NUMERICAL PREDICTION OF HURRICANE MOVEMENT WITH THE USE OF A FINE GRID

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(Original manuscript received 22 October 1959; revised manuscript received 29 February 1960)

ABSTRACT

A numerical model is described for integrating the barotropic prediction equations to obtain forecasts of the displacement of a hurricane center. The model employs a fine grid which is centered over the hurricane and covers only the vortex circulation. Forecasts for Connie and Diane (1955) and Betsy (1956) are presented. Reduction of truncation errors by the use of a smaller grid mesh results in improved trajectory forecasts.

1. Introduction

A problem characteristic of numerical prediction of the movement of hurricanes is that the scale of these disturbances usually is relatively small compared to those with which the numerical forecaster is presently concerned in high latitudes. Because of the small scale, truncation errors are excessive when predictions are made with standard numerical models.

Several numerical methods, of varying degrees of objectivity, have been devised for the prediction of hurricane movement. Most of these cope with the problem of small scale by treating the hurricane, in some manner, as an entity separate from the total flow pattern. They usually involve the computation of an environmental or steering flow and the displacement of the vortex in this flow. The action of the hurricane on its environment is either completely neglected or only partially taken into account.

In the graphical-numerical method developed by Riehl and Haggard (1956), the storm center is displaced with the mean geostrophic wind computed across a large rectangular box superimposed upon the storm; this effectively averages out the circulation of the storm itself. Sasaki and Miyakoda (1954, 1956) developed a technique for subtracting a vortex represented by an analytic function and computing the instantaneous velocity of the center of the residual or steering flow with the aid of the barotropic vorticity equation. The vortex center is extrapolated with this velocity. Numerical forecasts on a digital computer made by Kasahara (1957, 1959a) employed a somewhat similar method in which the steering flow is obtained by subtracting a radially averaged vortex from the total field. More recently, Kasahara (1959b) has incorporated the technique into a two-level baroclinic model. Morikawa (1956) and Levine (1958) describe a numerical model in which the hurricane is replaced by a point vortex; the vortex is displaced in the steering flow. In these methods the vortex is assumed not to change intensity or shape in the forecast interval.

The numerical model, described in the present paper, which uses the barotropic vorticity prediction equation predicts the movement of the hurricane by treating it as an integral part of the larger-scale flow. By placing a fine net of points over the vortex, truncation errors are reduced to the extent that it is possible to represent the larger features of the hurricane circulation with some degree of clarity. In so far as the vortex is resolved by the fine net, interaction between the storm and its environment is incorporated.

2. Description of the numerical model

The numerical model was coded for the IBM 704. The time extrapolations are carried out over two grid nets. A "coarse" grid of 25×22 points with an interval of $d = 300$ km at the standard latitudes covers the region shown in fig. 1 (the projection is a Lambert conformal; standard latitudes are 30 deg and 60 deg). The "fine" grid is a mesh of 15×15 points with grid interval equal to one-half the large grid interval, or $d' = 150$ km. It covers a region approximately 2000 km on a side, centered over the hurricane. Since $d' = \frac{1}{2}d$, every other point of alternate rows of the fine grid is common to the coarse grid. In the subsequent discussion, subscripts i, j are used to designate points of the coarse grid ($0 \leqslant i \leqslant p$ and $0 \leqslant j \leqslant q$), and subscripts r, s are used for the fine grid ($0 \leqslant r \leqslant p'$ and $0 \leqslant s \leqslant q'$).

The barotropic prediction equations (with the geostrophic approximation) in their usual form are

