ON THE ROLE OF THE SUBTROPICAL JET STREAM OF WINTER IN THE ATMOSPHERIC GENERAL CIRCULATION

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ABSTRACT

Calculations of budgets of heat, angular momentum and kinetic energy for the tropical latitudes are obtained in a coordinate system following the subtropical jet stream axis at the 200-mb surface.

Comparisons of the fluxes of various quantities with those required at certain latitudes show that the vertical mass circulation has a very important role in the tropics and that the daily eddies are important on the poleward side of the jet stream.

Large fluxes of various atmospheric properties are computed for 3 different months. The month of January, 1956, shows very large export of kinetic energy and angular momentum from tropics into middle latitudes.

Statistics of the middle-latitude zonal and meridional motion are studied. It is shown that the month of January, 1956, was one of very low zonal index, blocking, and very slow eastward wave motion. Possible connection between lower and higher latitude circulations is suggested.

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1. Introduction

This paper is a continuation of an earlier work (Krishnamurti, 1961), on the structure of the subtropical jet stream of the winter of 1955–56. In the earlier paper the author described the average structure of various atmospheric variables in a coordinate system following the jet stream axis at the 200-mb surface. Here an attempt will be made to utilize the data and calculate fluxes of various atmospheric properties and to compare them with the balance requirements on these fluxes. The method will be somewhat similar to those utilized by the General Circulation Project of MIT (1954, 1957) and UCLA (1955, 1957).

It would be pertinent to recall here some of the main points of the data-processing in the jet stream coordinate system. Each day the axis was divided into nine regions (table 1). Each of the 116 stations was located into its coordinates s, n, where s stands for the region and n for the distance (in degrees latitude) normal to the jet stream axis. The regions $s = 1$ to $s = 9$ give us nine cross sections of atmospheric variables each day. It must also be noted that the data were averaged following a moving jet stream axis such that individual stations were located each day with respect to their distance from the jet axis for that day.

Table 1. The s, n, p coordinate system in which all calculations were carried out. (See Krishnamurti 1961).

s	n Degrees latitude from jet axis	p Pressure (mb)
1. South-west current over United States	7.5	1000
2. Westerly current over Eastern United States	5.0	850
3. North-west current over Atlantic	J	700
4. South-west current over West Africa	− 5.0	500
5. Westerly current over North Africa	−10.0	300
6. North-west current over Arabia	−15.0	200
7. South-west current over India	−20.0	100
8. Westerly current over Japan	(Each value n, p applies to each zone, s.)	
9. North-west current over Pacific		

Let $Q(s, n, p, t)$ represent any such variable where p is the pressure level and t the day. The following further definitions were used:

$\bar{Q}(s, n, p)$ Five day mean value of $Q(s, n, p, t)$.
$\overline{Q}(s, n, p)$ Monthly mean value of $Q(s, n, p, t)$.
$\tilde{\bar{Q}}(n, p)$ Mean value of $\bar{Q}(s, n, p)$ around the world.
$\tilde{\overline{Q}}(n, p)$ Mean value of $\overline{Q}(s, n, p)$ around the world.

Absence of stations and missing observations at any point give rise to difficulties in any averaging. The following rules were observed for this purpose:

$$\bar{Q}(s, n, p) = \frac{1}{N} \sum_{t=1}^{N} Q(s, n, p, t) \qquad (1)$$

where $N =$ the maximum number of observations, *i.e.*, between 0 and 5. Similarly

$$\overline{Q}(s, n, p) = \frac{1}{N} \sum_{t=1}^{N} Q(s, n, p, t) \qquad (2)$$

where $N =$ the maximum number of observations between 0 and 30.

It is pertinent to mention here that the choice of the 9 cross-sections was such that it enabled us to obtain a representative structure for the monthly mean and the 5-day mean distribution of atmospheric variables. The detailed structure is described in Krishnamurti (1961). Missing data had an adverse effect on various calculations of fluxes, especially those by the so-called "daily eddies." It is felt that in regions of very strong winds, where the balloons stopped reporting, a certain unavoidable bias has entered. This point is discussed later in the relevant sections dealing with the calculations. It can only be said here that lack of data in the tropics does not enable us yet to incorporate the contributions of daily disturbances.

Since there was no *a priori* justification for giving more weight to any of the nine regions in the s integration, the following similar scheme was observed:

$$\tilde{\bar{Q}}(n, p) = \frac{1}{N}\sum_{s=1}^{N} \bar{Q}(s, n, p) \qquad (3)$$

and

$$\tilde{\bar{\bar{Q}}}(n, p) = \frac{1}{N}\sum_{s=1}^{N} \bar{\bar{Q}}(s, n, \mathrm{p}) \qquad (4)$$

where N is the maximum number of s regions between 0 and 9.

Since we shall be interested in the evaluation of fluxes of various quantities, the relevant equations are described here to avoid repetition later. Table 2 lists the symbols used in this paper.

TABLE 2. List of symbols.

Symbol	Meaning of symbol
$\boldsymbol{V}_2$	Total horizontal wind at any point.
$\boldsymbol{V}_3$	Three dimensional wind vector.
C_s	Component of wind speed along the jet axis.
C_n	Component of wind speed normal to the jet axis. Taken positive to the left looking downstream along jet axis.
C_{sg}	Component of the geostrophic wind along the jet axis.
C_{ng}	Component of the geostrophic wind normal to the jet axis. Taken positive in same direction as C_n.
z	Height of the constant pressure surface.
T	Temperature at any point.
q	Specific humidity of air.
ω	Vertical velocity in isobaric coordinates.
K	Kinetic energy per unit mass of air.
$\boldsymbol{F}$	Frictional force per unit volume of air.
$\boldsymbol{F}_x$	Frictional force per unit volume of air in the zonal direction.
u	Component of zonal wind at any point.
g	Acceleration of gravity.
L	Latent heat of evaporation.
a	Mean radius of earth.
ϕ	Latitude at any point.
ρ	Density of air at any point.
τ	Volume of integration, 5 degrees latitude wide, 900 mb in depth and going around the world, following the jet stream coordinate.
μ	Coefficient of viscosity of air.
k	Surface drag coefficient.
V_0	Total surface wind.
dA	Horizontal area element.
C_p	Specific heat of air at constant pressure.

Let $Q(s, n, p, t)$ stand for any of the following quantities:

(i) $gz + C_pT + Lq$ Total heat content per unit mass,

(ii) Lq Latent heat per unit mass,

(iii) $\frac{1}{2}(C_s^2 + C_n^2)$ Kinetic energy per unit mass,

(iv) $ua\cos\phi + \Omega a^2\cos^2\phi$ Angular momentum per unit mass.

The transport by the monthly mean wind field is expressed by the relation

$$\frac{1}{g}\int_p\oint_s \bar{\bar{Q}}\,\bar{\bar{C}}_n\,dsdp.$$

We can write $\bar{\bar{Q}} = \tilde{\bar{\bar{Q}}} + \bar{\bar{Q}}'$ where $\tilde{\bar{\bar{Q}}} = \frac{1}{S}\oint_s \bar{\bar{Q}}\,ds$ and the primes denote deviations from the mean along s. Hence we can write

$$\frac{1}{g}\int_p\oint_s \bar{\bar{Q}}\,\bar{\bar{C}}_n\,dsdp = \frac{1}{g}\int_p\oint_s (\bar{\bar{Q}}\,\tilde{\bar{\bar{C}}}_n + \bar{\bar{Q}}'\,\bar{\bar{C}}_n{}')dsdp. \qquad (5)$$

The right hand side of the equation has two terms: the first term (I) represents the transport by the monthly mean mass circulation; the second term (II) represents the transport by standing eddies.

