AN ANALYSIS OF THE TRAJECTORY OF A CONSTANT-PRESSURE BALLOON IN A LONG ATMOSPHERIC WAVE IN COMPARISON WITH THE NONLINEAR THEORY [1]

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1. Introduction

In recent years, the Naval Research Laboratory has conducted a series of experiments in flying the constant-pressure balloons (transosondes) over the Northern Pacific Ocean and the United States. Each of these transosondes was equipped with a transmitter which operated for 15 min every two hours at alternate frequencies of 6, 13 and 19 megacycles per sec. The trajectories of these balloons were determined by means of radio-direction-finding (RDF) bearings on these signals. Discussions of these flights have been presented by Mastenbrook and Anderson (1953; 1956). Meteorological data obtained from these flights have been analyzed, and meteorological applications of the trajectories of these balloons have been discussed by Neiburger and Angell (1956) and Angell (1959).

Attempts have been made to investigate oscillations and trajectories of a particle in some model pressure systems which characterize the pressure distribution in the atmosphere. The linearized system of the Lagrangian equations for a particle moving in a long pressure wave in a rotating fluid has been studied by Kao and Neiburger (1959). A comparison of the trajectory computed from the solution of the linearized system and the observed trajectory of a constant-pressure balloon in a long atmospheric wave has been made by Viezee (1959).

In a recent paper (Kao, 1960), the nonlinear characteristics of the oscillations and trajectories of a particle in a long wave of finite amplitude in a rotating fluid have been investigated, and the zeroth- and first-order solutions of the nonlinear system of the Lagrangian equations have been obtained. The objectives of this paper are to analyze the observed trajectory of a constant-pressure balloon in a long atmospheric wave and to compare it with the trajectory computed from the solution of the nonlinear system of the Lagrangian equations.

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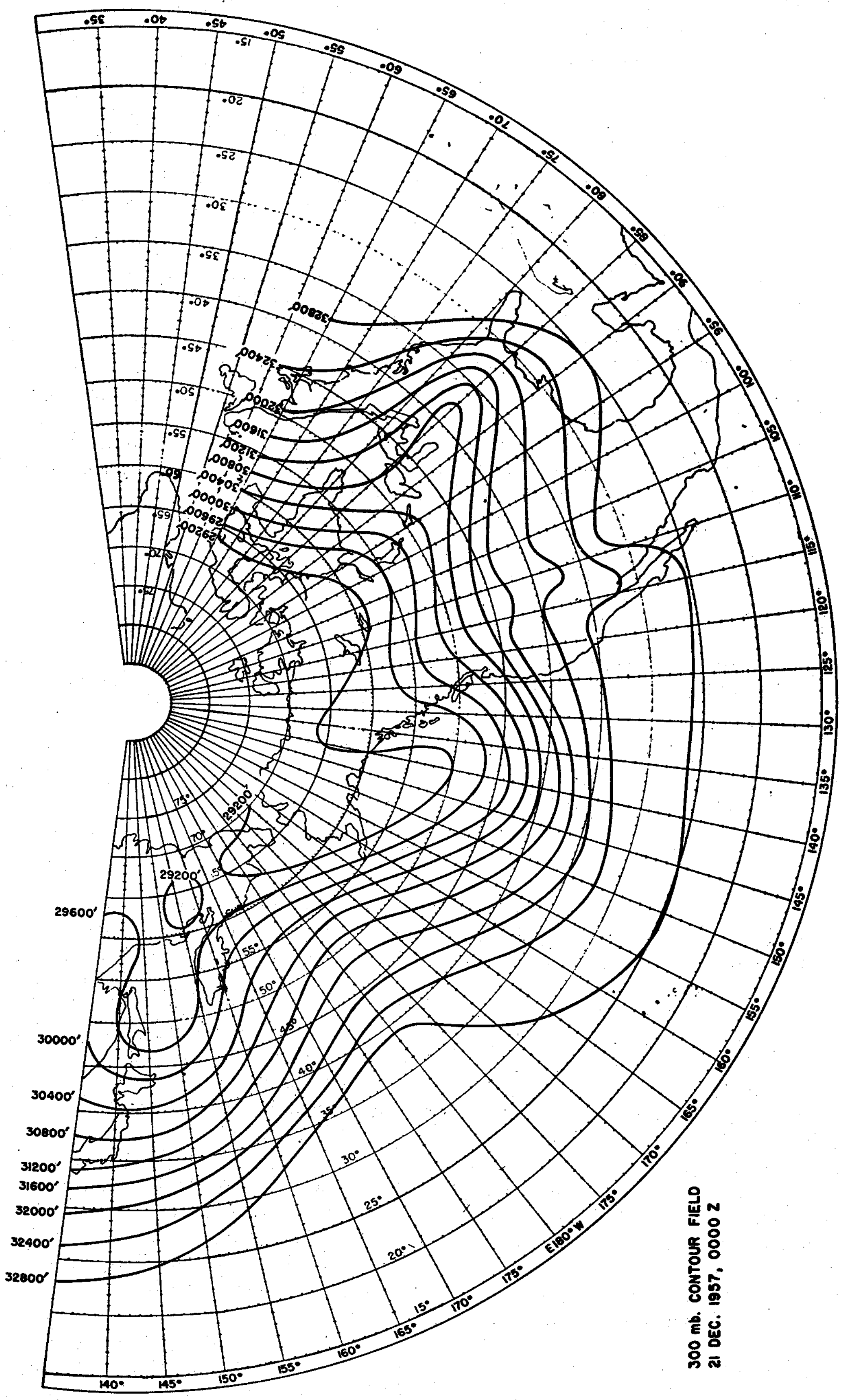