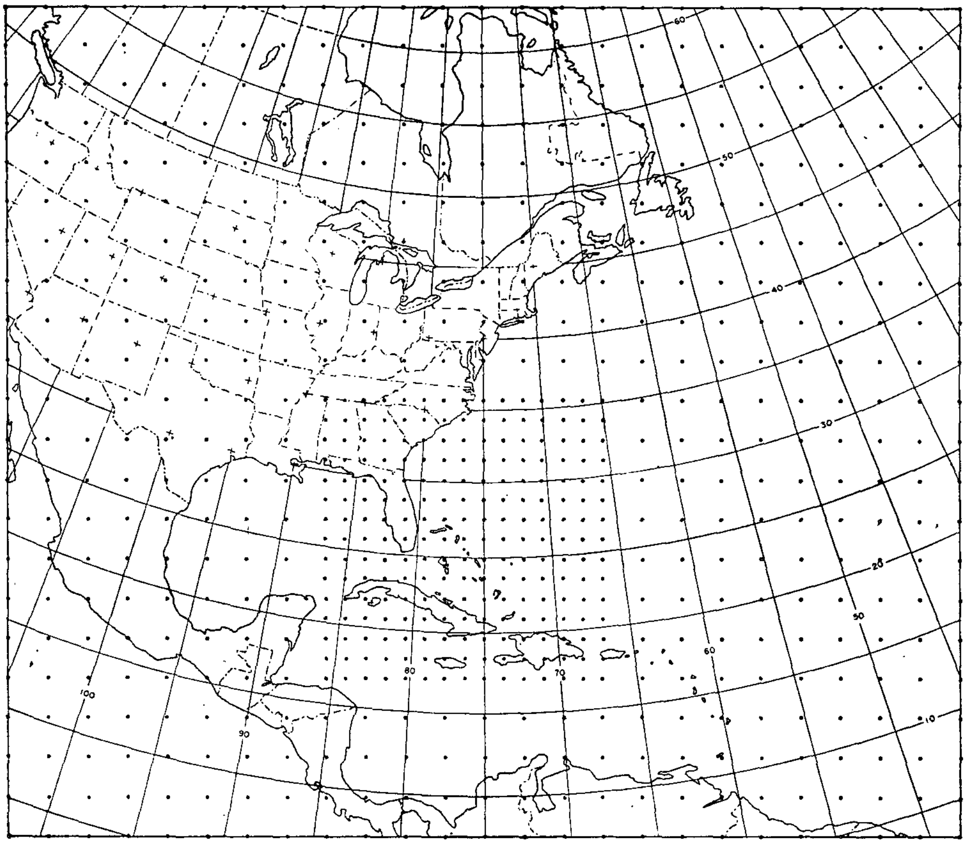

FIG. 1. Geographical region covered by the coarse grid. A typical position of the fine grid is shown.

$$\frac{\partial \zeta}{\partial t} = \tfrac{1}{4} M J(\zeta + f, z);$$

$$\zeta \equiv M \nabla^2 z; \quad M \equiv g m^2 / f d^2;$$

$$\zeta^{\tau+1} = \zeta^{\tau} + \Delta t \left(\frac{\partial \zeta}{\partial t}\right)^{\tau} \quad \tau = 0;$$

and

$$\zeta^{\tau+1} = \zeta^{\tau-1} + 2\Delta t \left(\frac{\partial \zeta}{\partial t}\right)^{\tau} \quad \tau > 0.$$

Here, ∇^2 is the finite-difference Laplacian operator

$$\nabla^2 z \equiv z_{l+1,m} + z_{l,m+1} + z_{l-1,m} + z_{l,m-1} - 4z_{l,m}$$

and $J(u,v)$ is the finite-difference Jacobian operator

$$J(u,v) \equiv (u_{l+1,m} - u_{l-1,m})(v_{l,m+1} - v_{l,m-1}) - (u_{l,m+1} - u_{l,m-1})(v_{l+1,m} - v_{l-1,m})$$

where l,m denotes i,j or r,s. The history of the motion is carried by ζ the relative vorticity; f is the Coriolis parameter, z is the height of the 500-mb surface, and m the map-projection scale factor. The numbers of grid points are $(p+1)(q+1) = 25 \times 22 = 550$ for the coarse grid and $(p'+1)(q'+1) = 15 \times 15 = 225$ for the fine grid. The extrapolation intervals are $\Delta t = 1$ hr for the coarse grid and $\Delta t' = 15$ min for the fine grid. The time interval $\Delta t'$ was chosen to be $\frac{1}{4}\Delta t$ to allow for the large velocities in the hurricane.

The computation procedure for a complete one-hour time step is as follows:

(1) By using the prediction equations applied to the coarse grid, the relative vorticity ζ and height z are extrapolated from time τ to $\tau + 1$ in the conventional manner (Charney and Phillips, 1953) if one assumes $\partial\zeta/\partial t = \partial z/\partial t = \zeta = 0$ for all time on the boundary. The presence of the fine grid is ignored in this step.

(2) For coarse-grid points on the boundary of the fine grid, the fine-grid tendency values $(\partial\zeta/\partial t)_{r,s}$ and $(\partial z/\partial t)_{r,s}$ are computed from the coarse-grid $\tau + 1$ and τ fields; for example,

$$(\partial z/\partial t)_{r,s} = (z_{i,j}{}^{\tau+1} - z_{i,j}{}^{\tau})/4\Delta t'.$$

From these values, the tendencies are computed at the intermediate fine-grid boundary points by linear interpolation. These tendencies are used in the manner explained in step 3.

(3) For each one-hour time step on the coarse grid, there are four 15-min steps on the fine grid: the first begins from time τ and the last ends at time $\tau + 1$.

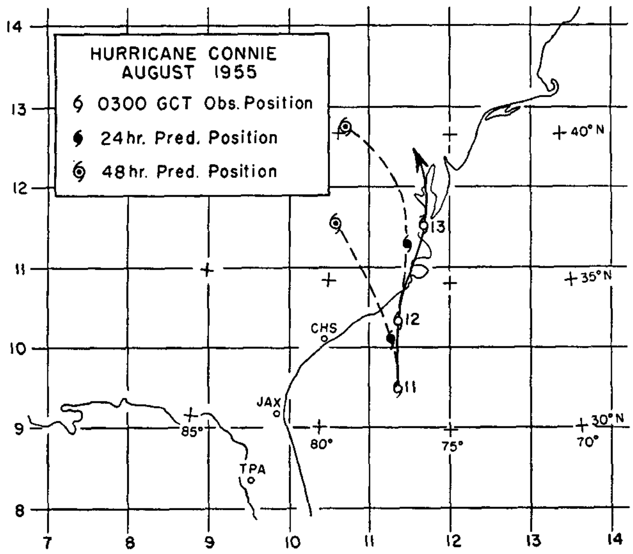

FIG. 2. Observed and predicted 24-hr and 48-hr 500-mb positions for Hurricane Connie, August 1955. The grid interval is 300 km at standard latitudes of 30 and 60 deg.

By using the prediction equations for the fine grid, each of these time steps is carried out in a manner identical to that used for the coarse grid, except that the boundary values of ζ and z are computed by using the tendencies found in step 2. For example,

$$z_{r,s}^{\tau+\frac{1}{2}} = z_{r,s}^{\tau} + (\partial z/\partial t)_{r,s}\Delta t'.$$

In other words, the boundary values at the intermediate time steps are found by linear time-wise interpolation from the coarse-grid values.