The expressions $\frac{1}{g}\int_p\oint_s \bar{Q}\bar{C}_n dsdp$ represents the total transport of $\bar{Q}(s, n, p)$ by the five-day mean wind field $\bar{C}_n(s, n, p)$. Each of the months, December 1955, January 1956, and February 1956, was divided into six five-day periods, and total transports for a whole month were obtained by averaging over the six five-day periods, expressed by $\frac{1}{6}\sum_{N=1}^{6}\left[\frac{1}{g}\int_p\oint_s \bar{Q}\bar{C}_n dsdp\right]$. The five-day eddies referred to in this text are then defined by the following term:

$$\frac{1}{6}\sum_{N=1}^{z}\left[\frac{1}{g}\int_p\oint_s \bar{Q}\,\overline{\bar{C}_n}\,dsdp\right] - \frac{1}{g}\int_p\oint_s \bar{\bar{Q}}\,\bar{\bar{C}}_n\,dsdp$$
$$= \frac{1}{g}\int_p\oint_s \bar{Q}'\,\overline{\bar{C}_n}'\,dsdp. \qquad (6)$$

Machine computations were also performed to obtain the transports by five-day mean mass circulations, which are given by the expression $\frac{1}{g}\int_p\oint_s \tilde{\bar{\bar{Q}}}\,\tilde{\bar{\bar{C}}}_n\,dsdp$. Further, the expression $\frac{1}{g}\int_p\oint_s\int_t QC_n dtdsdp$ denotes the net total transport of Q over a whole month. The daily eddies referred to in the text are given by the equation

$$\frac{1}{gt}\int_p\oint_s\int_t QC_n dtdsdp - \frac{1}{g}\int_p\oint_s \bar{Q}\bar{C}_n dsdp = \frac{1}{g}\int\oint_s Q'C_n'dsdp. \quad (7)$$

Summing up, we have the final result that the total flux of Q over a month is the sum of the four terms:

(i) Transport by mass circulation,
(ii) Transport by standing eddies,
(iii) Transport by five-day eddies,
(iv) Transport by daily eddies.

In the actual handling of the atmospheric data, the author found considerable stability in the calculations of fluxes by terms (i) to (iii) in that the sign of the fluxes for the three different months was the same.

All vertical integrations were carried out using the following equation:

$$\int_{p=1000}^{p=100} Qdp = [(2/3)Q_{1000} + (5/3)Q_{850} + (5/3)Q_{700} + 2Q_{500} + (3/2)Q_{300} + Q_{200} + (1/2)Q_{100}]10^5. \quad (8)$$

where Q_{850} would refer to a value of Q at the 850-mb surface. The weight factors are determined by the spacing in terms of pressure.

2. Heat and moisture balance

The first law of thermodynamics has been expressed by the following equation by various meteorologists. See, for example, Riehl (1959), Malkus and Reihl (1960).

$$\frac{\partial}{\partial t}\int_\tau [gz + C_pT + Lq]d\tau = \frac{1}{g}\int_p\oint_s [gz + C_pT + Lq]C_n dsdp\Big]_n^{n+5^\circ} + Q_e + Q_s + R. \quad (9)$$

The heat source within the volume, $H = Q_e + Q_s + R$ consists of latent (Q_e) and sensible (Q_s) heat exchange between ground and air, and the net tropospheric radiation R.

For steady state the total heat source must equal the total outward flux of heat from the volume. In integrations over periods as long as a month the local change is a very small residual between source and transport terms, and may therefore be neglected. Kinetic energy has been omitted in eq (9) for a similar reason; terms involving this energy form are two orders of magnitude smaller than the other terms.

Fig. 1 shows the flux of $(gz + C_pT + Lq)$ by the three terms mentioned in the earlier section. No calculations of the flux by daily eddies were carried out.

The results of these computations may be summarized as follows: (a) The transports by the ageostrophic mass circulation are larger than those by the two types of "eddies" calculated. (b) Between $n = -20$ deg and $n = -10$ deg latitude, the transports are carried almost entirely by mass circulation and standing eddies. (c) North of the jet axis shorter time scales become dominant.

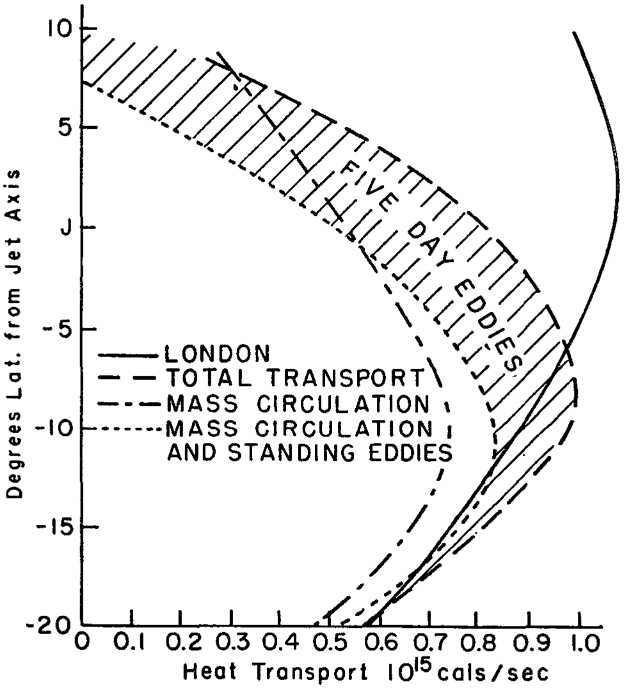

FIG. 1. Heat transport for winter 1955–56. London's estimate of the transport requirement is shown by the solid curve. The mean latitude of the jet axis is 27.5 deg lat.

Following many earlier attempts to compute the latitudinal distribution of atmospheric radiation (*cf.* Simpson 1929, Baur and Phillips 1935, Houghton 1951), a new attempt to determine the poleward heat fluxes from radiation requirements has been made by London (1957). His curve is shown in fig. 1 for comparison; it has been entered noting that the mean latitude of the jet stream is 27.5 N. South of $n = -5$ deg, our total transport and that of London agree fairly well. Between $n = -5$ deg and $n = -20$ deg our curve gives a net heat source of 0.40×10^{15} cal/sec, compared to London's value of 0.41×10^{15} cal/sec. North of $n = -5$ deg the two curves diverge, indicating that north of the jet axis the heat flux is carried largely by daily eddies, not considered here.

TABLE 3. Jacobs' estimates of heat source, and net heat source between $n = -5$ deg and $n = -20$ deg.