FIG. 1. The 300-mb contour map for 0000Z 21 December 1957.

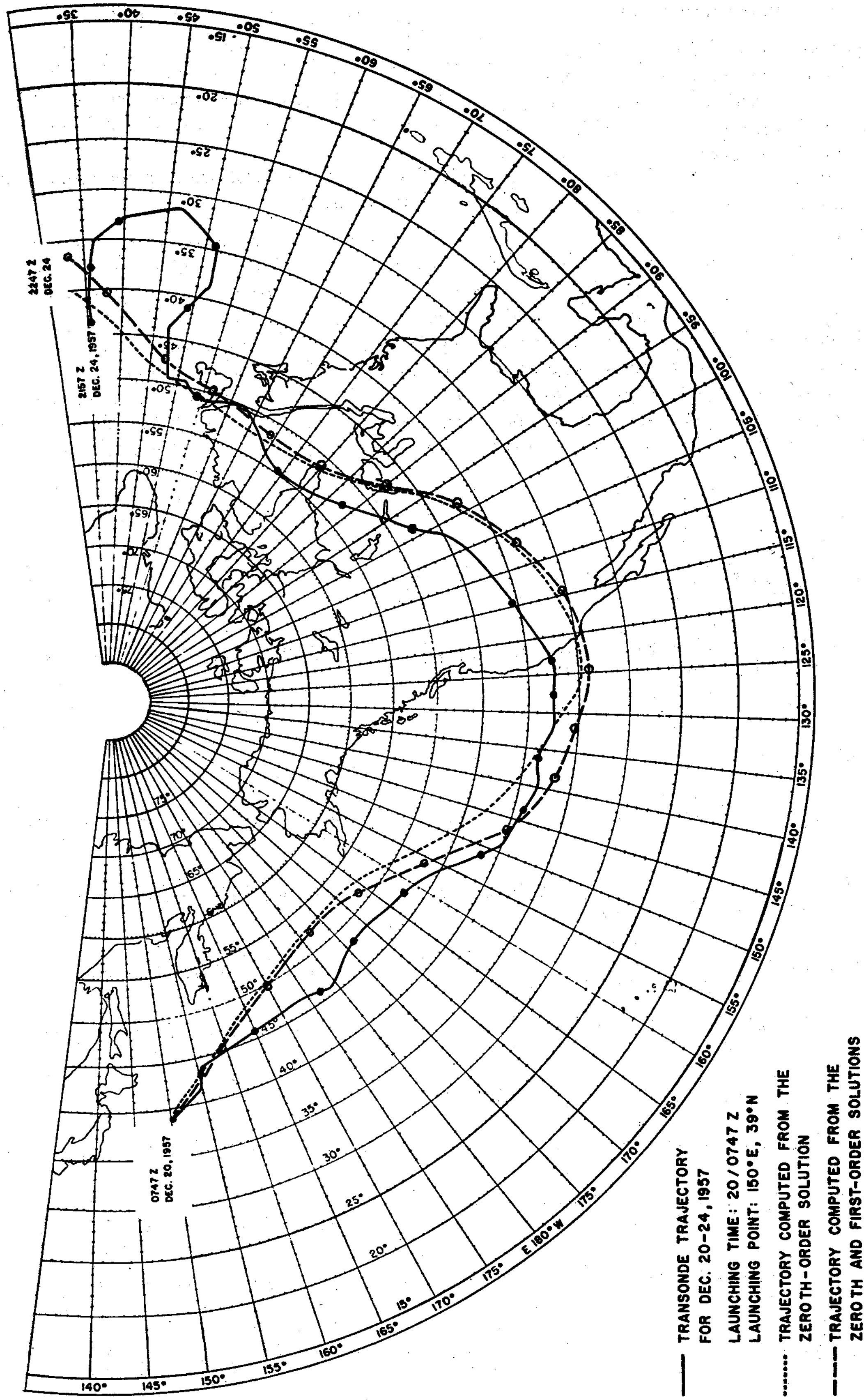

FIG. 2. The computed and observed trajectories at the 300-mb level from 0747Z 20 December to 2157Z 24 December 1957.

2. The observed trajectory of a constant-pressure balloon in a long atmospheric wave

During the period between 20 and 24 December 1957, the 300-mb contour maps over the Northern Pacific Ocean and the Northern American continent indicated that a quasi-stationary long wave prevailed. The wavelength of the long wave is of the order of magnitude of 5×10^6m, whereas its amplitude is of the order of magnitude of 5×10^5m. As a first approximation, the contour field at 300 mb during this period may be represented by a quasi-stationary, sinusoidal wave.

The 300-mb map (U. S. Weather Bureau analysis) for 0000Z 21 December 1957, from which various parameters are to be determined for the computation of the trajectory in the pressure system, is shown in fig. 1. The trajectory of the constant-pressure balloon at the 300-mb level from 0747Z 20 December to 2157Z 24 December 1957, transosonde flight no. 59, extending from 150E, 39N (near the East