Note on Wind Variability with Doppler Radar

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21 March 1962 and 13 April 1962

A scheme has been proposed recently by Lhermitte and Atlas (1961) to measure the horizontal and vertical motions of the detectable particles in widespread rain or snow conditions by pulse doppler radar. It is assumed that the wind at each altitude is uniform over a radius of a few miles, and that the spectrum of particle fall velocities is essentially invariant over this area. With these assumptions the horizontal wind velocity and the particle fall velocities may be deduced from the Doppler frequency shift measured at different azimuths, keeping the elevation angle (α) constant. A particular range interval at the distance R can be selected by gating the echo returned by the particles; the Doppler shift of this signal is analyzed and the radial speed is measured. By rotating the beam continuously in azimuth, this radial speed at a constant altitude $R \sin \alpha$ is displayed as a continuous function of azimuth on a display called "Velocity-Azimuth-Display" (VAD). The radar gives only the absolute value of the Doppler frequency shift. The patterns will therefore show two maxima, obtained when the radar beam is either coincident with or opposite to the wind vector azimuth. The radial component, $v_f \sin \alpha$, given by the vertical motion v_f, is added to the component $v_h \cos \alpha$, generated by the horizontal motion v_h, for the larger of these maxima. The smaller maximum, where the wind vector is opposite to the radar beam azimuth, is given by $v_h \cos \alpha - v_f \sin \alpha$. Therefore, $v_h \cos \alpha$ is derived from the average of the two maxima and $v_f \sin \alpha$ is obtained from their difference. Of course, there is always a spectrum of fall velocities due to the particle size distribution; a parameter of this spectrum must be considered as being represented by v_f.

This method is very useful in the study of air motions in widespread rain or snow conditions. The variability of the upper winds can be observed on a relatively small time scale, something which is difficult to accomplish with other techniques. An accuracy of 0.2 m sec^{-1} for speed and a few degrees for direction has been effectively obtained by using a suitable frequency analyzer and display technique. However, the method previously described was based on the display of the VAD patterns on an oscilloscope, with a photograph being taken at each altitude level. These photos then had to be analyzed in order to determine the wind and vertical velocity as a function of altitude.

This method has been improved recently by recording directly the radial motion, as a function of both azimuth and altitude. A continuously rotating beam and a slowly moving range gate are used. The time allowed for the gate to move from 0 to 40,000 ft (which is equivalent to an altitude change from 0 to 20,000 ft with an elevation angle of 30°) is six min. During this time, the radar beam performs 60 complete revolutions in azimuth, giving 60 VAD patterns each referring to successively higher altitudes. Since the altitude gate is continuously moving, the measurements will be related to the average wind in the altitude interval scanned by the gate during one complete revolution of the antenna. With our present equipment, this altitude change is of the order of the beam cross section and gate duration vertical component, (approximately 120 m at short range and 250 m at long range). The smoothing of the wind profile from this scheme is then less than that inherent in other techniques such as balloon tracking.

If the average radial velocity $\bar{v}$ is given as a voltage and recorded, each azimuth cycle will bring two maxima, one corresponding to $\overline{v_h}\cos\alpha-\overline{v_f}\sin\alpha$ and the higher one corresponding to $\overline{v_h}\cos\alpha+\overline{v_f}\sin\alpha$. Two curves as a function of height will be obtained by combining the higher and lower maxima directly on the record. Their average will be $\overline{v_h}\cos\alpha$ and their difference $2v_f\sin\alpha$.

However, the simplest way to express the signature of a frequency spectrum as a voltage is by use of a frequency meter or a device which counts the zeros of the time domain signal. The application to our problem of the theory of zero crossing for Gaussian noise (Rice, 1943) shows that the rate of half the zero crossing, or the frequency F given by the frequency meter, is equal to: $F=(\overline{f^2})^{\frac{1}{2}}=2\pi/\lambda(\overline{v^2})^{\frac{1}{2}}$ where $\overline{v^2}$ is the second normalized moment of the radial velocity spectrum and λ the radar wavelength. For the maxima of the VAD pattern F^2 will be:

$$F^2=4\pi^2/\lambda^2[(\overline{v_h}\cos\alpha+\overline{v_f}\sin\alpha)^2+\sigma_h^2\cos^2\alpha+\sigma_f^2\sin^2\alpha].$$

In this equation, $\overline{v_h}$ and σ_h^2 are the mean and the variance of the horizontal motion, $\overline{v_f}$ and σ_f^2 the mean and variance of the vertical motion ($\overline{v_f}$ is positive for one maximum and negative for the other one). If $\sigma_h^2\cos^2\alpha+\sigma_f^2\sin^2\alpha\ll(\overline{v_h}\cos\alpha-\overline{v_f}\sin\alpha)$ then F is equal to $2\pi/\lambda(\overline{v_f}\cos\alpha\pm\overline{v_f}\sin\alpha)$ to a very good approximation, and $\overline{v_h}\cos\alpha$ and $\overline{v_f}\sin\alpha$ will be derived from the two maxima of F in the manner previously described.