	From Jacobs' computations (1951)			Corresponding values in jet stream coordinates			From London's estimate
Latitude belt	Q_e cal/cm²/day	Q_s cal/cm²/day	$Q_e + Q_s$ cal/cm²/day	Latitude belt deg	$Q_e + Q_s$ cal/cm²/day	$Q_e + Q_s$ 10^{15} cal/sec	R 10^{15} cal/sec
0–5	155	−1	154	10–5	336	0.77	0.476
5–10	215	0	215	5–J	296	0.74	0.499
10–15	242	1	243	J−−5	276	0.72	0.517
15–20	245	10	255	− 5−−10	262	0.70	0.533
20–25	244	20	264	−10−−15	249	0.68	0.546
25–30	255	37	292	−15−−20	232	0.65	0.556
30–35	253	53	306	Sum between			
35–40	305	100	405	− 5−−20	743	2.03	1.635
40–45	209	94	303				

In the following, attention will be restricted to the region between −5 deg and −20 deg lat from the jet axis, essentially the trade wind belt and parts of the equatorial zone. In this region, which is largely oceanic, latent plus sensible heat flux from surface to air should balance the net export of 0.40×10^{15} cal/sec plus the radiation of 1.64×10^{15} cal/sec from London's curve, total 2.04×10^{15} cal/sec. The best available estimates of sensible and latent heat exchange between sea and air are those of Jacobs (1951), presented in detail in table 3. Adapting his computations to the jet coordinate system, the heat sources for the latitude belt considered are 2.03×10^{15} cal/sec.

Agreement between the two computations is good but not as perfect as the numbers indicate, because Jacobs worked with Atlantic and Pacific oceans only; he omitted the Indian Ocean and the land areas. It must be remembered that Jacobs' calculations, as well as those of this paper, are subject to a considerable per cent of error. Besides, only one winter has been considered here, which may not be fully indicative of average conditions. Large month-to-month variations of all transports were encountered, and these may not merely represent the seasonal trend. We may conclude that the comparison of Jacobs' and our calculations is satisfactory.

As already mentioned, the contributions arising from variations of the daily fields of temperature and wind were not considered; data were insufficient for this purpose. The five-day wind field tends to smooth out short wave troughs and ridges, hence their contributions are damped. It is interesting to note that as we go more and more northward, larger and larger transports by daily wind and temperature fields are required for balance. Alternately we can say that the tropical general circulation south of 22.5 deg latitude ($n = -5.0$ deg) can be approximated by a simple system where the heat transport is carried by the mean mass circulation. Farther north, short time-dependent eddies play an increasing role in the heat budget, probably largely through the moisture exchange. It is of interest that the standing eddies strongly transport heat equatorward on the jet coordinate system. This peculiarity arises because the C_n-component is positive in the ridge (at cold temperatures) and negative in the trough (at warm temperatures) above the lowest levels.

In this and the following section we shall be interested in making comparisons of flux calculations made in the jet stream coordinates and the conventional latitude-longitude coordinates. It would be pertinent here to show how far one is justified in a direct comparison of the two results. The reader is referred to Appendix 1 where we have attempted to answer this question.

Moisture balance. The preceding computations of total fluxes may be subdivided into fluxes of $gz + C_pT$ and of Lq; a separate moisture budget can then be calculated, given sources and sinks. Continuity of moisture is expressed by the relation

$$\frac{\partial \pi}{\partial t} = \frac{1}{g} \int_p \oint_s LqC_n ds dp \Big]_{n1}^{n2} + L(E - P) \qquad (10)$$

where π is latent heat energy, E evaporation, and P precipitation. The transport term of eq (10) has been calculated as before over volumes 5 deg latitude wide along the n-axis. For sources and sinks, Jacobs' (1951) estimates of energy exchange between ocean and at-

TABLE 4. Jacobs' estimates of latent heat flux (1951).

Jacobs' estimates (Dec–Feb) cal/cm²/day				Jacobs' estimates expressed in jet stream coordinates		
Latitude belt deg	LE	LP	$L(E\text{-}P)$	Latitude belt deg	$L(E\text{-}P)$ cal/cm²/day	$L(E\text{-}P)$ 10^{15} cal/sec
0–5	155	226	−71	10–5	138	0.32
5–10	215	224	− 9	5–J	155	0.39
10–15	242	122	120	J−−5	144	0.37
15–20	245	84	161	− 5−−10	151	0.40
20–25	244	88	156	−10−−15	151	0.41
25–30	255	118	137	−15−−20	124	0.35
30–35	253	142	109			
35–40	305	154	151			
40–45	209	160	49			

mosphere were used (table 4), where it should be noted that these cover the Atlantic and Pacific Oceans only. Besides the land masses, the Indian Ocean also is not included. Nevertheless, Jacobs' data have been adopted here as they appear to be the most recent and reliable, and as the omission of the Indian Ocean is serious only south of 20N.

As before, the local change may be omitted in eq (10), because the integration is carried over the whole winter. The transport by the three types is depicted on the left side of fig. 2. As the mass circulation is directed equatorward in the low levels where the moisture content of the air is large, a strong equatorward moisture transport results from the mean ageostrophic circulation. In this case the effect of standing eddies is negligible, but the five-day disturbances act to offset in part the transport by mass circulation, except in the far south. It follows that winds with positive C_n must be correlated with high moisture, winds with negative C_n with low moisture, as is reasonable (Riehl 1945).

The divergence of the flux in fig. 2 is less than the source as contained in Jacobs' calculations poleward of $n = -15$ deg lat. Assuming the latter to be correct or an under-estimate, the true moisture flux profile must be steeper than that of fig. 2. In this connection, it will be noted from fig. 1 that the required and computed transports at the northern extremity of the jet stream system show a discrepancy too large to be ascribed wholly to abnormalities of the 1955–56 winter or the margin of inaccuracy in London's calculation. This discrepancy still occurs at quite low latitudes where features such as interdiurnal temperature fluctuations can hardly be expected to improve the balance materially. There is a much better chance that daily disturbances affect the moisture flow distribution considerably, that addition of the daily eddies would produce a steeper profile of the flux curve in fig. 2, and that it would act to close the gap in fig. 1.

The integrated excess of $L(E-P)$ over the flux divergence in fig. 2 (right-hand side) is 1.9×10^{15} cal/sec from $n = -15$ deg lat northward. The northward flow at $n = +7.5$ deg is 0.30×10^{15} cal/sec from the left side of fig. 2; equatorward flow at $n = -15$ deg lat is -1.1×10^{15} cal/sec; divergence is 1.4×10^{15} cal/sec. Assuming the flux calculation at $n = -15$ deg lat to be correct, an additional 0.5×10^{15} cal/sec is then available for transport across $n = 7.5$ deg lat. From fig. 1 the gap at this latitude is 0.7×10^{15} cal/sec, and it is readily seen that the excess moisture source has the right magnitude to provide balance approximately.