Coast of Japan) to 35W, 45N (near the East Coast of the United States) is selected for comparing with the computed trajectory. The observed trajectory of the constant-pressure balloon is shown by the solid curve in fig. 2.

3. The zeroth- and first-order trajectories of the non-linear theory

In a recent paper (Kao, 1960), the zeroth- and first-order velocities of a particle in a quasi-stationary long wave of the form $\{Uy + Ak^{-1}\cos k(x - ct)\}$ in a rotating fluid have been obtained. It can be shown, by integrating these velocities with respect to time and applying the initial conditions, that for the case of the forcing frequency $k(U - c)$ being not equal to the multiples of the natural frequency, the zeroth- and first-order trajectories are respectively

$$\begin{aligned} Z_0(t) &= x_0(t) + iy_0(t) \\ &= Ut - \frac{A}{f[1-(k\lambda)^2]}\left\{\sin k[U_rt + x(0)] + \frac{i}{k\lambda}\cos k[U_rt + x(0)]\right\} \\ &\quad + \frac{i}{f}\left\{Z'(0) - U + \frac{A}{1-(k\lambda)^2}[k\lambda\cos kx(0) - i\sin kx(0)]\right\}e^{-ift} + \left\{Z(0) - \frac{i}{f}\left[Z'(0) - U - \frac{A}{k\lambda}\cos kx(0)\right]\right\} \end{aligned} \tag{1}$$