(4) To establish consistency between the two grids and to incorporate the effects of the fine grid into the history of the motion on the coarse grid, the height values of the coarse grid at $\tau + 1$ are replaced by the height values from the fine-grid extrapolation at the common points. By using this corrected height field, the coarse-grid vorticity is recomputed at these points.

These four steps are iterated for the total forecast interval. Since, in twenty-four hours, the movement of a hurricane may be several hundreds of kilometers, it is necessary to incorporate means to keep the fine grid approximately centered over the hurricane. After every one-hour time step over the coarse grid, the height field interior to the fine grid is scanned to find the height minimum. If the minimum height is located at any one of the four coarse-grid points closest to the center of the fine grid, no grid movement is assumed necessary. If this condition is not satisfied, a moving routine is initiated which moves the fine grid in such a way that the hurricane is again in the center of the grid.

3. Data preparation and model testing

To test the numerical prediction model, the storms Connie and Diane of August 1955 and Betsy of

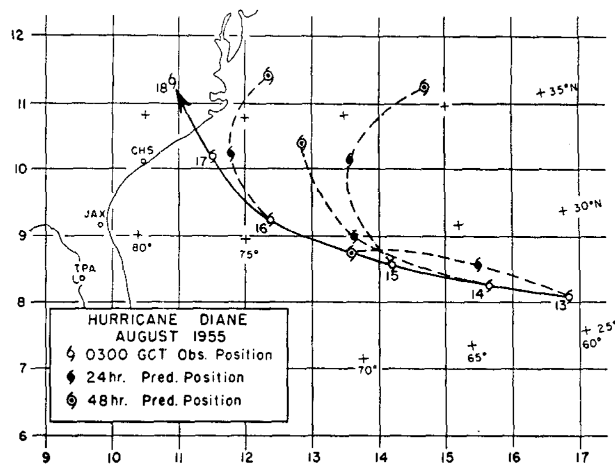

FIG. 3. Observed and predicted 24-hr and 48-hr 500-mb positions for Hurricane Diane, August 1955. Grid interval is 300 km at latitudes 30 and 60 deg.

August 1956 were used. Analyses of the surface, 500-mb and 200-mb maps for Diane and Connie were made by Professor Herbert Riehl of the University of Chicago by using time sections and differential thickness analyses over the ocean and subtropical regions (Riehl, 1955). For Betsy, 1000-mb, 700-mb and 500-mb analyses and numerous data for this storm were obtained from the National Hurricane Research Project in West Palm Beach, Florida.

For Connie and Diane, the pressure heights from the coarse grid in the vicinity of the fine grid were transferred to a larger scale map and the initial data for the fine grid were interpolated from a smoothed analysis. The purpose of the operation was merely to reduce reading errors. It was later found more satisfactory to read the fine-grid height field from the original map on which a careful analysis near the vortex had been performed, provided the scale of the map be at least 1:12.5 million.

4. Forecast results

Geostrophic forecasts up to 48 hr were made for 11–12 August 1955 (Connie), for 13–16 August 1955 (Diane) and for 13–17 August 1956 (Betsy). Figs. 2, 3 and 4 show the predicted trajectories compared with the observed trajectories. The location of the predicted center with respect to the observed center plotted on a compass rose is shown in fig. 5 for the 24-hr forecasts. Fig. 6 illustrates the error of the predicted displacement relative to the observed displacement: here the observed displacements have been represented by a single vector and the predicted center positions plotted relative to this.

Two statistical measures of the accuracy of the forecasts are presented in tables 1 and 2. The average $\bar{E}_v$ of the magnitude of the vector error is shown in

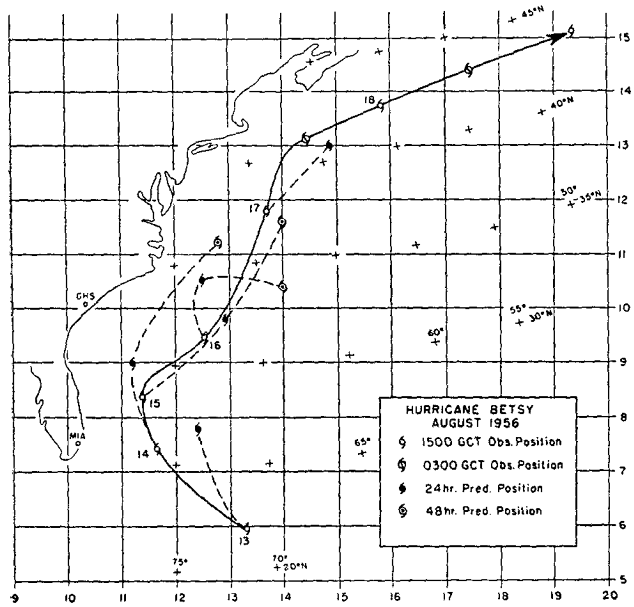

FIG. 4. Observed and predicted 24-hr and 48-hr 500-mb positions for Hurricane Betsy, August 1956. Grid interval is 300 km at latitudes 30 and 60 deg.