3. Angular momentum balance

The problem of the maintenance of the observed zonal circulation of the atmosphere has been discussed by many investigators in the light of angular momentum balance. While some disagreement exists about the nature of the transport processes (Starr and White 1954, Palmén 1951), nevertheless there is

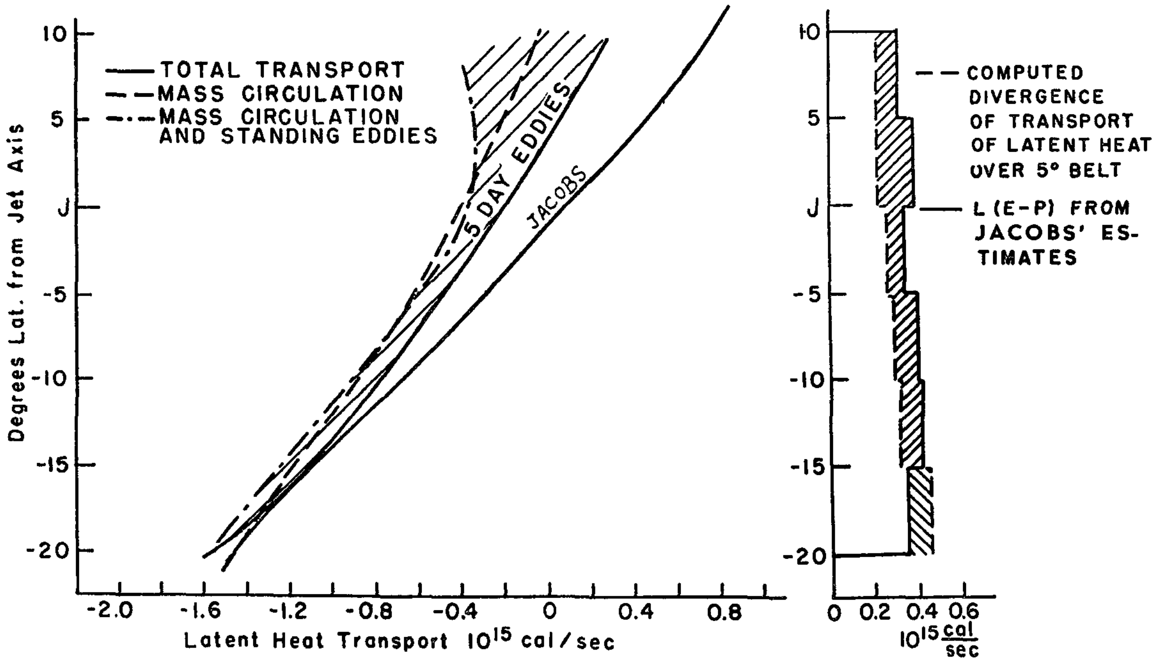

Fig. 2. Left: Latent heat transport for winter 1955–56. Jacobs' estimate of the transport requirement is also included. Right: Divergence of flux of latent heat, computed for winter of 1955–56, compared with the values obtained by Jacobs. Note that the Jacobs flux curve on the left was obtained with an assumed zero flux at $n = -17.5$.

general agreement that the tropics are the source regions of westerly absolute angular momentum for the troposphere and that the high latitudes are the sink. Since the subtropical jet stream is associated with large amounts of westerly angular momentum it should be an important link in the momentum exchange between tropical and polar zones.

In 1951, when wind data were quite sparse, Palmén suggested that the subtropical jet stream was formed as a result of accumulation of the westerly absolute angular momentum along the poleward margin of the tropical meridional cell. In 1952, Palmén and Alaka discussed the maintenance of the zonal circulation between 20N and 30N from this viewpoint.

Let M be the absolute angular momentum of a unit mass of air, then

$$M = ua\cos\phi + \Omega a^2 \cos^2\phi.$$

The momentum balance equation is obtained from the continuity and the zonal equation of motion (see Widger 1954).

$$\underbrace{\frac{\partial}{\partial t}\int_\tau \rho M d\tau}_{\text{I}} = \underbrace{\frac{1}{g}\int_A \rho M C_n dA}_{\text{II}} - \underbrace{\int_\sigma pa\cos\phi d\sigma}_{\text{III}} + \underbrace{\int_\tau a\cos\phi\rho F_x d\tau}_{\text{IV}} \quad (11)$$

where dA is an element of area of the boundary walls of the volume τ, and $d\sigma$ represents the projection of dA on a meridional plane passing through a given point. The terms I to IV are interpreted as follows:

I is the local change of absolute angular momentum in the volume during the period in question.

II represents the advection of absolute angular momentum into the volume τ across the boundaries.

III contributes to the absolute angular momentum in the volume by pressure torques.

IV determines gain or loss of absolute angular momentum in the volume by friction.

The following results were obtained:

Term I. Since data were used for the entire winter of 1955–56, the local change of absolute angular momentum was nearly zero and can therefore be neglected.

Term II. The integration was carried out between 1000 and 100 mb; vertical motion was assumed to vanish at top and bottom. Hence the term measures the convective transport of absolute angular momentum across the wavy walls of the jet stream coordinate system. It can be subdivided into two parts, relative and earth's angular momentum transports.

Transport of relative angular momentum. Fluxes by the three mentioned terms were calculated. As in the case of the heat transfer, daily fluctuations could not be taken into account. Flux by mass circulation came

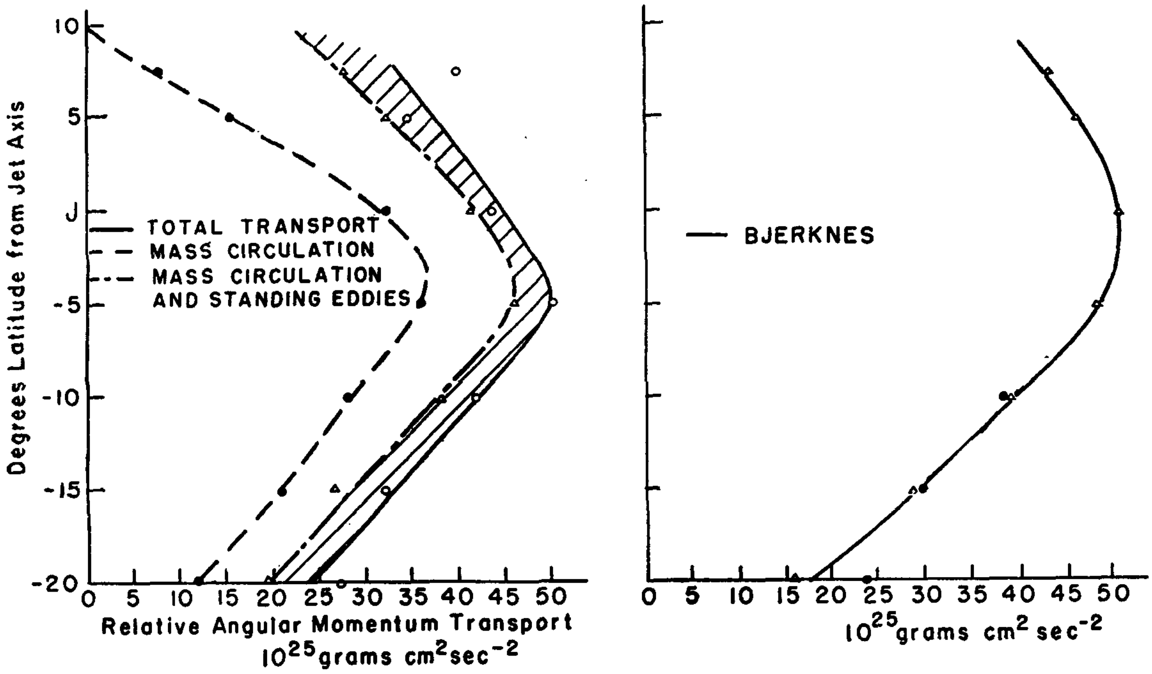

FIG. 3. Relative angular momentum transport for winter 1955–56. On the right the estimates from Bjerknes (1957) are shown.

out to be the largest. The transports for the entire winter 1955–56 are shown in fig. 3. Maximum total transport is about 50×10^{25} g cm^2 sec^{-2}, located near $n = -5$ deg (approximately lattitude 22.5 deg). The contribution by the standing eddies is quite large because at the ridges of the monthly mean wind field large values of $\overline{C_s}$ and $\overline{C_n}$ coincide, and this gives rise to large correlations. Contributions by five-day eddies are small except at $n \geqslant 5$ deg.

