0.49	0.48	0.48	0.50	0.48

Table 3. Interlevel correlation coefficients in summer, meridional components, 30N to 40N lat.

Level pairs, mb

Station	500 vs. 400		400 vs. 300		300 vs 200	
	r	Z	r	Z	r	Z
Albuquerque	0.782	1.050	0.801	1.101	0.794	1.082
Charleston	0.783	1.053	0.802	1.104	0.741	0.9527
Columbia	0.829	1.185	0.798	1.093	0.776	1.035
Dayton	0.840	1.221	0.806	1.116	0.776	1.035
Denver	0.827	1.178	0.854	1.271	0.797	1.091
Dodge City	0.815	1.142	0.834	1.201	0.773	1.028
Ely	0.846	1.242	0.861	1.297	0.829	1.185
Ft. Worth	0.782	1.050	0.719	0.9055	0.719	0.9056
Greensboro	0.827	1.178	0.825	1.172	0.805	1.113
Jacksonville	0.721	0.9097	0.777	1.038	0.686	0.8404
Lake Charles	0.726	0.9202	0.710	0.8872	0.620	0.7250
Little Rock	0.815	1.142	0.814	1.139	0.725	0.9181
Long Beach	0.810	1.127	0.827	1.179	0.791	1.074
Midland	0.739	0.9483	0.759	0.994	0.775	1.033
Montgomery	0.710	0.8872	0.720	0.9076	0.736	0.9417
Nashville	0.791	1.074	0.745	0.9616	0.731	0.9309
Norfolk	0.869	1.329	0.862	1.301	0.816	1.145
Oakland	0.892	1.432	0.878	1.367	0.782	1.050
Tucson	0.658	0.7893	0.628	0.7381	0.582	0.6655
Washington	0.831	1.192	0.869	1.329	0.832	1.195
Sums	15.893	22.0497	15.889	22.102	15.086	19.946
$\bar{r}$	0.795		0.795		0.754	
σ_r	0.060		0.063		0.066	
$r_{\bar{z}} = \tanh \bar{Z}$	0.802		0.802		0.760	

Note: Z is Fisher's hyperbolic arctangent transform, taken from Pearson and Hartley, (1954), p. 139.

MEAN INTERLEVEL WIND CORRELATION COEFFICIENTS

Meridional components above diagonals, zonal below

WINTER

LATITUDE	Pressure level (mb)	σ_v	950	850	700	500	400	300	250	200	150	100	80	50	30	σ_u
>50°	30	18.8	24	35	43	48	48	51	58	61	71	77	83	92		19.7
	50	18.3	26	39	48	54	55	57	64	68	78	84	90		92	17.2
	80	19.1	27	42	53	59	60	63	70	74	84	92		85	75	16.5
	100	21.1	28	44	57	64	65	67	74	78	89		87	73	63	17.3
	150	24.0	29	45	60	69	70	74	80	84		81	71	59	49	19.8
	200	27.6	33	48	66	77	78	82	91		78	67	57	46	38	22.8
	250	30.1	33	48	67	80	85	93		88	71	59	50	39	32	25.3
	300	32.9	31	45	65	81	89		90	77	62	50	42	31	25	28.6
	400	30.2	35	50	70	90		86	80	71	57	46	37	27	22	26.4
	500	25.3	39	57	77		86	75	72	67	54	43	35	25	20	21.9
	700	18.6	49	73		70	61	55	57	54	44	37	29	21	16	16.2
	850	14.9	66		62	45	38	34	35	35	30	24	19	14	12	13.2
	950	10.5		67	48	35	31	27	30	30	28	26	20	16	11	9.9
40-50°	30	10.0	05	12	22	29	30	32	35	36	46	50	59	77		19.4
	50	11.1	06	15	28	37	39	42	45	48	60	66	75		83	16.8
	80	14.0	08	21	38	50	52	56	60	64	77	85		76	63	15.9
	100	18.5	09	22	42	55	57	61	65	68	84		83	65	51	19.1
	150	23.9	14	29	51	66	69	72	76	79		82	71	54	39	23.5
	200	30.6	18	35	59	74	78	82	89		79	67	58	42	29	28.2
	250	34.9	19	38	61	79	85	92		87	72	59	50	35	22	30.6
	300	37.5	21	41	63	81	89		91	78	67	54	45	30	18	32.5
	400	32.0	28	47	70	91		88	82	74	65	53	44	29	18	27.3
	500	26.1	34	53	76		89	77	74	69	61	50	42	28	18	22.5
	700	17.8	54	75		74	68	59	58	57	51	42	36	24	16	15.6
	850	15.7	70		68	47	40	32	31	31	27	23	20	13	07	13.4
	950	10.7		62	43	27	22	16	14	13	11	10	09	05	03	9.4
30-40°	30	8.3	05	11	23	28	30	31	32	34	42	49	57	72		16.5
	50	10.3	07	16	32	39	41	42	44	48	58	66	73		74	15.2
	80	14.7	09	21	42	53	55	57	60	64	76	86		65	50	15.0
	100	19.4	09	23	46	58	61	63	66	70	84		79	55	40	18.5
	150	27.3	12	26	53	66	69	72	75	79		77	65	42	27	24.1
	200	33.6	14	28	58	73	77	82	88		74	61	51	32	18	29.3
	250	35.6	16	31	62	78	84	91		85	68	54	45	27	14	30.5
	300	35.9	17	33	65	81	88		89	75	63	49	41	24	11	30.9
	400	30.3	22	39	70	90		85	79	70	60	48	41	24	12	25.4
	500	25.0	26	45	76		86	72	69	63	56	45	39	23	12	20.6
	700	17.1	40	61		68	62	53	50	46	42	36	31	21	13	13.7
	850	13.3	56		54	37	31	25	22	21	19	16	13	10	05	10.7
	950	9.7		48	31	20	16	12	10	07	07	05	05	04	02	8.1
<30°	30	7.5	03	07	09	15	15	12	13	14	20	23	29	47		17.3
	50	8.2	03	08	12	20	21	21	21	23	32	37	46		54	13.8
	80	12.0	12	16	25	36	37	38	39	40	52	64		43	17	14.5
	100	15.0	10	16	25	38	41	44	46	48	65		70	29	10	17.5
	150	20.4	09	15	26	39	46	54	58	63		68	50	23	08	22.8
	200	23.5	07	15	29	43	50	64	77		69	52	39	17	06	25.5
	250	23.5	08	17	32	47	57	81		82	61	45	34	17	04	24.5
	300	21.4	10	20	36	54	68		84	69	54	41	33	16	02	22.8
	400	17.4	20	30	43	72		75	65	58	49	40	35	17	03	18.5
	500	13.9	25	36	49		79	62	54	48	42	37	32	17	05	15.3
	700	10.3	40	58		63	54	45	39	34	28	27	23	13	02	11.4
	850	10.9	74		60	43	35	27	25	23	19	15	14	11	06	9.0
	950	11.2		63	45	32	25	19	17	16	14	09	09	07	06	7.7