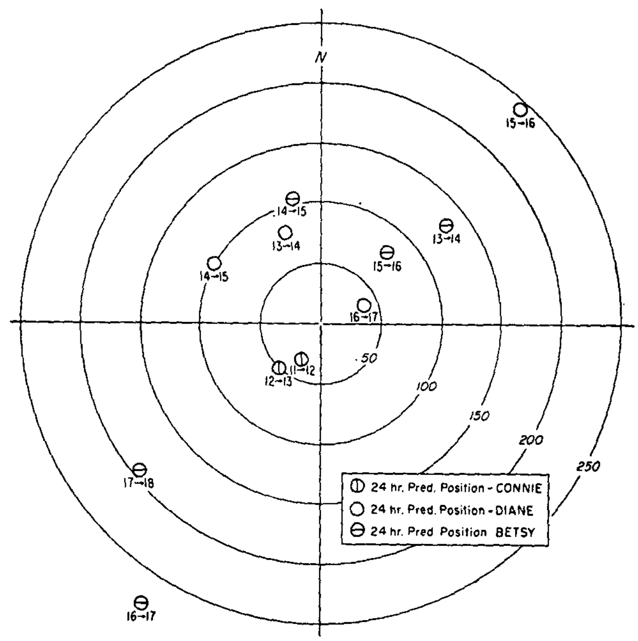

FIG. 5. Predicted 24-hr storm positions for Hurricanes Connie, Diane and Betsy with respect to the observed position (center). Distances are in nautical miles.

TABLE 1. Mean magnitude of error vector and mean observed displacement (nautical miles).

	Connie, Diane	Betsy	Total
$\bar{E}_v$ (24-hr)	92(6)	160(5)	126(11)
$\bar{S}_0$	220	332	270
$\bar{R}_v = \bar{E}_v/\bar{S}_0$	0.42	0.48	0.47
$\bar{E}_v$ (48-hr)	251(5)	320(3)	285(8)
$\bar{S}_0$	452	635	521
$\bar{R}_v = \bar{E}_v/\bar{S}_0$	0.56	0.50	0.55

TABLE 2. Mean error in magnitude of displacement vector and mean observed displacement (nautical miles).

	Connie, Diane	Betsy	Total
$\bar{E}_s$ (24-hr)	47(6)	136(5)	84(11)
$\bar{S}_0$	220	332	270
$\bar{R}_s = \bar{E}_s/\bar{S}_0$	0.21	0.41	0.31
$\bar{E}_s$ (48-hr)	57(5)	292(3)	144(8)
$\bar{S}_0$	452	635	521
$\bar{R}_s = \bar{E}_s/\bar{S}_0$	0.13	0.46	0.38

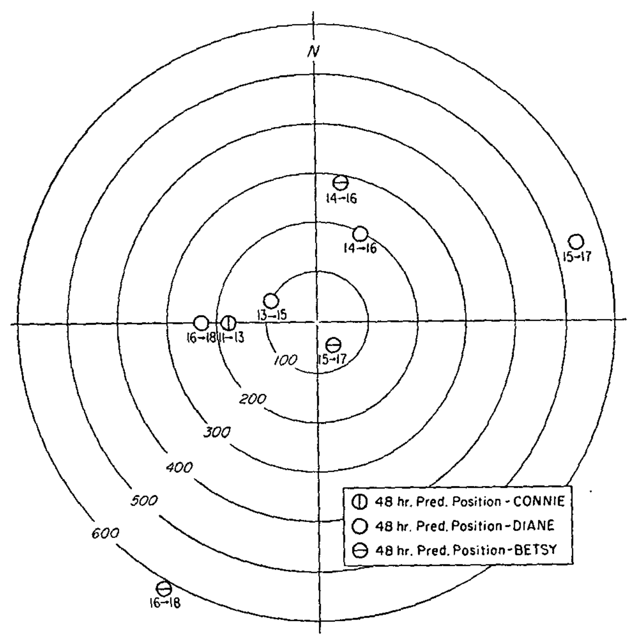

FIG. 6. Predicted 24-hr storm positions for Hurricanes Connie, Diane and Betsy relative to the observed storm displacements, after the latter have been reduced to a common scale. The dashed vector represents the mean predicted vector.

table 1 for the three storms. The number of cases is indicated in parentheses. The ratio $\bar{R}_v$ of the mean error vector to the mean observed magnitude $\bar{S}_0$ of the displacement is included. In table 2, the mean error $\bar{E}_s$ in the magnitude of the predicted vector displacement is presented. The ratio $\bar{R}_s = \bar{E}_s/\bar{S}_0$ is also given. $\bar{E}_v$ and $\bar{R}_v$ are measures of the error in the predicted speed of the storm, whereas $\bar{E}_v$ and R_v are influenced by errors in the predicted direction of movement. The statistical confidence limits of $\bar{E}_v$ and $\bar{E}_s$, computed for 80 per cent probability by using the appropriate "t values" (Davies, 1957), are presented in table 3.

