GEOSTROPHIC AND GRADIENT DEPARTURES IN JET STREAMS

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ABSTRACT

Wind measurements made by aircraft of Project Jet Stream during forty-eight flights are compared with geostrophic and gradient winds (computed on upper-air charts) in order to determine geostrophic and gradient departures. Due to random errors in radiosonde data and to a lesser extent in the aircraft winds, individual departures are not reliable. Therefore, the jet stream area, as represented on a vertical cross-section perpendicular to the wind flow, is arbitrarily divided into nine sectors. Average departures are computed for each sector and for certain combinations of sectors.

In *cyclonic* jet streams (*i.e.*, in the vicinity of upper troughs), these calculations gave the following results: observed wind speeds were, on the average, 27.5 kn (or 18.4 per cent) less than geostrophic speeds but were in excellent agreement with gradient speeds. Although the average gradient departure was approximately zero, these departures tended to be negative (observed winds less than gradient winds) on the south side of the jet core and slightly positive on the north side. The standard deviation of this population of true geostrophic departures was 20 kn, while the standard deviation of true gradient departures was 6 kn. Gradient winds were therefore considerably superior to geostrophic winds as an approximation to observed winds in cyclonically curved jet streams.

In *straight* jet streams, observed winds were 2.4 kn (2.8 per cent) sub-geostrophic (and sub-gradient) on the average; however, this departure should be considered as essentially zero. Sub-geostrophic flow was most pronounced in the layer immediately above the jet core. Sufficient data were not available for determining the departures in anticyclonic jet streams.

The statistical significance and theoretical consequences of the departures are discussed.

1. Introduction

There is a number of reasons for comparing actual winds in the free atmosphere with geostrophic and gradient winds. The first reason is that wind data in the upper troposphere and stratosphere are less numerous than height data, especially in regions of strong winds. In analyzing and forecasting wind fields at these levels, height gradients are commonly used to estimate wind speeds. This procedure is based on the observation that, in the lower troposphere, geostrophic departures are only a fraction of the total wind speed (see, for example, Neiburger *et al.*, 1948). However, at higher levels, the magnitude of geostrophic departures is not known with any precision so that further information is of practical interest. The second reason for computing geostrophic departures is that research in dynamical weather prediction has turned toward the use of non-geostrophic models and of the primitive equations of motion. Presumably, empirical knowledge of the magnitude of departures and of their synoptic distribution is pertinent to these studies. Thirdly, geostrophic departures are a measure of the instantaneous accelerations being experienced by air parcels, and give insight into the adjustment of the wind and pressure fields.

The principal reason for the lack of precise knowledge of the nature of geostrophic departures is that random errors in measuring winds and pressure (or height) gradients are of the same order of magnitude as the departures themselves. But the random character of the errors makes it possible to minimize their effect by computing *average* departures for data grouped according to some rational scheme. In the present paper, we utilize wind measurements made during forty-eight flights of the specially instrumented aircraft of Project Jet Stream (Rados, 1955; Endlich and McLean, 1957). The majority of these flights explored jet streams over the southeastern United States. Geostrophic and gradient departures are determined by comparing the aircraft winds with geostrophic and gradient winds computed on upper-air charts. Then average geostrophic and gradient departures are calculated for certain regions defined by their distances vertically and horizontally from the jet-stream core.

2. Recent studies of geostrophic departures in the upper troposphere

Measurements of geostrophic departures in the upper troposphere have been made in recent years by a number of investigators. For example, Murray and Daniels (1953) found apparent ageostrophic wind components as large as 10 kn toward lower pressure in jet entrances (regions of converging contours) and toward higher pressure in jet exits (regions of diverging contours).

As part of his studies of the "Nullschicht" (layer of zero average vertical motion and of maximum horizontal wind), Faust (1954) compared observed winds with gradient winds measured on 225-mb charts. When there was a sharp peak in the vertical wind profile, he found super-gradient components averaging about 12 kn with extreme values of 50 kn. The average wind speed of the sample was about 70 kn. Only fifteen cases having speeds greater than 100 kn were included, due, presumably, to the lack of observations under conditions of strong winds. Vertical profiles not having a pronounced peak were found to be quite consistently sub-gradient.

Murray (1954) compared observed and geostrophic winds at a number of levels for one month. At 300 and 200 mb, he found geostrophic departures having RMS (root-mean-square) values of 20 and 18 kn respectively. He pointed out that the effects of various errors were contained in these apparent departures and in the computed true departures which will be discussed later.

In a study of winds over the British Isles for the year 1953, Smith (1955) found average geostrophic departures of about one knot directed towards higher pressure at 300- and 200-mb levels. For wind speeds greater than 60 kn, deviations directed towards higher pressure occurred two-thirds of the time. Smith attributes this result to the predominance of jet exits over Great Britain.

In discussing the significance of the sharp peak in observed wind profiles through jet streams, Endlich, Solot, and Thur (1955) suggested that winds should tend to be super-geostrophic to the south of the jet core and sub-geostrophic to the north by values as large as 30 per cent.

Neiburger and Angell (1956) evaluated geostrophic departures at 300 mb from wind data computed from constant-pressure balloon trajectories. Accelerations were determined as the difference between successive winds experienced by the balloon. Geostrophic departures were deduced from a vector form of the horizontal equation of motion. By this method, departures are determined without recourse to radiosonde data. The average absolute value of 200 departures was found to be 19 kn. This value is 29 per cent of the average geostrophic wind speed of their sample. Neiburger and Angell state that only about half of this deviation can be accounted for by the curvature effect; *i.e.*, gradient departures would average about 15 per cent or 10 kn.

A study of geostrophic deviations at 300 and 200mb deduced from balloon positions accurate to within one to three miles has been made by Giles and Peterson (1957). Accelerations were computed from hourly average velocities in contrast to the four-hour average velocities used by Neiburger and Angell. The mean geostrophic departure was found to be 6 kn (11 per cent of the average geostrophic speed). Giles and Peterson do not give any comparison of geostrophic and gradient departures.

In discussing case studies of jet-stream structure as revealed by aircraft observations, Cunningham (1957), Endlich and McLean (1957) and Saucier *et al.* (1958) noted that observed wind shears were often considerably different from shears calculated from the observed temperature gradients by use of the thermal wind equation. This difference between observation and calculation implies that large geostrophic departures existed at either the top, the bottom, or, possibly, throughout the layers involved.

Zobel (1958) compared 200-mb winds at Crowley, England with geostrophic and gradient winds. Two hundred and thirty-six comparisons were made; of these, fifteen had observed speeds greater than 100 kn. For straight flow, the RMS vector deviation (difference between observed and computed wind) was 11 kn. For curved flow, the RMS geostrophic deviation was 15 kn, and the RMS gradient deviation was 11 kn. Zobel concluded that gradient winds are superior to geostrophic winds in representing the observed wind field.

In a comparison of rawins and geostrophic winds at 300 and 200 mb over the United States (1956 data), Kochanski (1958) found a mean absolute vector deviation of 16 kn. Of forty-nine comparisons made when rawin speeds were $\geqslant 80$ kn, all were found to be super-geostrophic.

The observations reported in the following sections of this paper will be compared with the results summarized above.