Transport of earth's angular momentum. The computation showed that this term had the order of 0.5×10^{25} g cm^2 sec^{-2}, therefore was one order of magnitude smaller than that of the transport of relative angular momentum. The reason for this lies at least partly in the fact that the wind field has been adjusted exactly for mass balance (see Krishnamurti 1961).

Term III. Since the area of computation lies south of the major mountain belts of the world, no actual mountain torques were evaluated. A calculation of this term revealed that the transport of angular momentum by pressure torques is directed toward lower latitudes everywhere. The order of magnitude of this transport is 10^{25} g cm^2 sec^{-2} and is much smaller than the transport of relative angular momentum, which has a magnitude ranging from 10^{26} to 10^{27} g cm^2 sec^{-2}.

Term IV. Term IV represents the gain or loss of angular momentum in the volume due to frictional interactions of the zonal wind with the ground. Evaluation of this term requires knowledge of the zonal stresses at the ground everywhere. Integration of the zonal stresses, multiplied by the distance from the axis, over the ground measures the momentum exchange between ground and atmosphere.

Priestley (1951) obtained values of zonal stresses by using ship observations, and they have been considered fairly reliable. In 1954, Palmén computed a momentum budget for the tropics and noted that Priestley's values could be an under-estimate by about 0.1 to 0.3 dynes/cm^2, which is roughly 10 per cent, fully within the margin of error of the computations. Bjerknes (1957) has also obtained a similar result in his model of the tropical general circulation. It was felt that these investigations had covered the subject thoroughly and that little would be gained by the extensive labor involved in a recalculation of the zonal stresses. Therefore, Priestley's values (for January over the oceans) were utilized.

It was assumed that Priestley's computations in polar coordinates could be applied to the s, n coordinate system using mean latitudes for each 5 deg interval along n. The frictional exchange of momentum between ground and air (M_F) is given by

$$M_F = \int_r (a \cos \phi) \rho F_x dr \qquad (12)$$

where F_x is the frictional force per unit mass. If τ_{xz} is the vertical shearing stress along x,

$$F_x = -\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}$$

and

$$M_F = -\int_n \oint_s a \cos \phi \tau_x ds dn. \qquad (13)$$

Values of τ_x and of M_F are contained in table 5. Total pick-up of momentum from the ground equatorward of the subtropical jet axis is 4.27×10^{25} g cm^2 sec^{-2}.

The following summarizing statements can be made:

Fig. 3 depicts, accumulatively, the n-distribution of the three transport terms. The contribution by mass circulation is large, as found also in the three-wave dishpan experiment by Riehl and Fultz (1958). Angular momentum transport by the ageostrophic mass circulation attains over 35×10^{25} gm cm^2 sec^{-2}, which is of the order of magnitude of total momentum exchange requirement across the subtropical belt. It follows that if the coordinate system for reference is properly chosen, the atmosphere of the lower latitudes is revealed as a simple circulation which attempts to affect not only the heat but also the momentum exchange between source and sink by means of a single circulation cell.

Table 5. Priestley's estimates, oceanic observations (1951).

		Corresponding values in jet stream coordinate from Priestley's estimates		
Latitude degrees north	Priestley's values of surface zonal stresses dynes/cm^2	Latitude belt deg	Stress over belt dynes/cm^2	Angular momentum transport 10^{25} g cm^2/sec^2
30	0.04	5–J	0.04	−0.5
25	−0.16	J– −5	−0.16	2.1
20	−0.44	−5– −10	−0.44	6.1
15	−0.75	−10– −15	−0.75	10.9
10	−0.86	−15– −20	−0.86	13.0
5	−0.44	−20– −25	−0.44	6.8
0	−0.24	−25– −30	−0.24	3.8

The standing eddies contribute little equatorward of the jet axis but become very important along the poleward margin, where they execute almost the whole transfer as the mass circulation dies out. The five-day disturbances contribute uniformly along the whole n-coordinate to poleward momentum flow.

On the right-hand side of fig. 3, the momentum transport of this paper and that obtained by Bjerknes (1957) are compared. The curves are very similar except at the poleward end where daily circulations may be expected to gain in importance.

4. Kinetic energy balance

The investigation of Pisharoty (1955) on the northern hemispheric kinetic energy budget has been an important source of information about this subject. Working with data for January and February of 1949, he found that the latitudes south of 30 deg are the principal source regions of kinetic energy of the tropospheric circulations. The maximum poleward flux of kinetic energy for the entire period was found to take place near 30N. Palmén, Riehl and Vuorela (1958) computed that the kinetic energy exported from the tropics is almost wholly produced by the mean meridional circulation.

The equations describing the kinetic energy budget have been discussed by many meteorologists. In particular reference may be made to the contribution by Starr (1951).

The kinetic energy equation over a volume τ may be expressed by:

$$\underset{\text{I}}{\int_\tau \rho\frac{\partial K}{\partial t}d\tau} = \underset{\text{II}}{-\int_\tau \nabla_3\cdot(K\rho\boldsymbol{V}_3)d\tau} \underset{\text{III}}{-\int_\tau \boldsymbol{V}_2\cdot\nabla_2 p d\tau} + \underset{\text{IV}}{\int_\tau \boldsymbol{V}_2\cdot\boldsymbol{F}d\tau}. \quad (14)$$

The terms of eq (14) may be interpreted as follows:

I = local change of kinetic energy in the volume,
II = divergence of transport of kinetic energy from the volume,
III = production of kinetic energy by pressure forces in the volume,
IV = total frictional dissipation of kinetic energy in the volume.

For steady state, term I can be omitted.

In the following we shall first investigate how far a kinetic energy balance can be realized from considerations of the monthly mean data, $\overline{Q}$, only. For this purpose the month of December, 1955, is selected.

For computational convenience, terms II, III and IV were expanded as follows:

$$\text{Term II} = -\int_\tau \nabla_3\cdot K\rho\boldsymbol{V}_3 d\tau.$$

Using Green's theorem, this volume integral can be converted to a surface integral of transport of kinetic energy across the wavy walls. After introduction of the hydrostatic equation, this term becomes

$$= -\frac{1}{g}\int_p\oint_s \overline{K}\overline{C_n}dsdp$$

$$\text{Term III} = -\int_\tau \overline{\boldsymbol{V}_2}\cdot\nabla_2\overline{p}d\tau$$

$$= \frac{1}{g}\int_p\int_n\oint_s f[\widetilde{\overline{C_n}}\widetilde{\overline{C_{sg}}} + \overline{C_n}'\overline{C_{sg}}' - \overline{C_s}'\overline{C_{ng}}']dsdndp$$

$$\text{Term IV} = \int_\tau \rho\boldsymbol{V}_2\cdot\boldsymbol{F}d\tau$$

This term represents frictional dissipation of kinetic energy in the volume τ. We can transform this term to give us explicitly the frictional dissipation in the surface boundary layer and in the free atmosphere, see Palmén (1958).

$$\int_\tau \overline{\rho}\overline{\boldsymbol{V}}_2\cdot\overline{\boldsymbol{F}}d\tau = -\int_\tau u\left(\frac{\partial\overline{v}}{\partial z}\right)^2 d\tau - k\rho_0\int_A \overline{v}_0{}^3 dA. \quad (15)$$

Dissipation of kinetic energy by internal friction in the free atmosphere is neglected because: (i) the investigations of Pisharoty (1955) and Palmén (1955) have shown that for large scale atmospheric motion it is much smaller than the other terms; (ii) our present knowledge of the eddy viscosity coefficients does not permit any exact determination of this term.