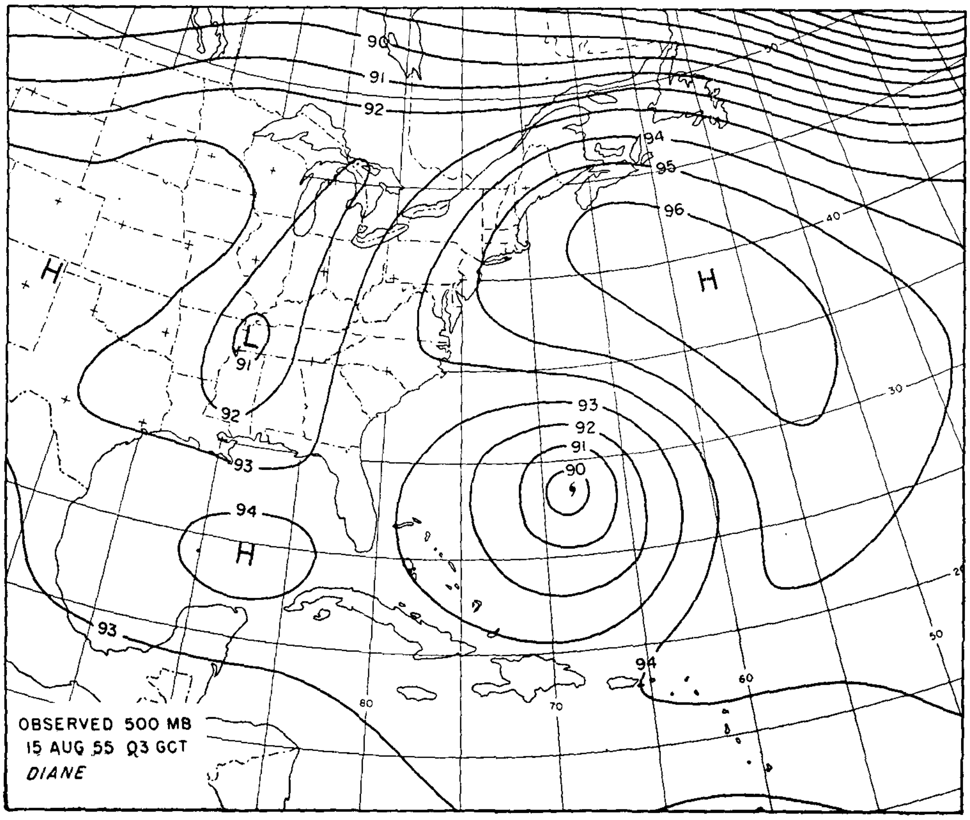

FIG. 7. Observed 500-mb height field for 03GCT 15 August 1955.

TABLE 3. Eighty per cent confidence limits for $\bar{E}_v$ and $\bar{E}_s$ in nautical miles.

24 hr:	$92 < \bar{E}_v < 160$	$53 < \bar{E}_s < 116$	11 cases
48 hr:	$181 < \bar{E}_v < 389$	$44 < \bar{E}_s < 246$	8 cases

By comparing, for the total number of 24-hr forecasts, $\bar{E}_v$ and $\bar{E}_s$ from tables 1 and 2, it is seen that a large part of the error in predicted displacement is due to error in predicted speed. From table 2, it is seen that the Betsy forecasts are largely responsible for this. In particular, the Betsy forecasts from the 16th and 17th of August have such large errors in predicted speed that the mean is strongly affected. Whereas the Betsy forecasts are characterized by large errors in predicted speed, the Diane forecasts show large errors predominately in the predicted direction of movement (fig. 2); they produce a rightward deflection of the total average relative predicted displacement vector (dashed vector) in fig. 7. Figs. 7, 8 and 9 show, respectively, the observed map for 15 August 1955, the 24-hr predicted map from the 15th and the observed map for 16 August 1955. It appears that the error in the direction of movement is largely caused by the erroneous prediction of large-scale features, in particular the Atlantic subtropical high-pressure cell. This failure must to a considerable extent be due to lack of data in the Atlantic Ocean. Kasahara (1957), in his steering-flow forecasts, observes an even more systematic deflection to the right of the observed trajectory for both the Diane and Betsy forecasts. The fact that the fine-grid forecasts for Betsy do not show this tendency suggests the possibility that there is more than one factor producing the erroneous rightward deflection.

To estimate the sensitivity of the prediction of movement on the initial analysis in the neighborhood of the vortex, two of the forecasts were repeated using analyses that had been independently prepared. For Diane, two 24-hr forecasts were made from 16 August as shown in fig. 10. For Connie, two 24-hr forecasts from 12 August were made as shown in fig. 11. Whereas the difference in predicted movement in Diane is not excessive, the Connie predictions are significantly different. On examination of the latter, the two initial analyses reveal a stronger northward current in the second analysis. Both of these analyses were consistent with the few observations that were available. The sensitivity must be to a great extent due to the fact that Connie was at the initial map time located in the saddle between two subtropical high cells; under these circumstances, small changes