3. Data

Modern airborne navigation sets are able to measure winds with a vector error whose standard deviation is 4 kn or less. Errors in GMD-1 wind measurements in jet-stream situations (Ference, 1951) are more than double this value. Therefore, the wind data gathered by aircraft of Project Jet Stream are comparatively well suited to the purpose of measuring geostrophic departures, but height gradients (or "D" gradients (Bellamy, 1945)) were not measured by the project aircraft due to the lack of sufficiently accurate absolute (radio) altimeters. As an alternative, geostrophic and gradient winds were computed on constant-pressure charts prepared independently of the aircraft winds. No attempt was made to separate the observed winds into components along and across contours. As pointed out by Riehl *et al.* (1955), if allowance is made for the subjectivity of upper-air analysis, strong winds appear to be parallel to the contours. Therefore, in this paper "departure" means the difference between the observed wind speed and a computed speed.

Geostrophic and gradient winds are, of course, subject to the errors of measuring height gradients. These errors are largely produced during the computation of each individual pressure-height sounding by errors in measuring temperature. The standard deviation of the resulting random error in geostrophic (or gradient) winds has been estimated at 10 kn at jet stream levels by Murray (1954). (This error is equivalent to independent temperature errors of 0.5C at stations 250 n mi apart.) Other small errors (having a standard deviation of about 5 kn) occur in map analysis and in scaling winds from contours. The overall variance of the error in geostrophic departures determined from aircraft winds and radiosonde height gradients is the sum of 4^2, 10^2, and 5^2, or about 140, giving a total error whose standard deviation is approximately 12 kn. If we assume that the errors are normally distributed about zero, the mean absolute error will be 0.8[1] of 12, or approximately 10 kn. Thus, due solely to the errors of measurement, a comparison of *aircraft winds* and *geostrophic* or *gradient* winds would show a mean deviation of 10 kn even if the winds were always in perfect balance with the pressure field. Comparisons of *rawins* and gradient winds (such as those made in several of the studies mentioned earlier) are believed to have an even larger mean deviation due to the larger errors of measurement in rawins than in aircraft winds. As mentioned earlier, reliable estimates of true departures must therefore be averages of a number of individual values so that random errors will cancel.

In general, research flights were either begun or terminated at synoptic hours to facilitate comparison of aircraft and synoptic data. Height contours were linearly interpolated from synoptic data in order to compute geostrophic winds for comparison with aircraft winds measured at non-synoptic hours. In computing gradient winds, it is, of course, necessary to measure the radius of curvature of the path being followed by a particular parcel. Normally, the curvature of the contours at a particular time is used as an estimate of path curvature; however, we wished to obtain a more accurate representation. The trajectory of a particular parcel for the two-hour period prior to the time (t_0) that the aircraft encountered it was constructed in two segments. Height contours were interpolated from synoptic maps for the times $(t_0 - 1\frac{1}{2})$ and $(t_0 - \frac{1}{2})$ hours. Trajectories from $(t_0 - 2)$ to $(t_0 - 1)$ and from $(t_0 - 1)$ to t_0, respectively, were assumed to be parallel to the interpolated contours. The positions of the parcel at the three times t_0, $(t_0 - 1)$ and $(t_0 - 2)$ determined the curvature which was measured by using a nomogram prepared by Gustafson (1950). This method of computing the radius of curvature of the contours (called R_{gr} in later sections) makes allowance for their movement and changes in shape.

[1] It is a property of the normal distribution that the mean absolute deviation is 0.8 of the standard deviation.

Due to the fact that the curvature of flow affects the magnitude and direction of the centrifugal force which, in turn, determines the difference between geostrophic and gradient winds, the data were classified according to curvature. When the radius of curvature of the trajectory was less than approximately 2500 mi, a particular case was classed as cyclonic or anticyclonic. Cases with larger radii of curvature were classed as straight. Twenty-two flights fell into the cyclonic category. As discussed by Endlich and McLean (1957), the majority of these flights were made in or just to the rear of upper troughs. The majority of the twenty-six straight flights were made on the east side of upper ridges near the inflection point from anticyclonic to cyclonic curvature. There were only three anticyclonic flights in the data sample being considered in this paper, an insufficient number for drawing any conclusions. Additional flights made during the winters of 1956–57 and 1957–58 will eventually be used to give greater reliability to the statistics and to permit further subdivision of the data by synoptic patterns.

4. Geostrophic and gradient departures

Most of the aircraft winds were measured during horizontal traverses flown perpendicular to jet streams. Geostrophic and gradient departures were computed for sixty-mile sectors measured from the jet axis by subtracting the average geostrophic or gradient speed in the sector from the average wind speed measured by aircraft. The latter average was usually based on three or four aircraft observations.

A total of 327 comparisons of observed winds and geostrophic and gradient winds was made. For this sample, the mean observed wind speed is 94 kn, the mean gradient speed is 95 kn, and the mean geostrophic speed is 113 kn. The departures are presented in tables 1 and 2 for straight and cyclonic flow separately for reasons given earlier. Each value in the tables is an *apparent* departure since it is the sum of an (unknown) true departure and a random error.

Inspection of the apparent geostrophic departures in table 1 shows that in some cases the values are comparable in magnitude to the wind speeds. The ten largest geostrophic departures range from −73 to −110 kn and their average value is −89 kn. The average observed wind speed for these cases is 116 kn,

TABLE 1. Geostrophic and gradient departures in cyclonically-curved jet streams. Asterisks denote B-29 flights; others are B-47 flights.

Flight no.	Sector	Observed wind speed (knots)	Gradient departure knots	Gradient departure %	Geostrophic departure knots	Geostrophic departure %
6	I	104	4	4	−26	−20
6	I	112	7	7	−28	−20
6	I	104	11	12	−26	−20
21	I	120	25	26	−10	−8
32	I	68	−2	−3	−27	−28
2	II	52	2	4	−18	−26
2	II	70	−10	−13	−17	−20
2	II	63	1	2	−17	−21
7	II	77	5	7	−28	−27
9	II	108	5	5	−2	−2
32	II	94	9	11	−21	−18
33	II	94	−2	−2	−56	−37
33	II	87	−3	−3	−63	−42
33	II	107	22	26	−43	−29
33	II	96	5	5	−44	−31
37	II	75	10	15	−13	−15
37	II	68	5	8	−17	−20
37	II	64	−1	−2	−6	−9
32*	II	154	4	3	−66	−30
33*	II	47	−1	−2	−33	−41
33*	II	66	6	10	−14	−18
33*	II	78	10	15	−32	−29
33*	II	105	15	17	−45	−30
33*	II	132	7	6	−48	−27
2	III	62	−4	−6	−24	−28
2	III	54	−6	−10	−22	−29
2	III	68	5	8	−42	−38
4	III	47	17	57	14	42
4	III	52	15	41	12	30
6	III	79	7	10	−16	−17
6	III	85	10	13	−15	−15
6	III	80	10	14	−20	−20
6	III	84	14	20	−41	−33
7	III	52	−15	−22	−48	−48
7	III	71	−29	−29	−89	−56
9	III	66	10	18	−4	−6
6*	III	60	−3	−5	−25	−29
6	IV	107	−9	−8	−23	−16
6	IV	109	−1	−1	−31	−22
7	IV	113	−32	−22	−47	−29
7	IV	122	4	3	−58	−32
7	IV	112	−26	−19	−38	−25
9	IV	119	−1	−1	−21	−15
9	IV	120	−22	−15	−40	−25
10	IV	158	−2	−1	−22	−12
21	IV	141	33	31	−9	−6
21	IV	124	4	3	−36	−23
21	IV	123	3	3	−37	−23
21	IV	147	2	1	−53	−27
2	V	71	−15	−17	−22	−24
7	V	111	−9	−8	−69	−38
7	V	110	4	4	−40	−27
9	V	124	−16	−11	−46	−27
9	V	130	−18	−12	−40	−24
19	V	80	−4	−5	−25	−24
19	V	67	−38	−36	−53	−44
19	V	87	−33	−28	−53	−38
19	V	100	−15	−13	−35	−26
19	V	112	7	7	−28	−20
19	V	114	14	14	−16	−12
19	V	94	1	1	−6	−6
19	V	86	−17	−17	−54	−39
24	V	105	−4	−4	−55	−34
24	V	127	10	9	−47	−27
24	V	118	4	4	−52	−31
24	V	108	−11	−9	−72	−40
29	V	142	−8	−5	−68	−32
29	V	153	−2	−1	−87	−36
29	V	136	−1	−1	−84	−38
29	V	135	−5	−4	−65	−33
29	V	136	−4	−3	−54	−28
32	V	107	14	15	−23	−18
32	V	93	3	3	−32	−26
32	V	114	0	0	−36	−24