Since all the terms can be computed, we shall be able to determine to what extent the kinetic energy budget is satisfied with use of monthly data alone. The results are shown in fig. 4.

Poleward of the jet axis, the computations yield a large imbalance, similar to that obtained by Riehl and Fultz (1958) in the three-wave dishpan experiment. The mean mass circulation is directed toward lower pressure everywhere, hence there is production of kinetic energy by means of this circulation on both sides of the axis. As the transport converges strongly poleward of the axis, divergence of transport and production by pressure forces have the same sign, leaving a large surplus of kinetic energy gain, which is one order of magnitude larger than dissipation by friction. Riehl and Fultz, who considered a steady state case, took the position that the kinetic energy gain had to be expended by increasing the eddy potential energy. Since the subtropical jet stream is not a wholly steady system, an attempt was made to consider contribution to the kinetic energy flow by the five-day disturbances.

This calculation was carried out for all three months and fig. 5 shows the kinetic energy transport for each month with accumulative totals from contribution by mass circulations, monthly standing eddies, and five-

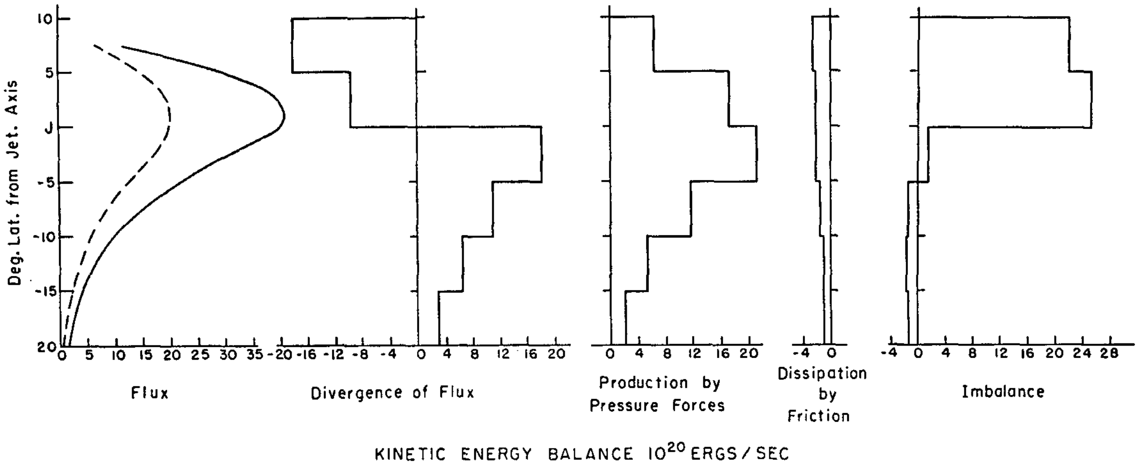

FIG. 4. Magnitudes of the various terms of the kinetic energy balance for December, 1955.

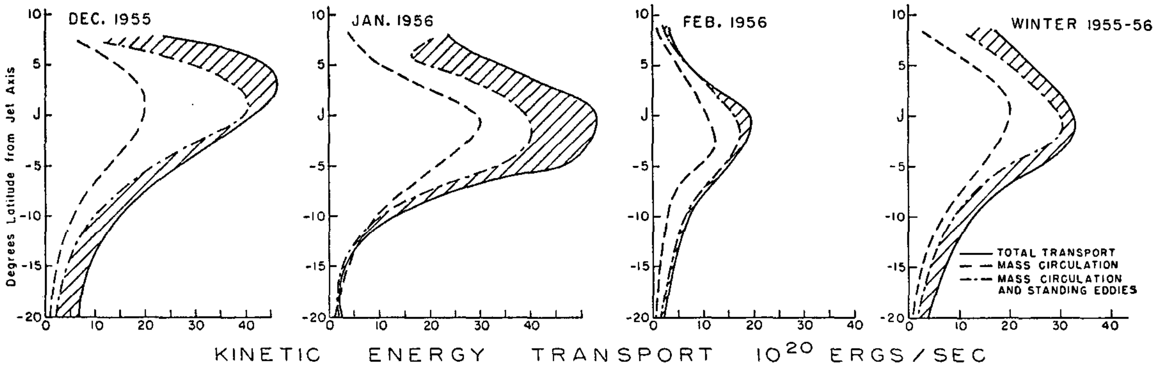

FIG. 5. Transport of kinetic energy for winter 1955–56.

day disturbances. It is a principal result that the transport in the three months differs considerably. In February the flow was half of that in other months, with all three types of transport making a smaller contribution. Further, it is striking that all three terms act to transport kinetic energy poleward and that the magnitude of the eddy contribution is not larger for the five-day transport than for the monthly semipermanent eddies. In December, 1955, the five-day disturbances contributed partly towards improving the balance north of the jet axis. But from an inspection of fig. 5 we see that this effect cannot be counted upon to provide kinetic energy balance. There remains the possibility of a strong flux divergence of daily disturbances. Barring this, the interpretation would have to fall in line with Riehl and Fultz (1958).

A calculation of flux of kinetic energy by the daily eddies was also carried out, as detailed in table 6. This calculation did not decrease the imbalance because of the unrepresentativeness of wind data.

The results for December, 1955, in the jet coordinate system may be compared with those of Pisharoty (1955), who determined the kinetic energy flux across different latitude circles for January and February, 1949. Also we may include here the calculations on kinetic energy balance south of 30N during winter made recently by Palmén, Riehl and Vuorela (1957) with estimates of the mean meridional circulation. The comparison is shown in table 7. Pisharoty's computations include contributions from daily winds. Palmén calculated the source and sink terms and deduced the flux of kinetic energy across 30N as a residual for a steady state. The use of the jet stream coordinate reveals a larger production and a larger export by the mean vertical mass circulation.

Palmén had used the geostrophic zonal wind at 30N which is considerably smaller in magnitude than the C_s component; this should explain the reason for the differences in the numbers.

5. Subtropical jet stream and the higher latitude circulation

In the preceding sections we have shown that the tropical circulations transport large amounts of energy and zonal angular momentum into the higher latitudes. It is naturally of great interest to find any relation-

TABLE 6. Monthly transport of kinetic energy during winter 1955–56. Units 10^{20} ergs/sec.

