SHORTER CONTRIBUTIONS

NON-PARAMETRIC METHODS OF ANALYSIS APPLIED TO LARGE-SCALE CLOUD-SEEDING EXPERIMENTS

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1. Introduction

In Australia, a program of experiments is in progress, the objective of which is to determine if the injection of silver-iodide smoke into suitable clouds can increase the precipitation over a specified area. The success or otherwise of silver iodide in producing an increase in precipitation is inferred, in part, from the results of statistical tests.

The design of these experiments involves the selection of two areas as climatologically similar as possible. The clouds in each area are subjected to treatment with silver iodide in the following manner. Time is divided into roughly equal periods usually of the order of 12 days, and in any one period the clouds in only one of the two areas are treated. A random sequence determines in which of the two areas operations are carried out in each period.

Estimates of the total precipitation in each area for each period are the basic data upon which subsequent statistical analyses are carried out.

This paper is concerned with a description of non-parametric or distribution-free methods of statistical analysis which are being applied to these experiments and with a discussion of such methods as compared with the more usual parametric methods. The results of a simulated cloud-seeding experiment are included merely as a convenient subject upon which to demonstrate the methods.

2. Parametric and non-parametric methods of analysis

Parametric methods of statistical analysis involve a knowledge of the form of the distribution of the variates upon which the analyses are made or at least a knowledge that the small sample available does not depart significantly from an assumed distribution (usually the normal distribution). This means that inherent in all results from such tests is the uncertainty of the truth of the assumptions of normality even when tests for normality have been carried out.

Non-parametric methods make no assumptions as to the form of the distributions of the variates and are independent of that form. In the methods to be discussed here, the only assumption made is that the periods during which a particular area was treated are a random sample from the total number of periods. This assumption is justified by the form of the experiment design.

It is usually stated that the disadvantage in using the non-parametric tests is that they are not as powerful as the parametric tests. The "power" of a test is a mathematically determinable figure which roughly corresponds to the intuitive idea of the ability of a test to distinguish between two alternative hypotheses. It is frequently forgotten, however, that in stating the "power" of a parametric test the assumptions regarding the form of the distribution are taken to be true. It is not the intention to discuss here by how much the uncertainty regarding the truth of the initial assumptions in parametric tests affects the confidence to be placed in the results, but it is suggested that the side-by-side use of non-parametric with parametric tests increases confidence in the results and affords a firmer basis upon which further inferences and judgments can be made, at least, within the region of agreement of the two types of tests.

3. Simulated experiment

As a convenient subject upon which the various methods of analysis can be demonstrated, the results of a simulated experiment were derived in the following manner.

The monthly rainfall total readings for five years of two towns, Manilla and Tamworth in N.S.W., Australia, were taken from the meteorological records. The two towns are approximately 30 mi apart and are situated just west of the Great Dividing Range which runs the whole length of the eastern side of the continent.

Successive months were designated Manilla seeded

(*MS*) or Tamworth seeded (*TS*) according to a random sequence. In those months designated *MS*, the Manilla totals (*M*) were increased by 10 per cent; in those designated *TS*, the Tamworth totals (*T*) were increased by 10 per cent. The resulting figures are given in table 1 and are taken to represent the results of an experiment on which cloud treatment with silver iodide is presumed to have increased precipitation in the treated area by 10 per cent.

TABLE 1. Results of simulated seeding experiment. Monthly total precipitation with designated values increased by 10 per cent.

Period designation	Manilla (inches)	Tamworth (inches)
MS	1.65	2.51
MS	0.55	0.71
TS	1.83	2.42
MS	1.07	0.76
TS	0.39	0.77
MS	5.36	5.57
TS	2.75	2.20
TS	1.12	1.03
MS	1.13	0.93
MS	3.31	3.89
MS	2.09	1.06
TS	2.42	2.86
TS	1.03	2.54
MS	0.62	1.34
TS	4.22	3.12
TS	2.67	3.27
MS	4.81	3.79
MS	4.25	5.26
TS	1.97	2.10
MS	1.31	1.18
TS	0.99	1.28
TS	0.28	0.45
MS	2.56	2.81
MS	4.97	4.29
TS	2.24	0.90
MS	0.20	1.03
MS	2.89	2.22
MS	2.10	1.46
TS	0.63	0.63
MS	1.29	0.80
MS	1.69	1.26
TS	0.65	1.24
MS	4.58	3.97
TS	2.44	2.46
TS	2.89	2.67
TS	2.95	2.49
TS	7.26	6.35
MS	0.70	1.10
TS	0.23	1.32
TS	0.51	0.21
MS	1.56	1.37
TS	2.54	2.82
MS	4.88	3.94
MS	1.88	0.96
TS	2.42	3.71
TS	5.78	4.88
TS	3.43	5.57
MS	3.63	3.65
MS	4.13	3.38
TS	4.58	5.86
MS	0.01	0.01
TS	1.61	1.73
MS	0.06	0.00
MS	1.06	1.47
TS	3.18	3.04
TS	2.41	2.83
TS	2.18	2.74
MS	4.27	4.52
TS	1.56	1.68
MS	4.27	3.38

4. Tests applied

Two parametric tests and three non-parametric tests were applied to these results. The null hypothesis in each case is that there is no increase in M and T during the *MS* and *TS* periods respectively.