TABLE 1—*Continued*

Flight no.	Sector	Observed wind speed (knots)	Gradient departure knots	Gradient departure %	Geostrophic departure knots	Geostrophic departure %
32	V	105	12	13	−35	−25
33	V	131	32	32	−39	−23
33	V	106	−14	−12	−34	−24
33	V	133	16	14	−32	−19
33	V	101	−37	−27	−49	−33
37	V	71	11	18	−4	−5
37	V	72	5	7	−8	−10
37	V	71	3	4	−7	−9
37	V	72	7	11	−3	−4
32*	V	177	−13	−7	−103	−37
32*	V	178	13	8	−44	−20
33*	V	152	9	6	−48	−24
33*	V	122	−23	−16	−58	−32
33*	V	154	−4	−3	−46	−23
33*	V	143	−2	−1	−37	−21
33*	V	123	−2	−2	−57	−32
2	VI	72	−18	−20	−58	−45
6	VI	100	15	18	−40	−29
6	VI	95	0	0	−55	−37
7	VI	110	−17	−13	−90	−45
7	VI	112	3	3	−38	−25
9	VI	112	4	4	−28	−20
9	VI	126	5	4	−14	−10
6*	VI	109	9	9	−11	−9
6*	VI	93	3	3	−27	−23
6*	VI	122	19	18	2	2
27*	VI	77	0	0	−13	−14
27*	VI	78	−10	−11	−37	−32
28*	VI	92	4	5	−38	−29
28*	VI	89	−8	−8	−61	−41
32*	VI	159	24	18	−51	−24
32*	VI	171	−4	−2	−69	−29
5	VII	80	−15	−16	−50	−38
5	VII	70	5	8	0	0
7	VII	107	−3	−3	−13	−11
7	VII	95	−7	−7	−15	−14
7	VII	84	−1	−1	−21	−20
10	VII	104	14	16	9	9
10	VII	88	5	6	0	0
21	VII	99	−23	−19	−81	−45
21	VII	108	−6	−5	−42	−28
21	VII	91	−34	−27	−79	−46
21	VII	100	−20	−17	−40	−29
21	VII	80	15	23	0	0
21	VII	33	3	10	−7	−18
21	VII	71	15	27	1	1
21	VII	73	8	12	−7	−9
21	VII	69	13	23	−1	−1
21	VII	86	8	10	−4	−4
7	VIII	93	−10	−10	−47	−34
7	VIII	85	−5	−6	−35	−29
9	VIII	121	−24	−17	−39	−24
19	VIII	101	12	13	−9	−8
19	VIII	78	−15	−16	−47	−38
20	VIII	56	−1	−2	−14	−20
24	VIII	111	1	1	−49	−31
24	VIII	99	−11	−10	−61	−38
24	VIII	106	1	1	−44	−29
24	VIII	106	−2	−2	−44	−29
24	VIII	93	−11	−11	−57	−38
24	VIII	89	−13	−13	−51	−36
24	VIII	101	−4	−4	−44	−30
24	VIII	93	−15	−14	−57	−38
24	VIII	85	−4	−4	−35	−29
24	VIII	83	−11	−12	−42	−34
24	VIII	77	−8	−9	−36	−32
29	VIII	121	−11	−8	−59	−33
29	VIII	104	0	0	−48	−32
29	VIII	105	−6	−5	−50	−32
29	VIII	111	13	13	−29	−21
29	VIII	93	6	7	−27	−23
29	VIII	87	−3	−3	−38	−30
29	VIII	72	−1	−1	−48	−40
32	VIII	99	−11	−10	−46	−32
32	VIII	94	6	7	−41	−30
32	VIII	86	−12	−12	−39	−31

TABLE 1—*Continued*

Flight no.	Sector	Observed wind speed (knots)	Gradient departure knots	Gradient departure %	Geostrophic departure knots	Geostrophic departure %
32	VIII	87	−8	−8	−48	−36
32	VIII	70	−25	−26	−60	−46
33	VIII	134	8	6	−21	−14
37	VIII	55	−9	−14	−15	−21
37	VIII	49	−11	−18	−14	−22
29*	VIII	100	20	25	15	18
32*	VIII	170	10	6	−35	−17
33*	VIII	132	7	6	−18	−12
5	IX	50	−10	−17	−17	−25
5	IX	56	−13	−19	−20	−26
5	IX	57	3	6	−3	−5
6	IX	91	−4	−4	−39	−30
6	IX	94	−16	−15	−31	−25
7	IX	91	1	1	−14	−13
7	IX	82	1	1	−8	−9
19	IX	98	16	20	−7	−7
19	IX	74	−1	−1	−16	−18
19	IX	83	8	11	−17	−17
19	IX	82	9	12	−3	−4
19	IX	63	−7	−10	−17	−21
19	IX	73	1	1	−7	−9
19	IX	62	0	0	−8	−11
19	IX	71	−9	−11	−19	−21
6*	IX	117	17	17	7	6
6*	IX	95	0	0	−25	−21
10*	IX	136	1	1	−24	−15
10*	IX	120	−7	−6	−30	−20
27*	IX	74	−21	−22	−41	−36
27*	IX	76	−25	−25	−44	−37
27*	IX	68	−9	−12	−13	−16
27*	IX	62	0	0	−2	−3
28*	IX	86	−34	−28	−94	−52
28*	IX	80	−46	−37	−110	−58
28*	IX	69	−28	−29	−47	−41
28*	IX	59	−14	−19	−21	−26
29*	IX	66	4	6	1	2
32*	IX	152	−13	−8	−73	−32
32*	IX	142	−8	−5	−46	−24
32*	IX	133	28	27	5	4

approximately 25 per cent higher than the average wind speed of the entire sample. The apparent *gradient* departures for the ten cases are all considerably smaller than the geostrophic departures and have an average value of −21 kn, which is approximately one-quarter of the corresponding geostrophic departure. Evidently, contour curvature is quite important in these wind computations. The apparent average gradient departure for this sample (−21 kn) is much larger than averages derived later for other data samples and, therefore, is apparently due in large part to errors in contour gradients given by radiosonde data. In summary, the largest apparent geostrophic departures are associated with strong winds, marked curvature, and large errors in measuring geopotential. Further statistics based on tables 1 and 2 are given below.

For cyclonic flow (189 comparisons), the variance of *apparent geostrophic departures* is 548 (knots^2). This total variance is due to the combined effect of the variance of true departures and the variance of errors of measurement; *i.e.*, σ^2 (apparent geostrophic departures) $= \sigma^2$ (true geostrophic departures) $+ \sigma^2$ (errors). The estimated value of the latter variance is 140, and thus accounts for one-fourth of the total variance. The estimated variance of *true geostrophic departures* is 408, giving a standard deviation of approximately 20 kn. Comparable values given by Murray (1954) are 13 and 8 kn for the 300- and 200-mb levels for average upper-air conditions. The variance of *apparent gradient departures* is 179 and is largely accounted for by the variance due to errors (140). The estimated variance of *true gradient departures* is therefore thirty-nine, giving a standard deviation of the true gradient departure of approximately 6 kn.

