A COMPARISON BETWEEN TWO TRANSFORMATIONS USED IN NORMALIZING METEOROLOGICAL DATA

Wallace E. Howell

W. E. Howell Associates

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Many meteorological quantities, such as precipitation for instance, occur in nature in frequency distributions far different from the Gaussian normal distribution. A number of transformations have been suggested for converting the natural frequency distribution of precipitation approximately to normality, the most generally successful among them being the cube root of the precipitation (Stidd, 1953), and its incomplete gamma function (Thom, 1957). The independent success of these two suggested making a comparison between them.

The cube-root transformation of a series of precipitation observations is accomplished by simply extracting the cube root of each term of the series. The incomplete-gamma-function transformation is accomplished by a curve-fitting procedure following the method of maximum likelihood, which expresses the transformed distribution in terms of two parameters, β and γ, which Thom calls respectively the scale and shape parameters. The product $\beta\gamma$ is equal to the mean of the fitted series, and the ratio of any precipitation amount x to the mean precipitation of the series is $x/\beta\gamma$.

A comparison between the two transformations was made by starting with the normal Gaussian distribution, performing the inverse of each of the indicated transformations upon it, and comparing the results graphically. In fig. 1, the incomplete-gamma distributions for several values of γ are shown as "normalized" by the cube-root transformation to a Gaussian distribution with standard deviation t. It is seen that the relationship is a linear one except at very low values of γ and large deviations, and even here the departure from linearity is small. Although the reason for this relationship is not obvious, it may be described empirically by the formula

$$\left(\frac{x}{\beta\gamma}\right)^{\frac{1}{3}} = 1 - \frac{1}{9\gamma} + \frac{t}{(9\gamma)^{\frac{1}{2}}}.$$

It follows from this result that the cube-root and incomplete-gamma-function transformations are practically indistinguishable over the principal range of variation in their ability to transform any observed frequency distribution to normality, although there may be some distinction between them at the extremities of the distributions, especially when γ is small. Likewise, properties attributable to one of them, such as its relationship to a physical theory of precipitation, must be almost equally applicable to the other. In particular, the slope of the characteristic curve on the Stidd graphs is identified with Thom's shape parameter.

REFERENCES

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Thom, H. C. S., 1957: *A statistical method of evaluating augmentation of precipitation by cloud-seeding.* Tech. Rep. No. 1, *Final Rep. of the Advis. Comm. on Wea. Control*, Washington, pp. 5–25.

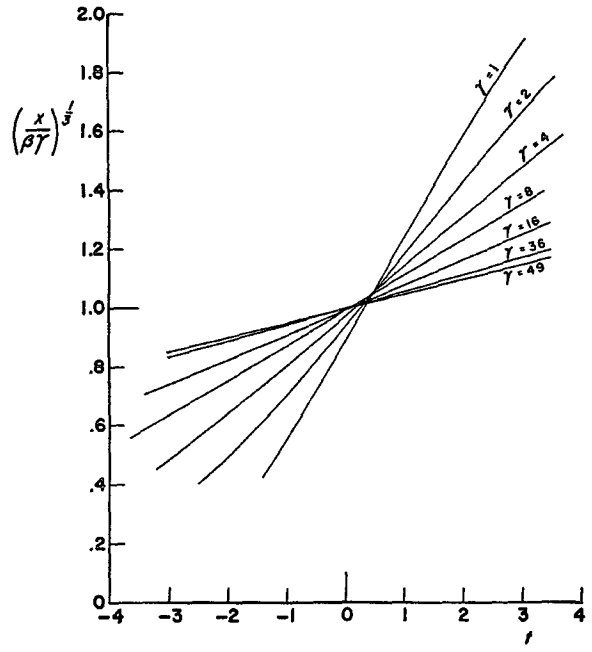

FIG. 1. Incomplete-gamma distributions "normalized" by a cube-root transformation.

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