and

$$\begin{aligned} Z_1(t) &= x_1(t) + iy_1(t) \\ &= \frac{Av_0}{fkU_r[1-(k\lambda)^2]}\{-k\lambda\cos k[U_rt + x(0)] + i\sin k[U_rt + x(0)] + k\lambda\cos kx(0) - i\sin kx(0)\} \\ &\quad + \frac{A}{4fkU_r[1-(k\lambda)^2][1-(2k\lambda)^2]}\{2k\lambda\sin 2k[U_rt + x(0)] + i\cos 2k[U_rt + x(0)] - 2k\lambda\sin 2kx(0) \\ &\quad - i\cos 2kx(0)\} + \frac{A(P^2+Q^2)^{\frac{1}{2}}}{f^2k\lambda(1+k\lambda)(2+k\lambda)}\{-(1+k\lambda)\cos[f(1+k\lambda)t + kx(0) - \cos^{-1}\beta] \\ &\quad - i\sin[f(1+k\lambda)t + kx(0) - \cos^{-1}\beta] + (1+k\lambda)\cos[kx(0) - \cos^{-1}\beta] + i\sin[kx(0) - \cos^{-1}\beta]\} \\ &\quad + \frac{A(P^2+Q^2)^{\frac{1}{2}}}{f^2k\lambda(1-k\lambda)(2-k\lambda)}\{(1-k\lambda)\cos[f(1-k\lambda)t - kx(0) - \cos^{-1}\beta] \\ &\quad - i\sin[f(1-k\lambda)t + kx(0) - \cos^{-1}\beta] - (1-k\lambda)\cos[kx(0) + \cos^{-1}\beta] + i\sin[kx(0) + \cos^{-1}\beta]\} \\ &\quad + i\frac{A}{f^2}(1-e^{-ift})\left\{\frac{v_0}{1-(k\lambda)^2}[k\lambda\sin kx(0) + i\cos kx(0)] + \frac{A}{2[1-(k\lambda)^2][1-(2k\lambda)^2]}\right. \\ &\quad \times[2k\lambda\cos 2kx(0) - i\sin 2kx(0)] + \frac{1}{k\lambda(2+k\lambda)}[(1+k\lambda)(P\sin kx(0) - Q\cos kx(0) \\ &\quad + iP\cos kx(0) + Q\sin kx(0)] + \frac{1}{k\lambda(2-k\lambda)}[(1-k\lambda)(P\sin kx(0) + Q\cos kx(0) \\ &\quad \left. - iP\cos kx(0) - Q\sin kx(0)]\right\} \end{aligned} \tag{2}$$

where $x(t)$ and $y(t)$ give the position of the particle with reference to a rectangular coordinate system with its x- and y-axis directed toward the east and north respectively; $k = 2\pi/L$ is the wave number, where L is the wavelength; $U_r = U - c$, where U is the mean velocity and c the phase velocity; $f = 2\Omega \sin\varphi$ is the Coriolis parameter, where Ω is the angular velocity of the earth and φ the mean latitude; $A = A_p k U$, where A_p is the amplitude of the contour of the wave; $z(t) = x(t) + iy(t)$ is the position vector of the particle at time t; $z'(t) = x'(t) + iy'(t)$ denotes the time derivative of $Z(t)$; $u_0 = x'(0)$ and $v_0 = y'(0)$ are respectively the x- and y-components of the initial velocity $Z'(0)$; and

$$\begin{aligned} P &= -\frac{1}{2}\left\{v_0 + \frac{A}{1-(k\lambda)^2}\sin kx(0)\right\} \\ Q &= -\frac{1}{2}\left\{u_0 - U + \frac{Ak\lambda}{1-(k\lambda)^2}\cos kx(0)\right\} \\ \lambda &= \frac{1}{f}(U-c) \\ \beta &= \frac{P}{(P^2+Q^2)^{\frac{1}{2}}}. \end{aligned} \tag{3}$$

Eq (1) and (2) are used to compute the first and second approximations, $Z_0(t)$ and $Z_0(t) + kZ_1(t)$ respectively, of the trajectory.