Month	Degrees latitude from jet axis	Transport by mass circulation	Transport by standing eddies	Transport by 5-day eddies	Transport by daily eddies	Total
Dec. 1955	7.5	7.3	4.3	16.1	−16.7	11.0
	5.0	14.5	13.7	16.0	−20.0	24.2
	J	19.6	12.2	8.8	− 2.4	38.2
	− 5.0	11.5	9.5	4.6	11.5	37.1
	−10.0	5.5	4.6	8.3	3.3	21.7
	−15.0	2.2	1.7	4.7	− 4.6	4.0
	−20.0	0.5	1.0	4.7	− 4.7	1.5
Jan. 1956	7.5	4.9	16.3	4.9	29.7	55.8
	5.0	19.2	7.0	−1.0	31.9	58.0
	J	29.4	8.9	14.4	11.5	64.2
	− 5.0	21.6	1.3	13.7	9.2	45.8
	−10.0	10.1	0.1	2.2	3.6	16.0
	−15.0	4.1	1.8	1.2	− 6.2	0.9
	−20.0	1.5	1.2	−0.9	1.1	2.9
Feb. 1956	7.5	1.2	2.0	−0.2	13.4	16.4
	5.0	4.1	3.2	−1.0	12.5	18.8
	J	10.0	2.1	3.3	4.9	20.3
	− 5.0	9.9	3.5	0.7	2.8	16.9
	−10.0	4.5	1.1	0.6	2.7	8.9
	−15.0	1.3	0.7	1.6	− 1.7	1.9
	−20.0	0.2	0.3	1.0	− 1.1	0.2

TABLE 7. Comparison of kinetic energy budget in the tropics. (Units 10^{20} ergs/sec.)

	Pisharoty	Palmén	Krishnamurti
Transport across 30N or equivalent	20	22	37 (without 5 day eddies)
Frictional dissipation in surface boundary layer	10–15	9	6
Production of *KE*	30–35	31	42

ships between the computed quantities and their effect, if any, on the circulation of the higher latitudes.

Such investigations have been carried out with some success in the past by Riehl (1950) and Cressman (1948). These reports deal with the subject of meridional propagation of certain trends, which can be used for forecasting 5-day averaged circulation features.

If such trends are indeed a feature of the general circulation, it would be of interest to compute certain statistics of the higher latitude circulation and then make a comparison with the events at lower latitudes. With such an aim in mind, various circulation parameters of the high and the low latitudes were calculated, and they are presented in fig. 6.

The following lower-latitude circulation parameters were computed:

(i) Latitude of the subtropical jet stream,
(ii) Five-day averaged flux of total energy across the jet stream axis,
(iii) Five-day averaged flux of absolute angular momentum across the jet stream axis.
(iv) Five-day averaged flux of kinetic energy across the jet stream axis.

The following higher-latitude circulation parameters were computed:

(i) Five-day mean geostrophic zonal wind U_g around 45N, at 500-mb surface,
(ii) Five-day mean value of the root mean square meridional geostrophic wind, around 45N at 500-mb surface,
(iii) Five-day averaged wave speed *C* around 45N at the 500-mb surface (Prepared from a detailed Hovmöller diagram),
(iv) Sea level zonal index between 35N and 55N (Taken from *Mon. Wea. Rev.*, 1956),
(v) Surface sea level pressure of the Siberian High.

The statistics refers to the winter of 1955–56, and all averages are integrations around the world.

First we shall discuss the higher latitude statistics of fig. 6.

Klein (1956) has presented a summary of the weather during January, 1956, which was a month of very low zonal index as shown from calculations at the sea level and at 700-mb. There were more storms around the northern hemisphere during this month than in normal Januaries or in December, 1955, or February, 1956. This index is included in fig. 6. Defant (1956, 1958, 1959) has investigated the behavior of the jet streams during this January and has shown that the circulation in the middle latitudes was unusually characterized by blocking situations.

A Hovmöller diagram was prepared for the entire winter to determine the speed of troughs and ridges around 45N at the 500-mb surface. Due to the large size of the chart it is not presented here. The results are summarized by the 5-day mean speed of the waves. During January a minimum in the speed is observed. In fact a negative speed of waves for a period around the 9th of January was found. This is the period of large blocking over the great part of the northern hemisphere. The daily northern hemispheric charts show this phenomenon rather clearly.

The mean zonal and the mean meridional motion of the northern hemispheric mid-latitude circulation is shown by the curves of (i) geostrophic zonal wind around 45N at 500 mb and (ii) root mean square meridional geostrophic wind at 500 mb. During the low index period the zonal wind was small as it should be and the meridional motions were large.

With these circulation parameters of the middle latitudes before us, the question can be asked whether there was any relation between these and the lower latitude calculations.

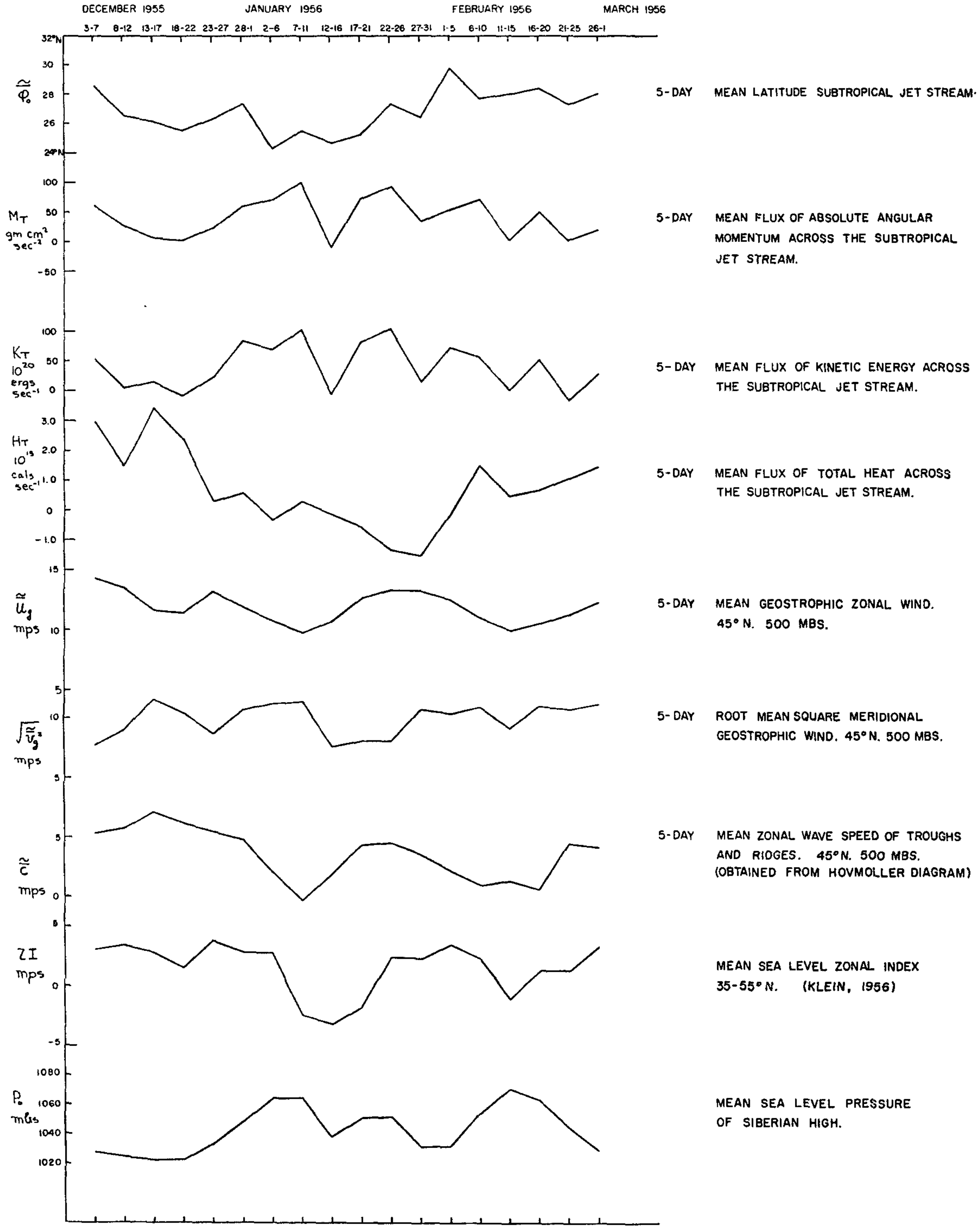

FIG. 6. Five day averaged statistics of various lower and higher latitude circulation parameters. Legend on the right.

