A NOTE ON THE INTERDIURNAL VARIABILITY OF METEOROLOGICAL ELEMENTS

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The interdiurnal variations of a meteorological element are sometimes expressed as the mean of the magnitudes of the differences between values of the element at the same observation hour on consecutive days. Some investigators prefer to use the mean of the magnitudes of one-day changes of *daily means* of the element. Other authors prefer to use the standard deviation of values of the element occurring at the same observation hour on consecutive days. Finally, the standard deviation of daily means is also used, at times, to express the interdiurnal variations of the element in question. To the author's knowledge, the relationships between these various statistics, supposedly measuring the same characteristic of climate, have not been previously discussed. An empirical study [1] of surface-air temperature at the North Atlantic Ocean Vessel Stations has shown that the spatial and temporal variations of $\overline{|D_{j+1} - D_j|}$ and $\sigma(D)$ (where D_j is the mean daily temperature on the jth day and $\sigma(D)$ is the standard deviation of the D_j) are not always in phase. It is, therefore, desirable and necessary to examine the relationship between these variables in order that their differences may be understood.

To avoid confusion, the following nomenclature is introduced. If ϕ_j is the value of a meteorological element at a particular observation hour (or the daily mean of the element) on the jth day of a series of N days, then $|\phi_{j+1} - \phi_j|$ will be called the interdiurnal variability (I_j) and

$$(N-1)^{-1} \sum_{j=1}^{N-1} |\phi_{j+1} - \phi_j|$$

will be called the mean interdiurnal variability ($\bar{I}$). The relationship

$$(N-1)^{-1} \sum_{j=1}^{N-1} (\phi_{j+1} - \phi_j)^2 \approx 2\sigma^2(\phi)[1 - R_1], \quad (1)$$

where $\sigma^2(\phi)$ is the variance of the ϕ_j and R_1 is the autocorrelation coefficient between ϕ's on consecutive days, is easily derived and has been used previously in meteorology (see [2]). It should be noted that equation (1) becomes exact as N becomes large. By use of the fact $(\phi_{j+1} - \phi_j)^2 = |\phi_{j+1} - \phi_j|^2 = I_j^2$, equation (1) can be written as

$$(N-1)^{-1} \sum_{j=1}^{N-1} I_j^2 = 2\sigma^2(\phi)[1 - R_1]. \quad (2)$$

We may also write $I_j = \bar{I} + I_j'$ and $I_j^2 = \bar{I}^2 + I_j'^2 + 2\bar{I}_j I_j'$. Averaging this last relationship gives

$$(N-1)^{-1} \sum_{j=1}^{N-1} I_j^2 = \bar{I}^2 + (N-1)^{-1} \sum_{j=1}^{N-1} I_j'^2 = \bar{I}^2 + \sigma^2(I). \quad (3)$$

If (3) is substituted into (2) and the resulting equation is solved for $\bar{I}^2$, one obtains

$$\bar{I}^2 = 2\sigma^2(\phi)[1 - R_1] - \sigma^2(I). \quad (4)$$

From equation (4), we see at once that two locations having the same $\bar{I}$ need not have the same $\sigma(\phi)$, since $\sigma(I)$ and/or R_1 may differ between the two locations. The following question naturally arises: which of the two statistics, $\bar{I}$ or $\sigma(\phi)$, is preferable for climatological summaries. From a computational point of view, it is far easier to obtain $\sigma(\phi)$ because only one subtraction is needed,[2] while $N - 1$ subtractions are needed to obtain $\bar{I}$. Furthermore, $\sigma(\phi)$ can be obtained from summarized frequency distributions of ϕ. However, daily values of ϕ, in chronological order, are needed to obtain $\bar{I}$. On the other hand, $\bar{I}$ is a very stable statistic [3], while $\sigma(\phi)$ is subject to fairly large sampling fluctuations. Hence, a shorter historical record is needed to compute an $\bar{I}$ which will be representative of future $\bar{I}$ than is the case for $\sigma(\phi)$. In the final analysis, however, the choice between $\bar{I}$ and $\sigma(\phi)$ will usually depend on the purpose of the summary. If, for instance, $\overline{|\partial\phi/\partial t|}$ is to be compared at different stations, then

$$\overline{\left|\frac{\partial\phi}{\partial t}\right|} \approx \frac{\overline{|\phi_{j+1} - \phi_j|}}{\Delta t} = \frac{\bar{I}}{\Delta t}.$$

Here, $\bar{I}$ is the preferable statistic. On the other hand, in dealing with winds, if the resultant-vector wind is considered to be a basic current upon which disturbances are superimposed, $\sigma^2(u) + \sigma^2(v)$ is proportional to the mean kinetic energy of the disturbances [4] and would be more meaningful than relations involving $\bar{I}(u)$ and $\bar{I}(v)$.

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[2] This can be verified by writing $\sigma(\phi) = [\overline{\phi^2} - \bar{\phi}^2]^{\frac{1}{2}}$. Hence, only the difference, $\overline{\phi^2} - \bar{\phi}^2$, is needed.

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