In a widespread rain, σ_f^2 will be given mainly by the variation in fall velocity caused by the distribution of particle sizes. On this basis a study of actual Doppler spectra obtained with a vertical beam (Lhermitte, 1960) shows that σ_f^2 is of the order of 0.05 to 0.2 m^2 sec^{-2} for snow, 0.4 m^2 sec^{-2} for light rain, and 1 m^2 sec^{-2} for heavy rain. This agrees well with the variance given by Hitschfeld and Dennis (1956) for snow, but is somewhat smaller than the variance they reported for rain.

The variance of the horizontal motion is more difficult to characterize. Our experience with the VAD patterns in widespread rain or snow indicates that the main cause of the horizontal spectrum width is the wind shear through the vertical extent of the scattering volume in which the Doppler shift is measured. The effect of the turbulence which is noticed on the VAD patterns in widespread precipitation is mainly a departure of the curve from the theoretical cosine function. This effect ranges from small oscillations in the snow region above the bright band to a complete departure of the pattern from the cosine function in the friction layer. Since the time response of the frequency meter is sufficiently fast to follow this actual change of the spectrum, this device will give an instantaneous value of $(\overline{v^2})^{\frac{1}{2}}$.

From the above equations it follows that $(\overline{v^2})^{\frac{1}{2}}=[(\bar{v})^2+\sigma_t^2]^{\frac{1}{2}}$ (where σ_t^2 is the total variance), must be a good approximation of $\bar{v}$. If we assume that $\sigma_t/\bar{v}=0.2$ ($\sigma_t^2=9$ m^2 sec^{-2} for $\bar{v}=15$ m sec^{-1}), $(\overline{v^2})^{\frac{1}{2}}$ will be greater than $\bar{v}$ by only 2 per cent or 0.3 m sec^{-1}. According to the preceding discussion, this condition can be met in widespread precipitation with the 30° elevation angle used in this work, even in heavy rainfall conditions. The estimate of $\bar{v}$ with a frequency meter is therefore feasible. This has been confirmed by several tests comparing the measurements obtained on the same Doppler Spectrum by a frequency analyzer and by a frequency meter. Agreement within 0.5 m sec^{-1} (and often 0.2 m sec^{-1}) was obtained. Since the accuracy of the recorder is better than this figure, 0.5 m sec^{-1} is considered to be the typical accuracy of the records.

Two records obtained by this method are presented in Figs. 1a and 1b. In Fig. 1a the two curves combining the two classes of maxima are surprisingly smooth with a departure from the mean which is less than ±0.3 m sec^{-1}. Although this smoothing is partly due to the altitude averaging mentioned above, it shows a good

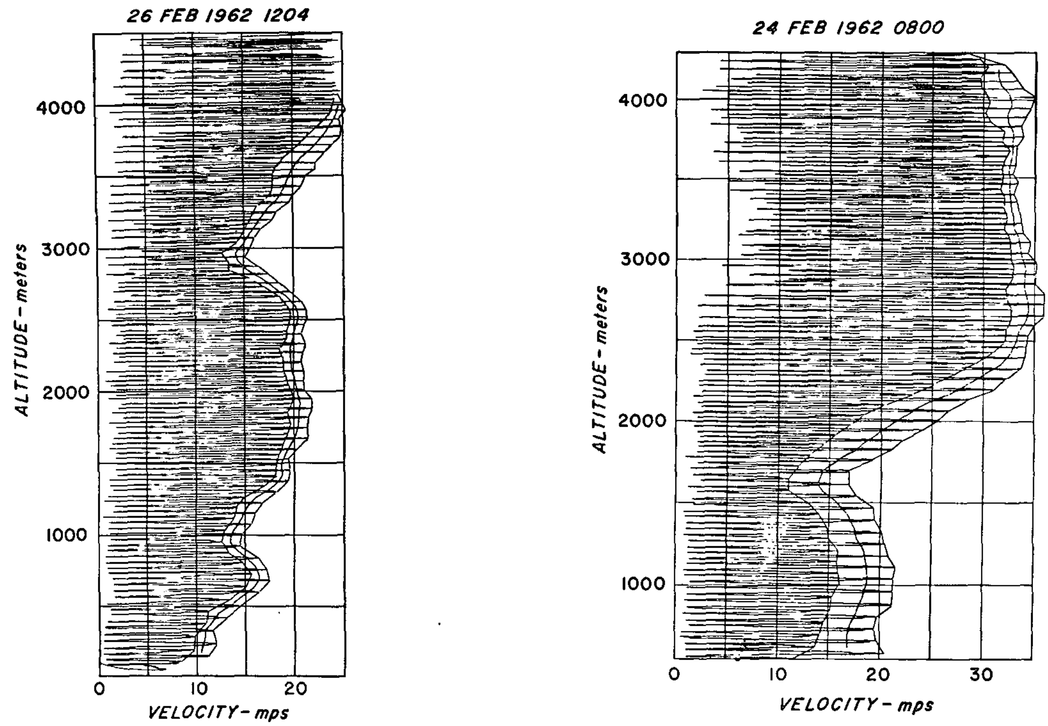

FIG. 1. Observed records of Doppler radial speed versus both azimuth and height on two separate occasions; (a) 26 February 1962 (on the left) and (b) 24 February 1962 (on the right).