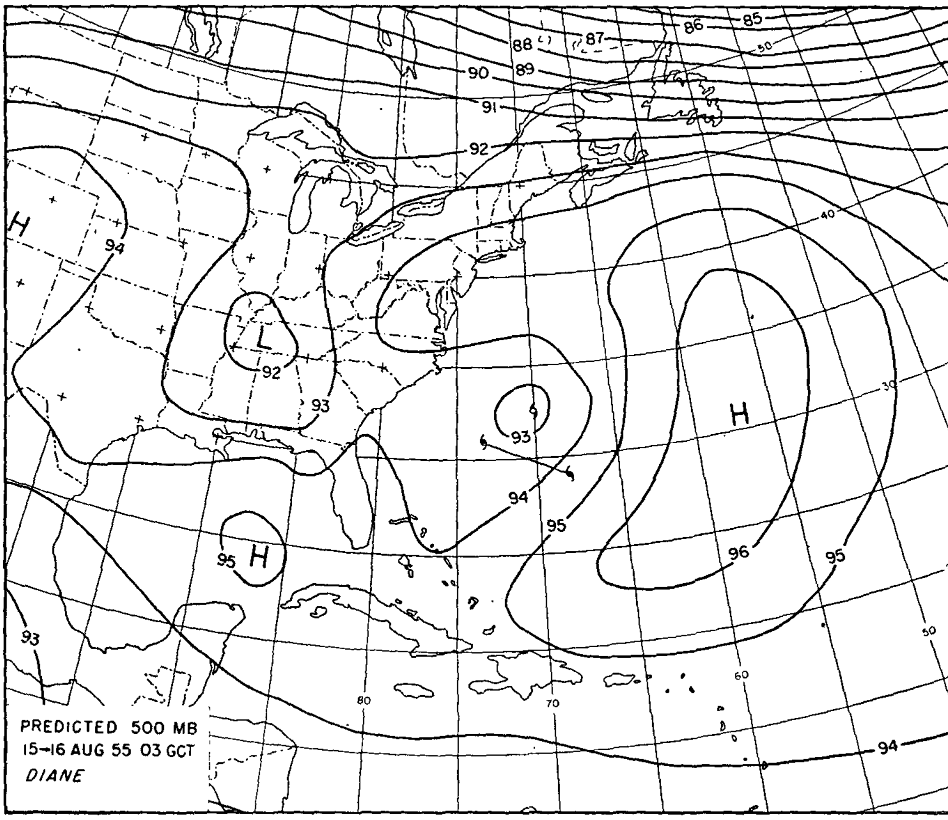

FIG. 8. Twenty-four hour predicted 500-mb height field valid 03GCT 16 August 1955.

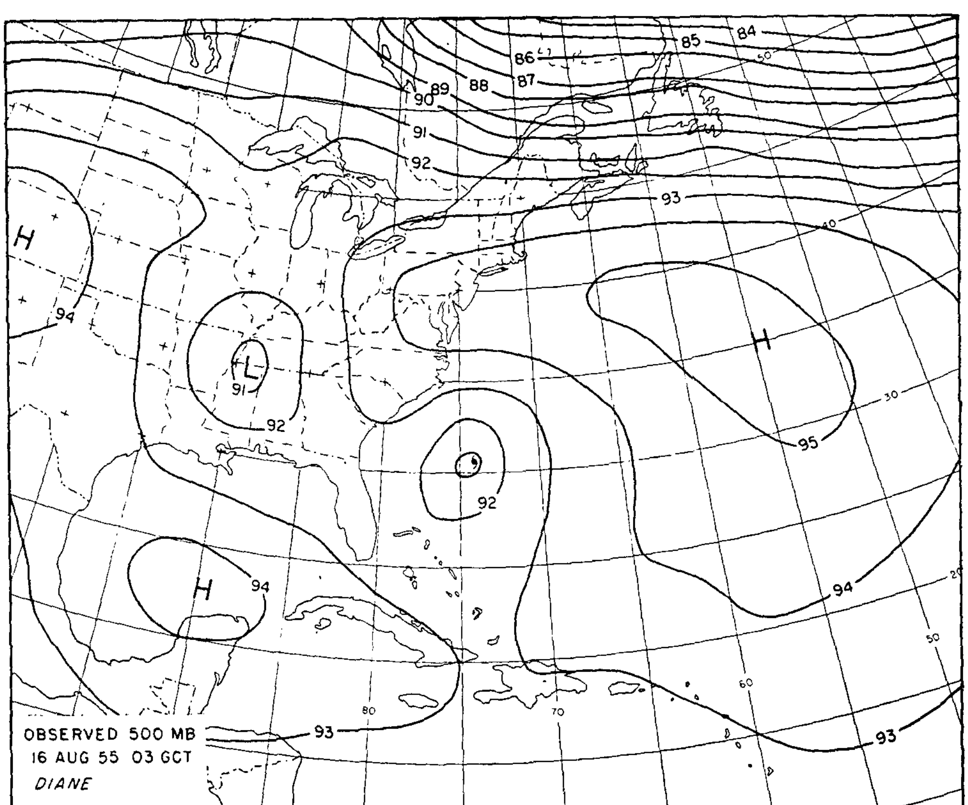

FIG. 9. Observed 500-mb height field for 03GCT 16 August 1955.