For straight flow (138 comparisons), geostrophic and gradient departures (and winds) are identical. We will refer to them as "gradient" to emphasize their similarity to the gradient departures for cyclonic flow. The variance of the apparent gradient departures is 158, giving an estimated variance of true departures of 18 and a standard deviation of true departures of approximately 4 kn.

It is also of interest to consider the total populations of geostrophic and of gradient departures formed by combining cyclonic and straight cases. The variance of apparent geostrophic departures of this population is 648, giving an estimated variance of true departures of 508 and a standard deviation of 22 kn. The variance of apparent gradient departures is 170 so that the estimated variance of true gradient departures is 30, and the standard deviation of true gradient departures 5.5 kn. It appears that gradient departures have a much smaller variance than geostrophic departures. Application of the "F" test to the two apparent departures shows that the variances are significantly different at the 1 per cent level.

Since a relation between geostrophic departures and geostrophic wind speeds had been found by Neiburger and Angell (1956) and a similar relation with observed wind speeds was noted in the discussion of large geostrophic departures given earlier, the relationship was investigated further. For the data in table 1, the coefficient of linear correlation between the absolute values of apparent geostrophic departures and geostrophic wind speeds is 0.79. The correlation of true geostrophic departures and geostrophic speeds would be even higher since the random errors of observations which are present tend to mask the true relationship. The extent of the masking is presumably related to the relative sizes of the variances of errors and of true departures which were given earlier. The corresponding correlation between apparent gradient departures and gradient wind speeds is only 0.20; however, the masking effect of the errors is now serious since the variance of errors is much larger than the variance of true gradient departures. In straight jet streams (table 2), the correlation of departures and computed speeds is 0.11, and reference to the appropriate vari-

TABLE 2. Geostrophic and gradient departures in straight jet streams. Asterisks denote B-29 flights; others are B-47 flights.

Flight no.	Sector	Observed wind speed	Gradient/Geostrophic departure knots	%
1	I	51	−24	−32
1	I	57	−28	−33
4	I	62	−19	−23
4	I	60	−25	−29
14	I	81	6	8
14	I	71	1	1
14	I	80	0	0
14	I	81	− 4	− 5
12	II	87	− 3	− 3
12	II	93	− 7	− 7
12	II	102	2	2
12	II	110	− 5	− 4
14	II	93	− 2	− 2
14	II	86	1	1
14	II	85	− 5	− 6
14	II	88	− 2	− 2
18	II	57	−13	−19
18	II	48	−16	−25
18	II	44	−14	−24
18	II	64	− 9	−12
29*	II	116	41	55
1	III	52	− 8	−13
1	III	44	−14	−24
4	III	60	8	15
12	III	75	−15	−17
12	III	66	−14	−18
14	III	87	7	9
14	III	70	0	0
14	III	68	− 2	− 3
14	III	70	0	0
14	III	66	0	0
14	III	73	− 1	− 1
14	III	68	− 2	− 3
14	III	83	3	4
13*	III	86	1	1
13*	III	82	− 3	− 4
30*	III	142	32	29
34*	III	64	11	20
41*	III	64	− 6	− 9
1	IV	61	−29	−32
4	IV	60	−25	−29
10	IV	157	− 3	− 2
10	IV	160	0	0
10	IV	146	− 4	− 3
14	IV	88	− 2	− 2
14	IV	88	− 2	− 2
12	V	117	7	6
12	V	119	− 1	− 1
12	V	127	7	6
14	V	103	3	3
14	V	96	− 4	− 4
36	V	92	7	8
36	V	73	− 8	−10
1	VI	42	−13	−24
4	VI	78	18	30
4	VI	81	6	8
12	VI	87	− 3	− 3
12	VI	87	2	2
12	VI	96	− 4	− 4
12	VI	95	5	6
14	VI	86	3	4
14	VI	90	3	3
14	VI	93	1	1
14	VI	87	− 1	− 1
13*	VI	98	− 2	− 2
13*	VI	93	− 2	− 2
13*	VI	101	1	1
13*	VI	100	0	0
16	VI	132	22	20
17*	VI	88	−12	−12
17*	VI	99	9	10
29*	VI	96	21	28
30*	VI	155	30	24
30*	VI	153	28	22

TABLE 2—*Continued*

Flight no.	Sector	Observed wind speed	Gradient/Geostrophic departure knots	%
33*	VI	124	−31	−20
34*	VI	75	0	0
34*	VI	81	1	1
41*	VI	106	11	12
41*	VI	116	26	29
7	VII	96	− 4	− 4
9	VII	115	−45	−28
10	VII	139	− 1	− 1
10	VII	134	4	3
10	VII	147	− 1	− 1
10	VII	145	5	4
10	VII	132	− 8	− 6
10	VII	137	− 3	− 2
10	VII	144	− 6	− 4
10	VII	117	17	17
14	VII	77	− 8	− 9
14	VII	85	5	6
9	VIII	110	0	0
12	VIII	114	4	4
12	VIII	105	5	5
12	VIII	97	0	0
20	VIII	63	− 7	−10
33	VIII	95	−15	−14
36	VIII	78	8	11
36	VIII	73	4	6
36	VIII	60	− 2	− 3
36	VIII	54	− 5	− 8
36	VIII	48	− 3	− 6
37	VIII	46	− 9	−16
37	VIII	45	− 7	−13
37	VIII	43	− 2	− 4
37	VIII	54	− 1	− 2
37	VIII	55	− 1	− 2
37	VIII	49	− 1	− 2
33*	VIII	114	−31	−41
6	IX	78	−12	−13
8	IX	119	11	10
9	IX	100	5	5
9	IX	114	14	14
12	IX	82	7	9
12	IX	101	1	1
12	IX	89	4	5
12	IX	88	8	10
14	IX	85	− 5	− 6
14	IX	75	− 5	− 6
14	IX	100	8	9
6*	IX	99	14	16
6*	IX	80	10	14
6*	IX	62	2	3
6*	IX	60	10	20
10*	IX	162	2	1
10*	IX	151	− 1	− 1
16*	IX	122	2	2
17*	IX	93	19	26
17*	IX	74	4	6
17*	IX	67	− 1	− 1
27*	IX	52	−16	−24
28*	IX	53	−22	−29
29*	IX	84	9	12
29*	IX	88	18	26
29*	IX	89	19	27
30*	IX	142	17	14
30*	IX	137	17	14
34*	IX	70	−10	−13

ances indicates that the masking effect of errors is even larger than in the previous correlations. We conclude that the magnitudes of geostrophic departures increase with geostrophic wind speeds and that gradient departures depend on gradient wind speeds; however, the extent of this latter dependence cannot be judged accurately due to the interference of errors.

In the following sections of this paper, we wish to compare departures measured in jet streams of various intensities. Therefore, we will express the departures in knots and also as percentages of the geostrophic or gradient speeds; however, significance tests will only be applied to the values expressed in knots.

5. The relation of departures to jet streams

It is of interest to determine whether geostrophic and gradient departures vary systematically in the vicinity of the jet core. Therefore, the jet-stream cross-section (fig. 1) was arbitrarily divided into the nine sectors shown. All departures were plotted in the sector to which they belonged. Generally, more than one departure obtained from a particular flight fell into a given sector. Such departures, measured in the same general region of a jet stream at short time intervals, are correlated with one another. Therefore, all departures of a particular flight falling into a given sector were averaged to give a single value. The resulting population of independent values was averaged to obtain a departure for each sector. These sector departures and the number of independent values going into each are shown in figs. 2, 3, 4, and 5.