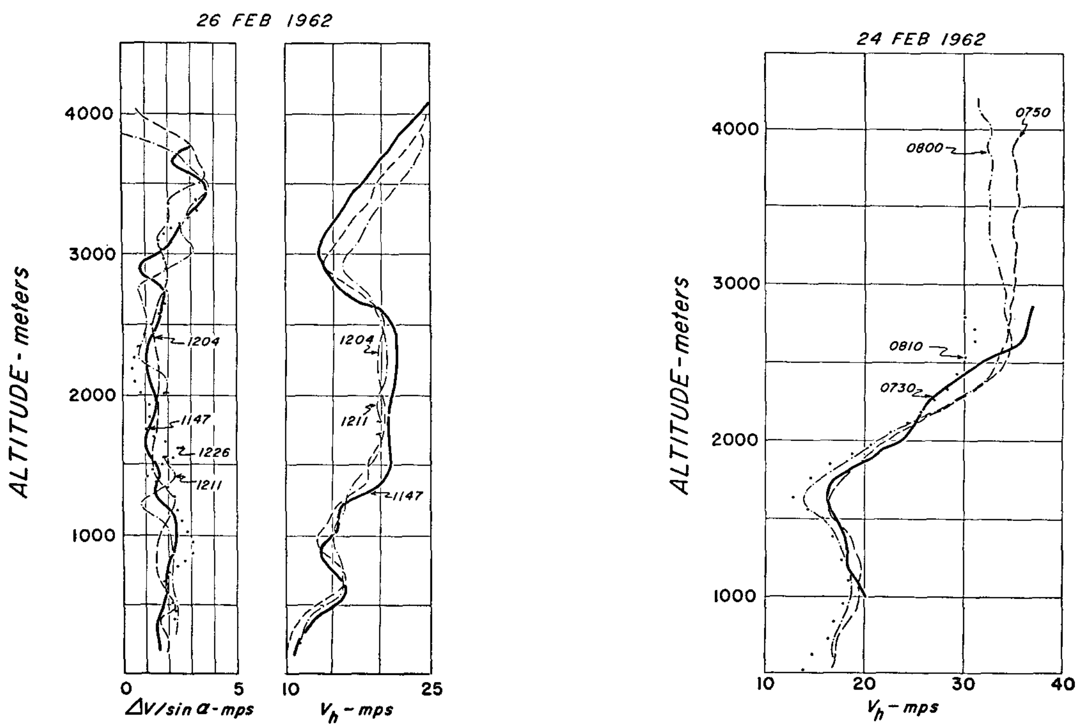

FIG. 2. Time variations of the horizontal and vertical speed profiles: (a) Vertical and horizontal speeds for 26 February 1962 (left diagram), (b) Horizontal speeds for 24 February 1962 (right diagram). The times for each curve (LST) are indicated in each diagram.

uniformity in the wind above the friction layer. This is confirmed by the very smooth shape of the VAD patterns above an altitude of 1000 ft. In the wind speed determination an additional averaging is produced by taking the mean between two points which are not at the same location on the horizontal plane. However, this does not seem to introduce more smoothing in the wind profile. In the other record, obtained on 24 February 1962 (Fig. 1b) with frozen droplets reaching the ground, somewhat higher fall speeds are noticed but the wind profile remains smooth.

For each of these situations the wind speeds obtained within approximately 10-min time intervals are plotted in Figs. 2a and 2b. In Fig. 2a the curves show good agreement, particularly in some "nodal" points (altitude 650 m, 1250 m, 2630 m) where the variations are limited to less than 0.1 m sec^{-1}. The larger variations are due to a regular and continuous change of the wind, especially in the region above 2000 m where the speed increases by 3 or 4 m sec^{-1} in only 24 min. On 24 February (Fig. 2b) the variability of the wind is greater. However, the time changes are also regular and thus undoubtedly real.

The particle fall velocities computed from the formula $\Delta v/2 \sin \alpha$, where Δv is the difference between the two Doppler maxima, are plotted in Fig. 2a for 26 February. The average of these curves shows a minimum of 1 to 1.5 m sec^{-1} in the altitude interval 1500–2700 m. Below this layer the fall velocities have an average value of 2 m sec^{-1}. This increase in fall speed is probably due to the growth of the snow crystals by their coalescence with droplets. The observation of snow pellets reaching the ground at this time supports this suggestion.

Fall velocities in excess of 3 m sec^{-1} are found above 3000 m. If we assume that the particles below 3000 m went through the same growth history as those above this level, then the large fall speeds must be attributed to downdrafts of the order of 1.5 to 2 m sec^{-1}. This could occur as a result of evaporative cooling. It seems highly unlikely that these fall velocities are indicative of aggregation since there is a consistent decrease in velocity below 3000 m.

The author is indebted to Dr. David Atlas for his suggestions and criticism in this work. Mr. Graham Armstrong provided valuable technical assistance. The radar used in this work was designed by Dr. Griffith Barry and Mr. George Dalrymple, of the M.I.T. Lincoln Laboratory, and furnished through the courtesy of that laboratory.

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