The month of January, 1956, was characterized by large flux of kinetic energy and momentum across the jet stream axis. The fluxes were larger than those computed for the other two months. This suggests that during periods of large flux from the lower to higher latitude, the high latitude circulations could be of a lower zonal index.

The heat flux from the tropics into middle latitudes is not correlated with other fluxes, and it is hard to find any direct relation with the middle latitude circulation.

It is not possible to find any significance of the short period fluctuations; this may be due to lack of data.

The 5-day mean position of the subtropical jet stream is also shown in fig. 6. The jet axis moves quite far north in late December, an unusual southward plunge was noticed in early January. Defant (1958) has also discussed this anomalous movement of the subtropical jet stream during January, 1956. It has not been possible to find any reasons for this movement of the jet stream or to relate it to the other circulations.

The author also made an examination of the surface pressure charts during the winter of 1955–56. It may be recalled that the winter circulation of the northern hemisphere is characterized by three centers on the surface map: The Aleutian low, the Icelandic low and the Siberian high. Statistics of the central pressure of the Siberian high are presented here to show possible relation with both lower and higher latitude circulations. It may be observed that the central pressure goes up to 1065 mb during periods of low zonal index, showing an inverse relation between the central pressure and the zonal index. Moreover, during periods of intensification of the Siberian high, the winter monsoon (the outflow of air from the high) is characterized by stronger northeast winds. The author believes that these are thus the periods when large production of kinetic energy, in the lower latitudes, takes place by air moving towards lower pressure. Also, as may be observed from fig. 6, these are the periods of large poleward fluxes. Of the 9 cross sections around the world, table 1, the one nearest to the Siberian high was found to contribute the largest in all flux calculations. It would not be wise to isolate any system like the Siberian high and look for various answers. It was not possible to obtain similar statistics for the Aleutian low and the Icelandic low. This inquiry may be considered somewhat in the lines of the work of Namias (1959), who has held the view that the earth's surface is a place to look to for clues on the variation of large scale zonal circulation. He has also discussed the role of snow cover and unusual rains as important anomalies in the distribution of heat sources and sinks affecting the future weather.

In conclusion, it may be mentioned that the presented calculations suggest that more work should be done to place the subtropical jet stream of winter in the general circulation of the atmosphere. This paper may be considered as a first attempt in this direction.

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APPENDIX I

On the differences in transport across the wave coordinate and the latitude coordinate:

At various places in this paper it became necessary to compare the fluxes of heat and momentum as obtained in the wave coordinate with that obtained across latitude circles by other workers. Such a comparison would be valid only if there were no differences in the fluxes across the two systems of coordinates. We shall examine these differences for heat and momentum fluxes and make estimates of errors introduced by this comparison. We shall consider one wave for this purpose.

Let *ACDB* (fig. 7) denote a wave, and straight line *AB* the latitude circle, on an isobaric surface.

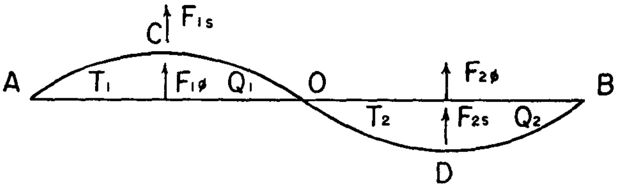

FIG. 7. Wave diagram for flux calculations.

We shall divide the wave into two regions *ACO* and *ODB*; these are now two volumes bound by the 1000- and the 100-mb surfaces.

We can separately write energy or momentum balance equations for these two regions. For *ACO*:

$$F_{1s} - F_{1\phi} + T_1 - Q_1 = 0$$

where F_{1s} is the northward flux across wavy wall, *ACO*, $F_{1\phi}$ is the northward flux across latitude circle, *AO*, T_1 is the upward flux on the top surface of volume, Q_1 is the upward flux on the bottom surface of volume. Similarly we have for *ODB*:

$$-F_{2s} + F_{2\phi} + T_2 - Q_2 = 0.$$

Subtracting the two equations we get

$$(F_{1s} + F_{2s}) - (F_{1\phi} + F_{2\phi}) = (T_2 - T_1) + (Q_1 - Q_2).$$

The left hand side of this equation represents the differences in the fluxes in the two coordinates in question; and the purpose of this enquiry here is to see how far we are justified in neglecting the right hand side.

We shall consider the heat and momentum fluxes separately.

Heat flux. $T_2 - T_1$ is the difference in the upward fluxes in regions 1 and 2. On the average, region 1 is about 5 deg north of region 2. A rough calculation of this term can be performed from London's estimates of latitudinal variation of radiation into space. Referring to his tables, we find that his estimate gives a difference of 0.016 langley/minute across a 5 deg latitude belt. This corresponds to $T_2 - T_1$ of about 0.06×10^{15} cals/sec, which is one order of magnitude smaller than the terms of the left hand side of the equation we are considering.

A rough estimate of $Q_1 - Q_2$ can be readily obtained from inspecting Jacobs values of the heat source $Q_s + Q_e$ (table 3). The order of magnitude of this term, again for a 5 deg latitude separation, is about 0.02×10^{15} cals/sec, which is smaller than the transport terms of the left hand side. We can therefore neglect the various terms of the right hand side.

Momentum flux. $T_2 - T_1$ is the upward convective flux of momentum at 100-mb surface and can be neglected. In order to arrive at an estimate of $Q_2 - Q_1$ we could look at table 5, which lists the angular momentum pickup over 5 deg latitude belts. We notice that $Q_2 - Q_1$ is not a large term but is not small enough to justify omission. For this case we can write $F_{1\phi} + F_{2\phi} = (F_{1s} + F_{2s}) - (Q_1 - Q_2)$.

The values of $F_{1\phi} + F_{2\phi}$ are listed in table 8 and compared with those of Priestley and of Bjerknes. It is interesting to compare tables 5 and 8. The large discrepancy at quite low latitudes does not appear in table 8. This has not improved the situation on the poleward side of the jet axis.

Table 8. Flux of angular momentum (10^{25} gram cm^2 sec^{-2}).

Degrees latitude *from* jet axis	$F_{1s}-F_{2s}$	Q_1-Q_2	F_1-F_2	Priestley 1951	Bjerknes and Venkateswaran
5	31.3	−0.5	31.8	42.2	46.0
0	39.6	2.1	37.5	42.7	50.5
− 5	46.4	6.1	40.3	40.6	48.0
−10	38.2	10.9	27.3	34.5	39.0
−15	32.5	13.0	19.5	23.6	29.0
−20	23.8	6.8	17.0	10.6	16.0

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