For cyclonic flow, it is immediately apparent that geostrophic departures (fig. 2) are quite large throughout the area explored. But the number of cases in certain sectors is small, so that the data must be combined further to increase the reliability of the statistics. Averages were therefore obtained for each row and column of fig. 2. The row averages pertain to the groups "above the jet core," "within 1000 ft of the jet core" and "below the jet core," while the column averages pertain to the groups "north of the jet core," "within sixty miles of the jet core," and "south of the jet core." These six averages are plotted in the margins of fig. 2. Student's "t" test was used to determine the "chance probabilities"[2] of the averages. The probabilities of all six averages are less than 1 per cent. The largest departures (both in knots and per cent) appear to be at the jet core (*i.e.*, for the groups "within sixty miles of the jet core" and "within 1000 ft of the core"); however, the departures for these two groups do not differ significantly from the other groups. We then computed an average geostrophic departure for each flight. The mean of these twenty-two independent values is -27.5 kn (or -18.4 per cent). This average (subgeostrophic) departure has a chance probability less than 1 per cent.

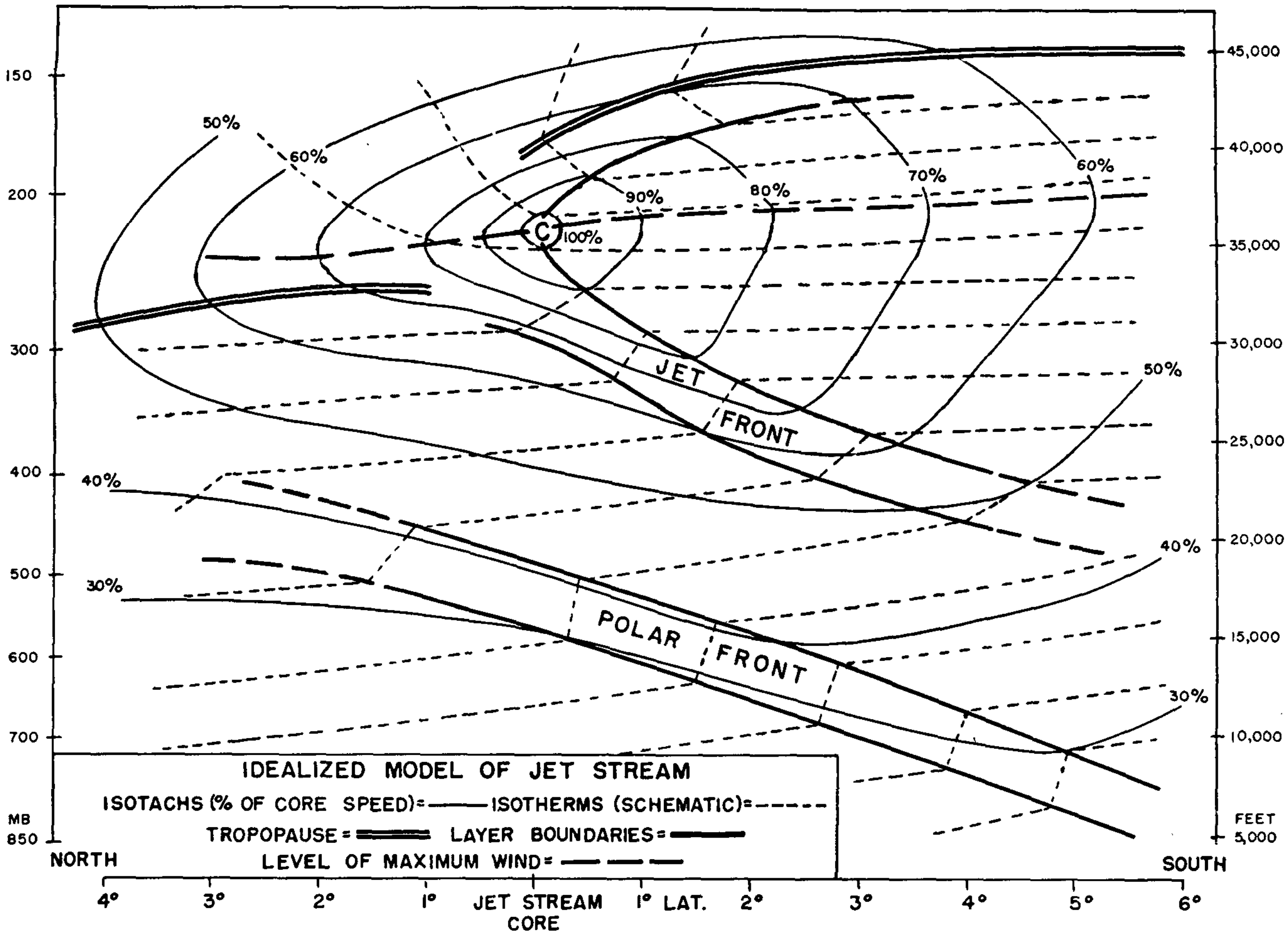
The computations were then repeated for gradient departures and the results are given in fig. 3. The departures are much smaller than the corresponding geostrophic values. The average for the group "north of the core" (5.5 kn) has a chance probability of 6 per cent while the probabilities of the other averages are greater than 10 per cent. The chance probability of the difference (8.9 kn) between the averages for the groups "north of the core" and "south of the core" is 2 per cent. On the basis of these statistics, it appears that gradient departures tend to be positive on the north side of the jet core and negative on the south side. (It will be shown later that the positive departures imply consequences which are not acceptable on synoptic grounds. Therefore, their existence must be considered open to question.) Computing an average gradient departure for each of the twenty-two flights and then obtaining the grand average gradient departure for cyclonic jets, we arrive at a value of -0.6 kn, which is essentially zero.

At this point, let us compare geostrophic and gradient departures in cyclonic jet streams. Geostrophic departures have an average value of -27.5kn, a standard deviation of apparent departures of 23 kn, and a standard deviation of true departures of 20 kn. The corresponding values of gradient departures are, respectively, zero, 13, and 6 kn. Gradient winds are therefore considerably superior to geostrophic winds in representing the true wind field in these regions.

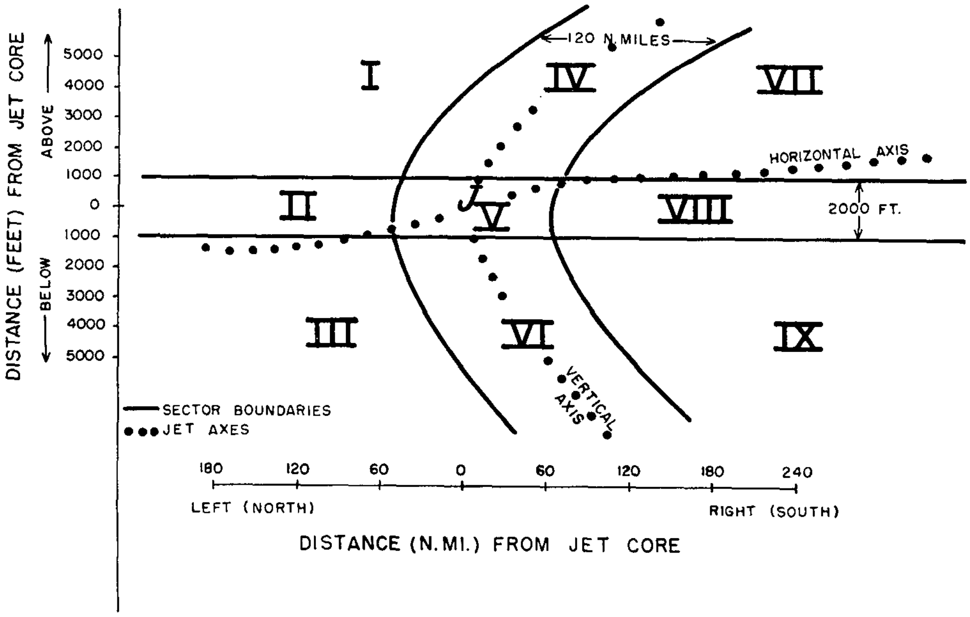
The departures for straight flow are shown in fig. 4. Only the departure for the group "above the jet core" (-16.7 kn) has a chance probability less than 10 per cent (actually 8 per cent). The chance probability of the difference (17.6 kn) between the groups "above the jet core" and "below the core" is 4 per cent, indicating that sub-geostrophic components increase with height at levels near the jet core. As before, we computed an average departure for each flight and averaged these twenty-six independent values, obtaining a mean departure for straight jet streams of -2.4 kn (sub-geostrophic), which is essentially zero.