SUMMER

LATITUDE	Pressure level (mb)	σ_v	950	850	700	500	400	300	250	200	150	100	80	50	30	σ_u
>50°	30	5.7	05	16	20	24	26	24	26	28	32	37	41	64		8.2
	50	5.8	10	23	30	34	36	33	37	42	48	56	64		67	7.6
	80	7.7	14	30	43	50	51	50	54	60	70	81		54	38	8.6
	100	10.8	16	31	47	56	59	58	63	70	82		77	45	32	11.1
	150	17.4	19	36	55	66	69	69	74	83		81	65	39	27	17.8
	200	23.2	17	36	57	71	76	77	85		83	69	56	32	23	23.2
	250	26.5	14	34	58	75	82			87	74	62	49	27	19	27.7
	300	27.3	15	36	60	79	89		89	79	69	57	45	23	17	28.3
	400	22.0	19	43	68	88		88	82	76	69	57	46	25	19	22.4
	500	17.5	23	52	77		88	78	74	71	66	55	45	24	18	17.5
	700	12.9	36	71		77	68	59	56	57	52	45	39	22	17	12.4
	850	10.9	60		68	50	41	33	32	33	31	29	25	15	12	9.2
	950	8.5		68	43	29	23	15	15	17	19	18	15	05	02	7.7
40-50°	30	5.0	02	04	04	07	08	07	08	09	10	12	17	33		7.4
	50	5.9	05	10	17	24	24	24	25	27	27	34	45		46	8.4
	80	8.7	08	16	30	44	47	48	50	53	61	71		35	22	10.4
	100	12.4	07	17	37	53	57	60	63	67	78		67	29	15	13.2
	150	22.3	08	20	44	63	69	73	76	80		74	54	20	06	21.0
	200	28.7	10	23	48	68	75	81	88		77	61	45	17	06	26.7
	250	28.8	14	26	52	69	82	91		87	73	58	43	16	06	26.4
	300	26.3	16	31	57	77	86		90	78	68	55	40	14	04	25.0
	400	20.4	25	39	65	87		86	80	73	66	53	41	14	03	19.9
	500	16.8	32	46	73		86	75	71	65	60	48	38	13	04	16.3
	700	12.8	47	65		73	63	54	50	47	43	35	28	10	03	12.9
	850	11.2	66		67	47	38	29	24	21	17	15	14	04	01	10.7
	950	7.6		62	40	27	21	15	11	08	05	04	05	00	-03	7.4
30-40°	30	5.7	02	03	05	09	08	07	06	06	07	09	15	32		8.0
	50	6.0	03	06	10	13	15	13	14	15	18	23	30		57	8.2
	80	7.7	06	12	23	31	34	34	35	38	45	55		47	36	11.2
	100	10.4	06	15	30	41	47	49	51	54	64		66	39	28	14.2
	150	17.8	05	16	37	55	63	69	73	77		72	56	31	23	21.6
	200	21.5	05	16	38	56	66	75	85		84	65	50	28	21	25.0
	250	20.5	09	20	41	60	73	87		90	81	62	48	27	19	23.3
	300	18.2	11	23	44	65	78		91	83	77	61	47	27	20	20.6
	400	14.3	18	32	55	80		85	81	76	71	59	47	27	19	16.4
	500	12.3	23	38	64		84	73	70	66	63	52	44	25	18	13.5
	700	10.1	31	53		68	60	51	48	45	43	38	34	21	15	10.2
	850	7.6	46		51	38	31	23	21	17	17	18	18	11	09	7.9
	950	4.9		47	28	19	13	07	05	02	01	04	06	05	04	5.4
<30°	30	8.4	02	02	01	01	02	03	03	03	01	05	09	30		12.0
	50	7.2	01	01	03	03	05	03	03	04	06	09	16		51	11.6
	80	7.2	04	06	07	10	13	13	13	15	21	26		40	29	11.1
	100	8.0	01	06	07	20	28	27	30	29	35		50	29	24	13.1
	150	13.5	-08	-03	04	22	38	47	55	65		50	30	17	14	19.5
	200	15.6	-07	-02	05	25	44	63	78		73	40	25	13	12	20.4
	250	13.8	00	03	09	30	53			83	66	39	24	14	12	17.0
	300	11.4	06	10	16	38	64		83	74	60	37	26	16	13	14.4
	400	8.9	16	20	26	57		71	62	54	49	39	25	17	14	11.3
	500	7.6	25	29	40		70	49	41	34	32	30	20	13	12	9.6
	700	7.1	43	57		53	37	21	15	09	08	15	12	09	10	8.4
	850	6.5	73		58	32	23	10	05	00	00	09	13	08	10	6.4
	950	6.1		56	39	22	16	05	03	-01	-02	05	06	03	05	5.7