4. The computation of the first and second approximation of the trajectory

To compute the first and second approximations of the trajectories, we first determine from the 300-mb map for 0000Z 21 December 1957 the values of the various parameters in eq (1) and (2). The wavelength, amplitude and phase velocity of the sinusoidal wave are determined from the average values of the wavelengths, amplitudes and phase velocities for the troughs and ridges in the area under consideration (the Northern Pacific Ocean and the United States). The initial coordinates, $x(0)$ and $y(0)$, of the particle are determined from the launching point (150E, 39N) and the orientation of the pressure wave at the launching time (0747Z 20 December 1957). This orientation is extrapolated from the 300-mb map for 0000Z 21 December 1957 by assuming that the wave is quasi-stationary and is moving with the constant velocity determined previously. The mean velocity is assumed to be equal to the mean geostrophic velocity. The values of these parameters so determined are listed as follows:

$L = 7.5 \times 10^6$m $x(0) = 1.3 \times 10^6$m
$k = 8.4 \times 10^{-7}$m^{-1} $y(0) = 1.7 \times 10^5$m
$U = 37$ms^{-1} $u_0 = x'(0) = 45$ms^{-1}
$c = 7.0$ms^{-1} $v_0 = y'(0) = 22$ms^{-1}
$A_p = 7.7 \times 10^5$m $U_r = U - c = 30$ms^{-1}
$A = 23.9$ms^{-1} $P = 3.5$ms^{-1}
$f = 9.4 \times 10^{-5}$s^{-1} $Q = 2.0$ms^{-1}.

The first and second approximations of the trajectory, computed from eq (1) and (2), are respectively shown by the dotted and dashed curves in fig. 2.

5. A comparison between the computed and observed trajectories

Before we compare the computed trajectories with the observed trajectory of the constant-pressure balloon, let us first examine the factors which affect the accuracy of the computed trajectory. It is obvious that the accuracy of the trajectory computed from (1) and (2) depends on that of the values of the parameters in these equations, whereas the accuracy of the latter depends greatly on that of the pressure field from which these parameters are determined. The contour field at 300 mb is generally accurate over the United States, but it is less accurate over the Northern Pacific Ocean where the upper-air data are sparse. The error of the values of the various parameters determined from the 300-mb map may amount to 10 per cent of the actual values.

In the theoretical model, we assume that the wave is simple sinusoidal and quasi-stationary. Although the pressure field, during the period and area under consideration, was close to the idealized case, waves of shorter wavelengths and higher frequencies existed. The effects of these waves on the trajectory have not been taken into account in the computation. Furthermore, the accuracy in the position fixes of the constant-pressure balloons, which is of the order of magnitude of 10^5m, should also be taken into consideration.

In view of the accuracy involved in the pressure field over the Northern Pacific Ocean and in the position fixes of the constant-pressure balloon, and the deviation of the actual pressure distribution from the theoretical model, the computed trajectories agree rather well with the observed, except the eastern end of the latter which was affected by a deepening trough off the East Coast of the United States on 24 December 1957.

It is seen from fig. 2 that both the wavelengths and amplitudes of the computed trajectories are slightly greater than that of the observed. These are respectively due to the overestimate of the values of the quantities $k(U - c)$ and $A\{fk\lambda[1 - (k\lambda)^2]\}^{-1}$. An adjustment of the values of these quantities would improve the results. The second approximation is an improvement over the first approximation in the first half of the trajectory, and there is no significant difference in their accuracies thereafter.

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