We may combine the data in figs. 3 and 4 in an attempt to obtain the average gradient departure in the vicinity of jet streams, remembering that the results may be biased by the lack of data for anticyclonic jets. The resulting departures are shown in fig. 5. The average for the group "above the jet core" (-9.5 kn) has a chance probability of 4 per cent and the average for the group "south of the core" (-3.2 kn) has a chance probability of 8 per cent. Probabilities of the other four averages are all greater than 10 per cent. The chance probability of the difference (7.4 kn) between the groups "north of the jet" and "south of the jet" is 5 per cent and the probability of the difference (9.6 kn) above-to-below the jet is 4 per cent. The mean gradient departure for the forty-eight flights is -1.6 kn which is essentially zero. The comparable mean geostrophic departure (based on figs. 2 and 4 combined) is -13.9 kn which has a chance

[2] The terms "chance probability" and "probability" are used herein to refer to the probability that a sample average differs from zero (or from another average) by the stated amount due to chance.



(a)



(b)

FIG. 1. a: Idealized cross-section perpendicular to a jet stream blowing into the page (after Endlich and McLean, 1957). b: The nine sectors of the jet-stream cross-section for which average geostrophic and gradient departures were computed.

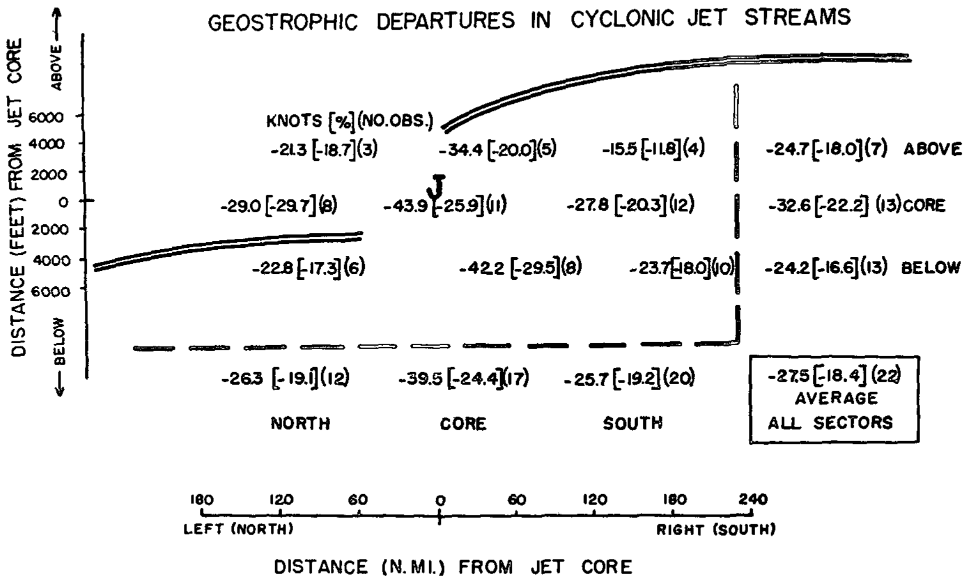


FIG. 2. Average geostrophic departures (in knots and per cent) in various sectors of a cross-section through cyclonically-curved jet streams. Double solid lines are tropopauses of fig. 1a. "J" marks the jet core.

probability less than 1 per cent. We may conclude that, on the average, jet-stream winds are in excellent agreement with gradient winds; however, there appears to be a tendency for subgradient flow above and on the south side of the jet core.

One may hesitate to accept the statistics given above since the average gradient departures are small and are based on data containing large errors. Certainly, more comparisons are needed and the accuracy of measured pressure gradients needs to be improved. The development of airborne radio and pressure altimeters with high sensitity (on the order of a few feet) would permit the measurement of "D" values (and geostrophic winds) with the same accuracy that winds are now measured by automatic navigators. Meso-scale departures could then be determined in particular situations of interest.

Our confidence in the result that jet streams are *not* supergradient, on the average, is increased if we consider the following factors. Horizontal wind profiles across jet streams are peaked, as shown by Riehl *et al.* (1955) and by Endlich and McLean (1957). If the pressure gradient is in balance with such profiles, the distance between contours must be a minimum at the jet core and increase to the north and south. But upper-air stations are spaced several hundred miles

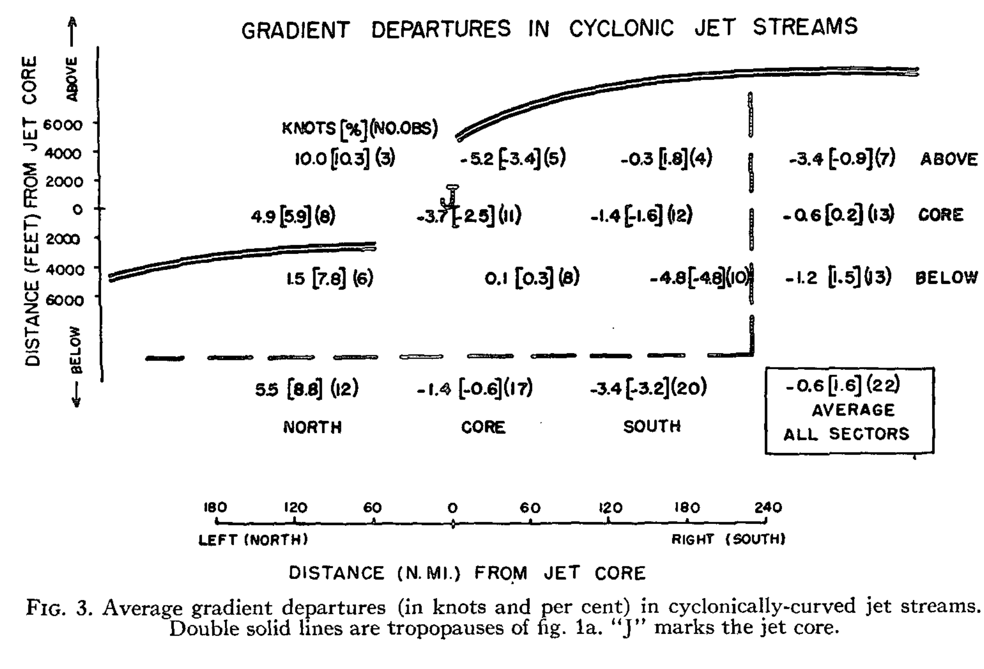


FIG. 3. Average gradient departures (in knots and per cent) in cyclonically-curved jet streams. Double solid lines are tropopauses of fig. 1a. "J" marks the jet core.