σ_v and σ_u are the standard deviations of the meridional and zonal wind components, respectively, in knots.

Correlation coefficients expressed in hundredths individual correlation coefficients used in averaging. It lists the meridional correlation coefficients for three adjacent altitude increments in summer, for each of the stations included in the 30N to 40N lat band. The hyperbolic arctangent transformations (Fisher's Z, from Pearson and Hartley (1954)) are also listed, as this method of combining correlation coefficients seems to be widely favored. The contention is not made, however, that the individual correlation coefficients are drawn from the same population (indeed, they are known to be from populations possessing different means and variances), but rather that the average correlation coefficients are representative of the individual values. For consideration of this point, the standard error of estimate (σ_r) in the arithmetically-averaged correlation coefficients ($\bar{r}$) is perhaps more revealing and is seen to be about 0.065 for the cases shown. Although some of the individual values do differ significantly from the average (Tucson, for example), the latitudinally-averaged correlation coefficients appear to the writer to be adequately representative of most of the individual values. In addition, table 3 shows that $\bar{r}$ underestimates tanh $\bar{Z}$ by about one per cent, a difference not considered important in ordinary applications. Although similar quantitative studies were not made for other points used in preparing fig. 2, visual examination indicated the qualitative similarity of most of the individual correlation coefficients when stratified by season, altitude increment, and latitude.

The patterns appearing in fig. 2 show the existence of a mid-troposphere level which is relatively highly correlated with other levels, as shown by Durst (1957) to be prevalent over England. A similar feature also appears in winter, although not as strongly, in the lower stratosphere. The general relative increase of interlevel correlations at altitudes above 30 mb, indicated by the spreading of the isopleths, is perhaps related to the wind persistence at these altitudes, but further speculation should await the availability of more frequent observations at higher altitudes.

Perhaps the most general indications of the patterns in fig. 2 are the greater vertical coherences occurring in winter, and the apparent symmetry of the zonal and meridional vertical coherences.

Table 4 lists the average interlevel correlation coefficients that were computed from the basic pressure-height wind data. Tabulations of all individual correlation coefficients are available from the U. S. Weather Bureau (Anon., 1958).

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A METHOD FOR LOCAL TEMPERATURE EXTRAPOLATION[1]

By ***W. R. Henson***

Yale University

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Introduction. The use of climatological data in ecology frequently involves the estimation of temperature at some field location from the records of a station in the climatological network. It is very unusual for ecological work to be done at the exact location of a climatological station, and the well known horizontal and vertical variations in temperature make the direct use of station records suspect. Such recent workers as Lindsey and Newman [5] have been able to apply station records directly in phenological investigations. However, in areas of broken topography and in work on short-term processes, it is usual to attempt some refinement in the extrapolation of temperature data.

Frequently, the records of a temporary field station are compared with those of a regular station. Mean

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[1] This paper is based on a contribution given at the 170th meeting of the American Meteorological Society, New Haven, 1958; it is a joint contribution from the Yale School of Forestry and the Forest Biology Division (No. 506) Science Service, Department of Agriculture, Ottawa, Canada.