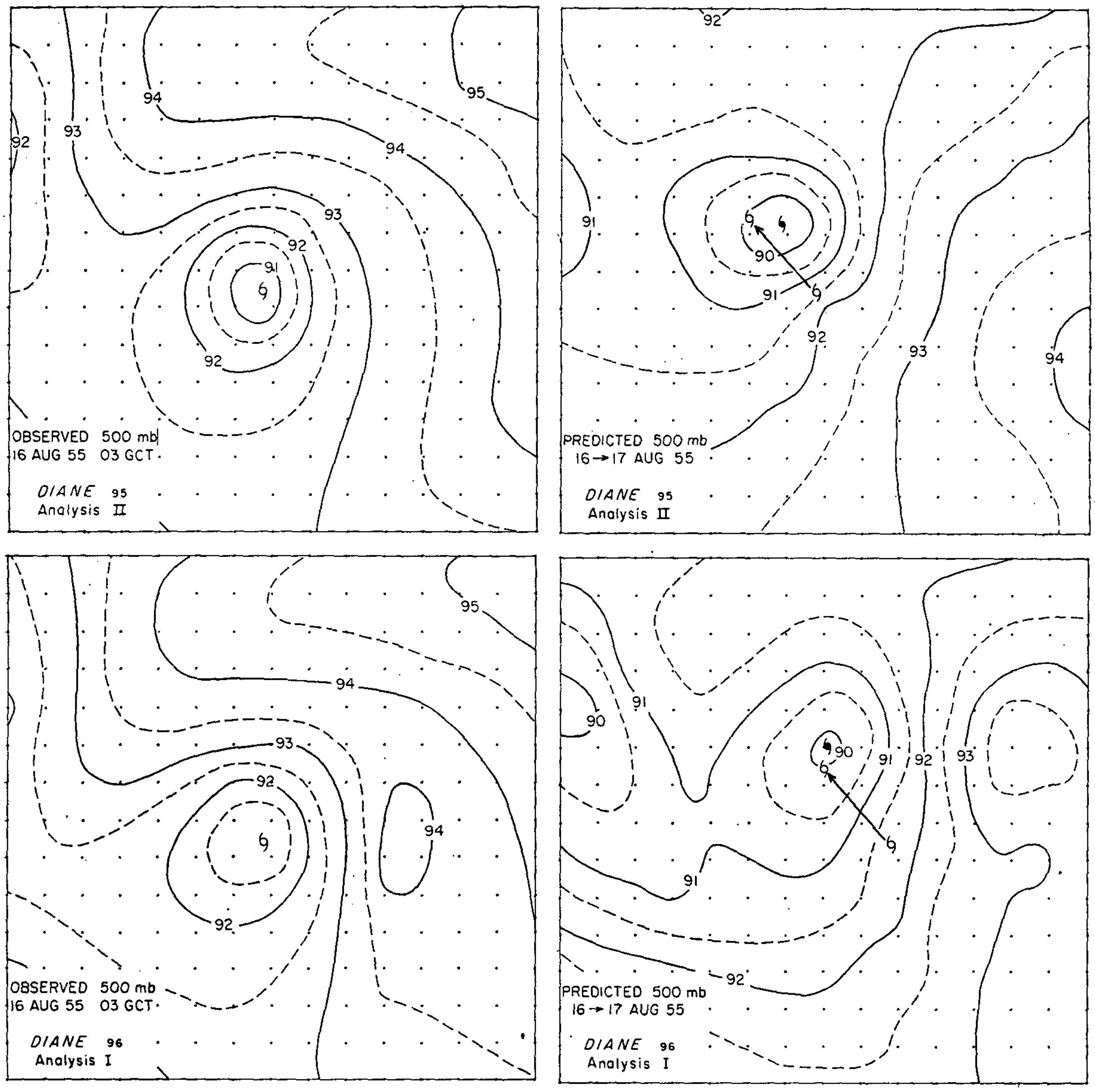

FIG. 10. On the left: two analyses of the 500-mb height field on the fine grid for 03GCT 16 August 1955 (Diane). On the right: the corresponding 24-hr predictions valid 03GCT 17 August 1955.

in the initial analyses could produce large changes in the predicted path.

From the same set of analyses for the three storms, forecasts using the standard 300-km grid interval were made; in addition, forecasts using a 210-km grid were made, Comparisons of $\bar{E}_v$ and $\bar{E}_s$ for the three sets of forecasts are shown in fig. 12. Also, the steering model forecasts obtained by Kasahara (1957) are included. The apparent reduction of error by reducing the grid interval indicates that truncation errors contribute significantly to the errors of predicted movement of the hurricane.

The average predicted displacement relative to the observed displacement for the fine-grid predictions is shown in fig. 7 by a dashed vector and indicates a small underprediction of displacement. The forecasts with the two aforementioned grid sizes show somewhat larger mean underprediction of displacement. For the eleven 24-hr forecasts, the mean underprediction in displacement found by computing the algebraic mean of the individual forecast errors is 26 n mi for the 150-km grid, 68 n mi for the 210-km grid and 106 n mi for the 300-km grid forecasts. Such error may be in part attributed to the "dispersive" characteristics of the solution of the finite-difference vorticity equations. In the solution of the linearized finite-difference vorticity equation, it can be shown that the phase speed of a wave perturbation superimposed on a uniform current always is less than the