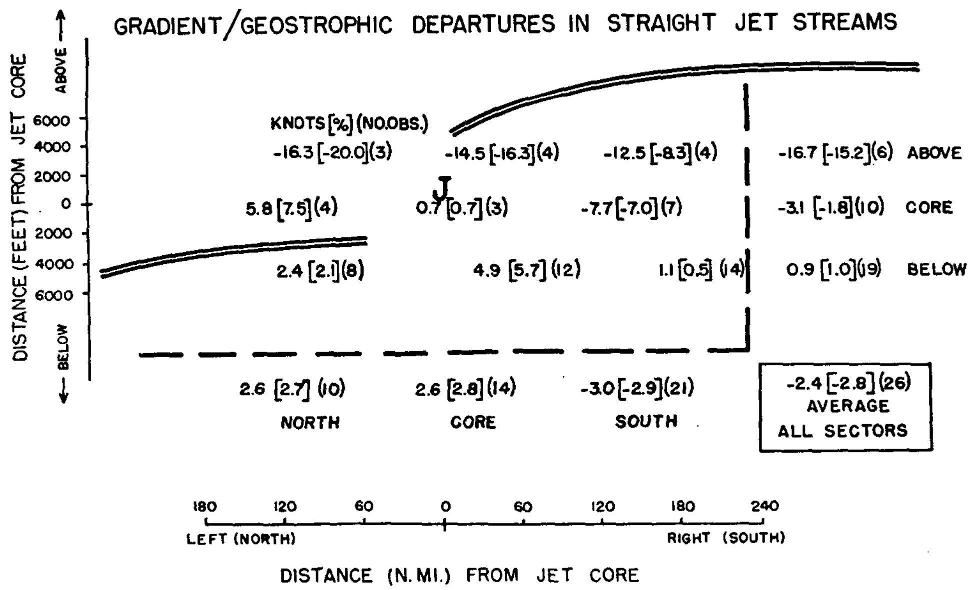


FIG. 4. Average gradient (and geostrophic) departures in straight jet streams. Double solid lines are tropopauses of fig. 1a. "J" marks the jet core.

apart, and analysts ordinarily interpolate contours more or less linearly between them. Therefore, the contour spacing approximately balances the *average* wind between stations. The above-average winds near the jet core will therefore *appear* to be super-gradient, while below-average winds will appear to be sub-gradient. Since fig. 5 shows approximately zero departure at the jet core in spite of the above-mentioned bias,[3] we conclude that these winds are not super-gradient.

[3] The existence of this bias was pointed out to the writers by Dr. J. P. Kuettner.

6. Comments on various studies

On the basis of the various studies which have been cited, the standard deviation of *apparent* geostrophic departures in the upper troposphere determined from observed winds and height gradients is in the range from 11 kn (Zobel) to 23 kn (present study). The latter value applies to cyclonically curved jet streams. Values derived from balloon trajectories are 8 kn (Giles and Peterson) and 21 kn (Neiburger and Angell). The standard deviations of *true* geostrophic departures have been given as 13 kn at 300 mb and 8 kn at 200 mb by

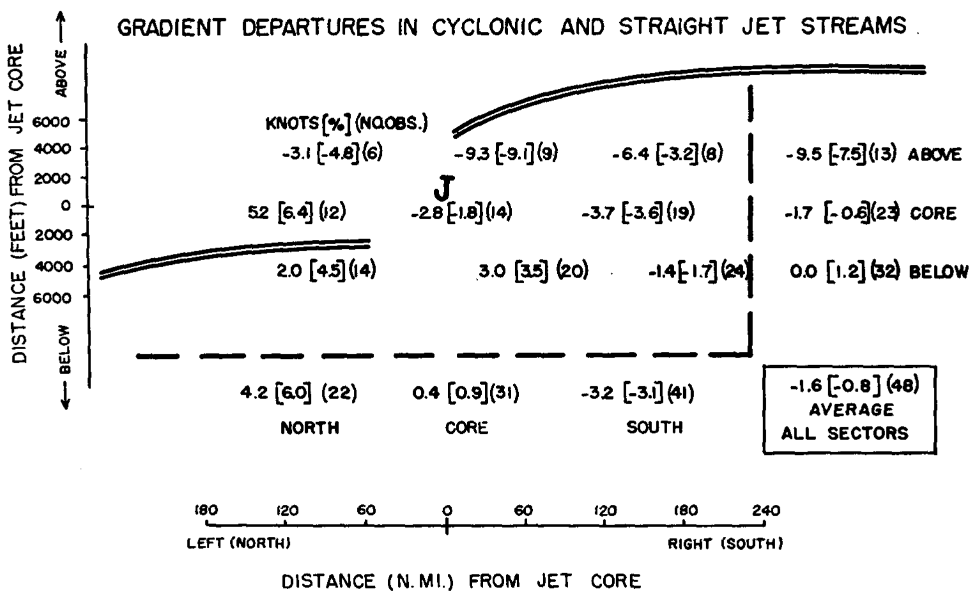


FIG. 5. Average gradient departures in cyclonic and straight jet streams. Double solid lines are tropopauses of fig. 1a. "J" marks the jet core.

Murray, and as 4 kn in straight jet streams and 20 kn in cyclonic jet streams in the present paper.

The writers believe that Faust's conclusion (summarized in an earlier section) that peaked vertical wind profiles are super-gradient may be influenced by errors of measurement. Reiter (1958) showed that, in the upper troposphere, teletype wind reports (U. S. data) must be smoothed to remove the fictitious peaks and valleys caused by instrumental errors before any reasonable analyses of the wind field can be performed. If Faust's data are subject to the same types of error, peaks would tend to represent positive errors in speed and might thus give a spurious indication of super gradient winds.

The pattern of geostrophic departures in jet streams suggested by Endlich, Solot, and Thur is not borne out by the results of the present paper. The super-geostrophic speeds called for to the south of the jet core in their model are not observed, probably because upper frontal zones (*e.g.*, the "jet front" in fig. 1) give strong vertical shear of the geostrophic winds (through the thermal wind equation) and bring the geostrophic winds into fairly close balance with observed winds. The sub-geostrophic speeds to the north of the jet core in their model are the result, as later data have shown, of too large horizontal wind shear in this region of the jet stream.

The observations of marked ageostrophic flow in particular jet streams reported by Endlich and McLean, Cunningham and Saucier *et al.*, are not invalidated by the present data; however, such departures are evidently meso-scale phenomena and vary considerably from case-to-case so that *average* values are not large.

Our data appear to be in direct contradiction to Kochanski's result that forty-nine cases of winds $\geqslant 80$ kn were all supergeostrophic. Perhaps Kochanski's data are biased toward anticyclonic flow in contrast to our sample which is biased towards cyclonic jet streams, so that the two samples are not really comparable.

7. Theoretical discussion

We now wish to determine whether the departures shown earlier can be accounted for dynamically. Geostrophic and gradient departures are, of course, related to forces not included in the geostrophic and gradient equations. The relations are given in the Appendix by deriving the governing equations and retaining all of the terms. By definition, the centrifugal force determines the difference between geostrophic and gradient departures (figs. 2 and 3 have shown that this force is of major importance near upper troughs). It is shown in the Appendix that if the actual curvature is more cyclonic than the curvature of the contours ($R < R_{gr}$), winds will be sub-gradient. Moreover, cross-contour components and vertical motions (properly correlated with wind direction) tend to produce sub-gradient flow. Using equations (2) and (4) of the Appendix, we can calculate the magnitudes of vertical currents (w), of the cross-contour flow angle (β), and of the difference between the actual radius of curvature (R) and R_{gr} (or the ratio R/R_{gr}) required to produce a negative departure of 3 kn (as given in fig. 5 for the south side of the jet). We find that $w = 2$ m sec^{-1}, or $\beta = 15°$, or $R/R_{gr} = 0.8$ gives the desired departure. Vertical motions of the above magnitude must be rejected on the basis of synoptic experience (*e.g.*, Fleagle, 1947; Endlich, 1953) which shows that currents covering significant areas are an order of magnitude smaller. However, the calculated values of β and of R/R_{gr} appear reasonable and can account for the sub-gradient departure specified above.