Let M_M denote the *MS* period values of Manilla precipitation, M_T denote the *TS* period values of Manilla precipitation, T_M denote the *MS* period values of Tamworth precipitation, T_T denote the *TS* period values of Tamworth precipitation.

Parametric tests: (i) A simple "t" test on the differences of *MS* and *TS* precipitation means about a regression line assumes normality and uniform variance of the variate about the regression line. A square-root transformation applied to the raw data satisfied these requirements to better than the conventional 5 per cent limits. A "t" test was then applied to $(\sqrt{\overline{M}_M} - \sqrt{\overline{M}_T}) - b_1(\sqrt{\overline{T}_M} - \sqrt{\overline{T}_T})$ where the bar notation indicates means and b_1 is the regression coefficient of $\sqrt{M}$ on $\sqrt{T}$.

(ii) Moran (1959) suggested that a more appropriate test for this design of experiment is a "t" test on $(\bar{U}_M - \bar{U}_T) - b_2(\bar{V}_M - \bar{V}_T)$, where $U = \sqrt{M} - \sqrt{T}$, $V = \sqrt{M} + \sqrt{T}$, and b_2 = regression coefficient of U on V.

Non-parametric tests: (i) The 60-period ratios M/T were arranged in order of magnitude. Of these ratios, 30 were *MS* and 30 *TS*. If we assume equal probability for all divisions of the 60 ratios into 30 "*MS*" ratios and 30 "*TS*" ratios, the probability of obtaining the observed or greater number of "*MS*" values above a median line common to both *MS* and *TS* values may be obtained by a simple combinational method (Kendall, 1943) or by the χ^2 approximation. The assumption of equal probability for every arrangement is equivalent to the assumption of *MS* values being a random sample from the total, which is assured by the design of the experiment.

The division of the 60 periods into exactly equal numbers of *MS* and *TS* periods was fortuitous. This test and the two succeeding ones, however, are still valid in cases where the original division gives unequal numbers in the two categories.

(ii) A Wilcoxon test (Wilcoxon, 1945; Mann and Whitney, 1947; Kendall, 1955) was also applied to the ordered array of M/T values. This test compares the ranking of all values in the *MS* sample with the ranking of all values in the *TS* sample. The assumption of equal probability of all arrangements of the *MS* and *TS* values applies to this test as well as to the succeeding one.

(iii) A natural approach to finding out whether the seeding had any effect is to compare the ratios of mean values of Manilla and Tamworth precipitations taken over *MS* and *TS* periods. On the null hypothesis, the expected value of $\bar{M}_M/\bar{T}_M - \bar{M}_T/\bar{T}_T$ is zero, and one may obtain the significance of an observed difference by computing the value of $\bar{M}_M/\bar{T}_M - \bar{M}_T/\bar{T}_T$ for all possible divisions into 30 "*MS*" and 30 "*TS*" periods and counting the proportion of these divisions which has a value equal to or greater than that observed.

The number of arrangements involved, however, is enormous (10^{17}) but an approximation to the required ratio may be obtained by taking a large number of random samples and computing the relevant ratio from these. In the present case, 10,000 random samplings were made with an electronic computor, and the fraction of this number having a value equal to or greater than the observed value of $\bar{M}_M/\bar{T}_M - \bar{M}_T/\bar{T}_T$ is quoted as a significance level. It is noteworthy that the observed value of $\bar{M}_M/\bar{T}_M - \bar{M}_T/\bar{T}_T$ gave a figure of 7.4 per cent for the overall increase in treated area precipitation — an underestimate of the 10 per cent increase put into the results.

5. Discussion of results

The results of the five tests are given in table 2. The probabilities vary from 0.016 for the Wilcoxon test on ratios to 0.043 for the standard "*t*" test on regression, but they all agree that the differences between *MS* and *TS* period precipitations are unlikely to have arisen from random sampling alone. It is true that, while such a conclusion would have been reached from the results of any one test, the sum total of the results is more convincing and gives a firmer basis for further inferences.

One reason, not usually recognized, for such differences between results is that the different tests emphasize different aspects of the data. The regression tests, because of the particular transformations they use, inflate the importance of the low precipitation values, while the method using random sampling on ratios of mean values gives more weight to those periods with large precipitations.

TABLE 2

Test	Significance level (one-sided)
Regression 1	0.043
Regression 2	0.032
Median Ratio	0.021
Wilcoxon	0.016
Random Sampling	0.034

In an artificial experiment with uniform percentage increases all round, it is therefore not surprising that the differences between the results of the various tests are not great. In a real experiment, there may be differential effects, such as increases in precipitation occurring only when large amounts of precipitation would have fallen naturally, and therefore there may be large discrepancies between the results of the various tests applied. In such a case, a closer look at the data and at what the tests are actually testing should be indicated rather than the acceptance or rejection of the null hypothesis on the results from the tests of higher "power." It would, in fact, strongly indicate that the null hypothesis was incorrectly posed.

6. Conclusions

The answer to the question of whether cloud seeding has increased precipitation in an area is derived by inference from, among other things, the results of statistical tests.

Parametric methods of statistical analysis depend upon assumptions, the validity of which in applications to precipitation data may be in doubt. The confidence with which the results of such tests may be regarded is therefore reduced. Non-parametric methods do not depend upon such assumptions, but they are usually regarded as less "powerful" than parametric methods.

The non-parametric tests are therefore presented as additional rather than alternative tests.

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