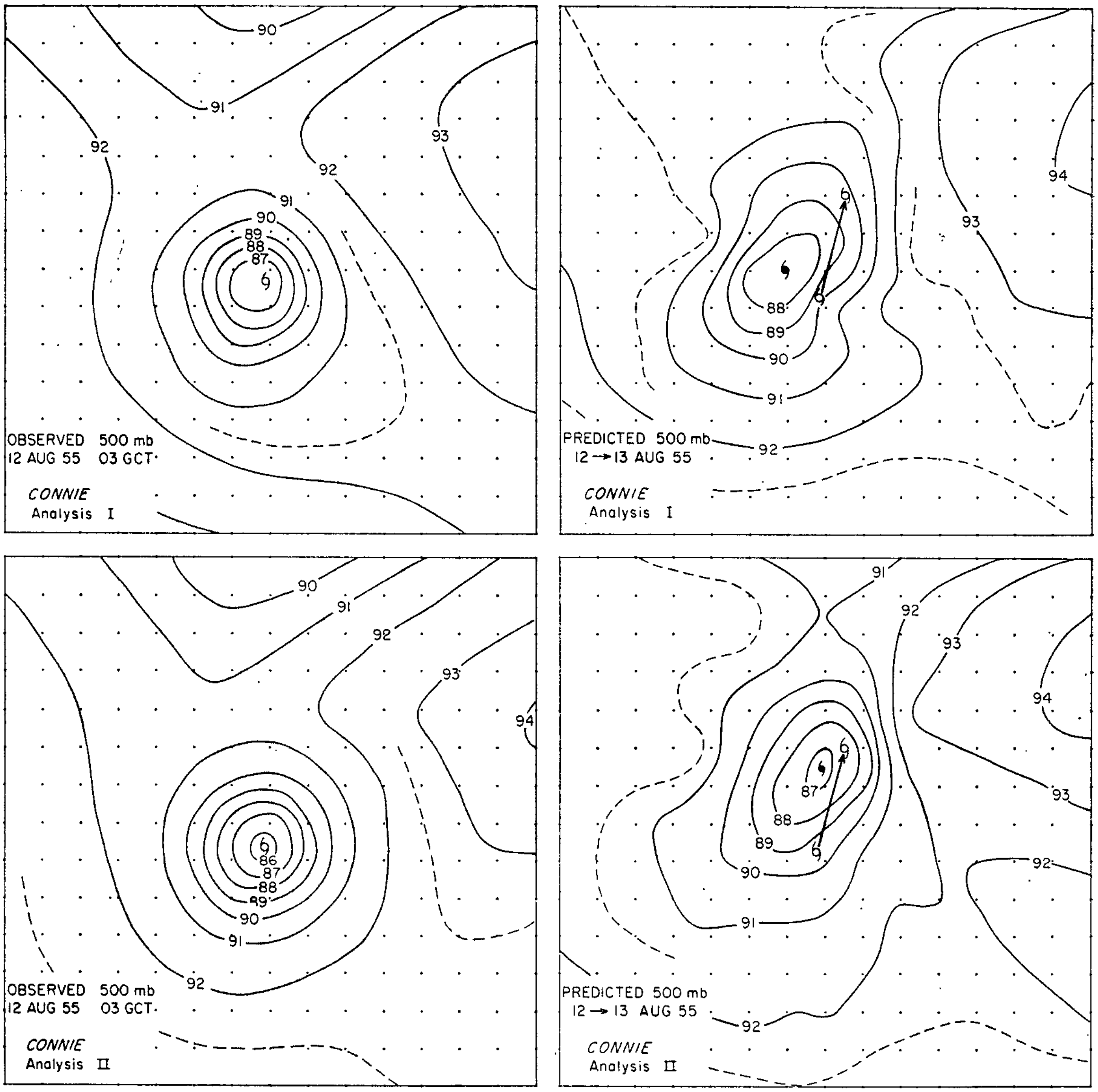

FIG. 11. A comparison of error $\bar{E}_v$ and $\bar{E}_s$ of predicted displacement for three hurricanes, Connie (1955), Diane (1955) and Betsy (1956), for four numerical models. The number of cases is indicated in parenthesis.

true speed and that the discrepancy varies strongly with Δx (for example, see Gates, 1959).

The influence of the artificial boundaries of the region of integration must certainly be large in the forecasts particularly after 48 hr. This is readily seen when one considers that a particle crossing the boundary at $t = 0$ and moving along a grid row with constant speed of 21 m per sec would pass the center of the grid in 48 hr. It is apparent that, for reliable 48-hr predictions, a larger region of integration must be used.

5. Conclusions

Through the use of a 150-km net of points centered over the hurricane, truncation errors arising in the numerical prediction of hurricane movement have been reduced by a significant amount. By using the simplest of numerical models (a quasi-geostrophic barotropic atmosphere), reasonably good 24-hr forecasts of the center displacement are obtained. Forty-eight-hour forecasts show considerable scatter, arising mainly from uncertainties in the initial data, truncation errors and artificial boundary conditions. Errors imposed by limitations of the physical model are of course present, but, because of the magnitude of errors arising from the above sources, it is difficult to examine them separately.

6. Acknowledgments

The writer wishes to thank Professor George W. Platzman for his advice and assistance in this research and in preparation of the manuscript. Many helpful discussions with Dr. A. Kasahara also are appreciated.

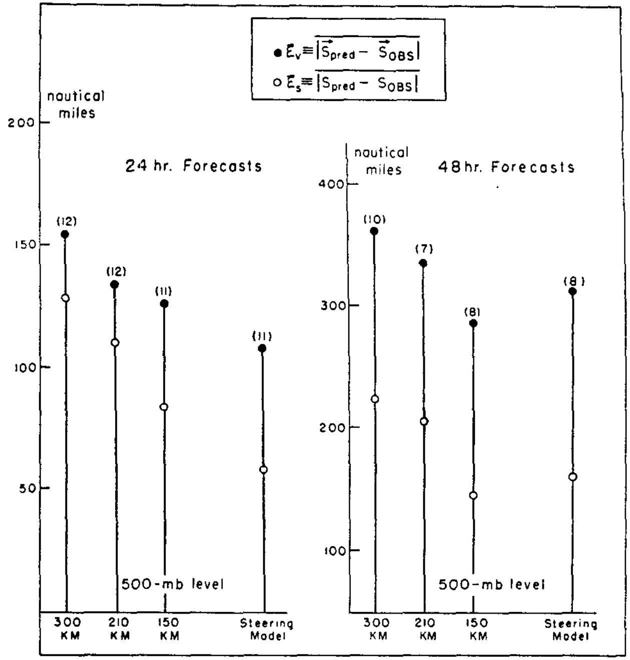

FIG. 12. Comparison of errors.

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