The positive gradient departure of 5.5 kn on the north side of cyclonic jet streams (fig. 3) could be accounted for on the basis of vertical motions or because the actual curvature is, on the average, less than the curvature of the contours. Calculation shows that the required values are $w = 4$ m sec^{-1}, or $R/R_{gr} = 1.5$. As before, vertical velocities of this magnitude are rejected. Since the calculated ratio of the actual radius of curvature to the radius of the contours departs from unity by a large amount, we are led to suspect that the positive departure of 5.5 kn is an unrepresentative value which may have been produced by an accidental combination of errors.

8. Summary and conclusions

Empirical data have been presented which indicate that the relations between observed winds in a number of jet streams and gradient and geostrophic winds were as follows: in cyclonic jet streams, observed wind speeds were 27.5 kn (18.4 per cent) less than geostrophic speeds on the average. The standard deviation of true geostrophic departures was 20 kn. By comparison, the average gradient departures appeared to be slightly positive to the north of the jet core and negative to the south of the core. But considering the entire cross-section through cyclonic jets, the average gradient departure was approximately zero and the standard deviation of the true gradient departures was approximately 6 kn. The superiority of the gradient assumption, compared to the geostrophic, appears to be considerable in jet streams in the vicinity of upper troughs.

In straight jet streams, the tendency for positive and negative departures to the north and south of the jet, respectively, was less marked than in cyclonic jets; however, sub-gradient (and sub-geostrophic) flow appeared to exist on the average above the jet core. Considering the entire cross-section, the average gradient departure was approximately zero. The standard deviation of true gradient departures was approximately 4 kn.

The data for cyclonic and straight jet streams were combined in an attempt to determine average departures without respect to the curvature of flow. The resulting patterns were similar to those discussed above. There was a tendency for sub-gradient flow above and to the south of the jet core and for essentially zero departures to the north of the core. The overall mean gradient departure was approximately zero.

On the basis of the available data, we conclude that, on the average, the winds in jet streams differ by only small amounts from gradient winds.

APPENDIX

The following discussion reviews the derivation of the geostrophic and gradient equations (following Holmboe *et al.*, 1945) and shows the forces which produce geostrophic and gradient departures.

The gradient and geostrophic equations are, of course, derived as special cases of the complete horizontal equation of motion. In (x, y, p) coordinates, this latter equation can be expressed in the following form:

$$\frac{d\mathbf{V}}{dt} = \frac{dV}{dt}\mathbf{t} + \frac{V^2}{R}\mathbf{n} = -fV\mathbf{n} - \nabla_p\phi + \mathbf{F} \qquad (1)$$

where $\mathbf{V} = V\mathbf{t}$ is the horizontal velocity, $\mathbf{t}$ is a unit vector along s, the tangent to the path, $\mathbf{n}$ is a unit vector perpendicular to $\mathbf{t}$ oriented to the left of the path, $f = 2\omega \sin\theta$, R is the radius of curvature of the trajectory chosen as positive for cyclonic curvature and negative for anticyclonic, $\phi = gz$, $\nabla_p = (\partial/\partial s)\mathbf{t} + (\partial/\partial n)\mathbf{n}$, and $\mathbf{F}$ represents the combined effects of the Coriolis term $-2\omega\cos\theta w\mathbf{i}$ ($\mathbf{i}$ directed eastward) and of frictional forces acting on the parcel. We may separate equation (1) into two scalar equations by taking dot products with $\mathbf{n}$ and $\mathbf{t}$. The first scalar equation is

$$V^2/R + fV = fV_g\cos\beta - 2\omega\cos\theta w\mathbf{i}\cdot\mathbf{n} + \tau_n \qquad (2)$$

where the geostrophic wind $V_g = (l/f)(\partial\phi/\partial 1)$, l is distance measured perpendicular to the contours, β is the angle between the wind vector and the contours, and τ_n is the component of friction in the direction of $\mathbf{n}$. The second scalar equation is

$$\frac{dV}{dt} = -\frac{\partial\phi}{\partial s} + \mathbf{F}\cdot\mathbf{t}. \qquad (3)$$

If $\mathbf{F}$ is assumed to be negligible in comparison with other terms, and if the air parcel is moving horizontally at constant speed, $dV/dt = \beta = w = 0$ and the flow is called gradient. Then equation (3) is identically zero and equation (2) becomes

$$V_{gr}^2/R_{gr} + fV_{gr} = fV_g \qquad (4)$$

where V_{gr} is the gradient wind and R_{gr} is the radius of curvature used in its computation.

In this paper, we have compared actual wind speeds (V, as measured by aircraft) with V_{gr}. Let us compare equations (2) and (4) with V_g fixed in order to examine the non-gradient effects of various terms. First consider the effect of a difference between R and R_{gr} by setting $\beta = w = \tau_n = 0$. Then V will be $\gtrless V_{gr}$ as R is $\gtrless R_{gr}$. Thus, if the actual radius of curvature differs from the computed radius, the wind tends to be either super- or sub-gradient in our comparisons. Next, we examine the effect of β by setting $w = \tau_n = 0$ and taking $R = R_{gr}$. Then, if $\beta \neq 0$, the right hand side of (2) is less than the right hand side of (4); hence, $V < V_{gr}$. Therefore, flow towards either higher or lower pressure tends to produce sub-gradient winds in our comparisons. The effect of the second term on the right side of equation (2) is determined by setting $\beta = \tau_n = 0$ and taking $R = R_{gr}$. For winds from the north, west (or east), and south, $\mathbf{i}\cdot\mathbf{n}$ equals 1, 0, and -1 respectively. For the first of these alternative directions, $w < 0$ (downward motion) tends to make the right-hand side of (2) larger than the right-hand side of (4); consequently, $V > V_{gr}$. Conversely, $w > 0$ tends to produce sub-gradient winds. For the second alternative direction, the second term in equation (2) is zero. In case of the third alternative (*i.e.*, south winds), upward motion tends to produce super-gradient winds, while downward motion tends to produce a sub-gradient effect. In regard to the last term on the right side of (2), we will only say that friction will tend to slow down the jet core, thus tending to make it sub-gradient.

In summary, sub-gradient flow can be the result of the following factors: $R < R_{gr}$, $\beta \neq 0$, $w \lessgtr 0$ for winds from the $\left\{\begin{matrix}\text{south}\\\text{north}\end{matrix}\right\}$, and the jet core, due to frictional drag. Super-gradient flow may be the result of $R > R_{gr}$ or of $w \lessgtr 0$ for winds from the $\left\{\begin{matrix}\text{north}\\\text{south}\end{matrix}\right\}$.

Computations reported in the main body of the paper assume typical values of V_g, V_{gr}, f, $2\omega\cos\theta\mathbf{i}\cdot\mathbf{n}$, and R_{gr} as 120 kn, 100 kn, 10^{-4} sec^{-1}, 10^{-4} sec^{-1}, and 1350 n mi respectively.

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