The Density, Pressure, and Particle Distribution in a Lightning Stroke near Peak Temperature

MARTIN A. UMAN
Department of Electrical Engineering, The University of Arizona

AND

RICHARD E. ORVILLE AND LEON E. SALANAVE
Institute of Atmospheric Physics, The University of Arizona

(Manuscript received 22 November 1963)

ABSTRACT

The first determination of the density, pressure, and particle distribution in a lightning stroke near peak temperature is presented. Relative amounts of atomic nitrogen, atomic oxygen, and singly ionized nitrogen are determined from line-intensity measurements in an optical spectrum of a return stroke. This information in conjunction with the previously measured stroke temperature from the same spectrum and theoretical tables of the thermodynamic properties of air is sufficient to determine the following properties of the lightning stroke: at a temperature of 24,000K the mass density of the stroke is about 0.1 of the mass density of air at STP; the stroke pressure due to atoms, ions, and molecules is about 18 atmospheres; the stroke pressure due to electrons is about 14 atmospheres; and the electron density is about 4.3×10^{18} cm^{-3}. The number of electrons per air atom (neutral, ionic, or comprising a molecule) is about 0.81; analogous quantities for 25 species of air particles present in the stroke at 24,000K are presented. The validity of the assumptions used in the analysis is considered.

1. Introduction

The thermodynamic properties of dry air at high temperature have been calculated by Gilmore (1955) for a wide range of temperatures and densities. The use of these data in conjunction with optical spectra of lightning strokes makes possible a calculation of the density, pressure, and particle distribution in those strokes. The lightning channel analyzed in the present paper is a return stroke for which Prueitt (1963) found an average excitation temperature of 24,400±800K. Uman (1964) has shown that this temperature value is in all probability within 10 per cent of the peak temperature of the stroke.

In order that the theory to be advanced in the next section be applicable to the lightning stroke under consideration, the following assumptions must be made: 1) The lightning stroke is optically thin. 2) Thermodynamic equilibrium is achieved within the stroke in a time short compared to the time in which the parameters of the stroke change. 3) The temperature and particle densities in the stroke at a given time are approximately constant across the cross section of the stroke. 4) The thermodynamic properties of dry air and of moist air do not differ appreciably. 5) The spectral lines of OI, NI, and NII considered in the analysis are primarily emitted at temperatures near 24,000K.

2. Theory

In thermal equilibrium at absolute temperature T, the number of atoms of one species in energy level i is given by

$$N_i=(Ng_i/B)\exp(-\mathcal{E}_i/kT), \qquad (1)$$

where N is the total number of atoms of the species, g_i is the statistical weight of the ith energy level, B is the partition function, $\mathcal{E}_i$ is the excitation potential of the ith energy level, and k is Boltzmann's constant. Now consider two atomic species, say atoms C and D. The number of C in the ith energy state and the number of D in the jth energy state can be written,

$$N_{Ci}=(N_C g_{Ci}/B_C)\exp(-\mathcal{E}_{Ci}/kT) \qquad (2)$$

$$N_{Dj}=(N_D g_{Dj}/B_D)\exp(-\mathcal{E}_{Dj}/kT). \qquad (3)$$

The measured intensity of an emission line due to transitions between two atomic energy levels can be written

$$I=KN_iAh\nu, \qquad (4)$$

where A is the Einstein emission coefficient for the transition, K is a geometrical factor, h is Planck's constant, ν is the frequency of the emission line, and N_i is the number of atoms in the upper energy level. For the case of atomic species C and D, the measured intensities can be written, respectively,

$$I_C = K N_{Ci} A_C h\nu_C \quad (5)$$

$$I_D = K N_{Dj} A_D h\nu_D. \quad (6)$$

Equations (2), (3), (5) and (6) can be combined to yield

$$\frac{N_C}{N_D} = \frac{I_C}{I_D} \frac{g_{Dj} B_C A_D \nu_D}{g_{Ci} B_D A_C \nu_C} \exp[(\mathcal{E}_{Ci} - \mathcal{E}_{Dj})/kT]. \quad (7)$$

Equation (7) gives the ratio of the number of atoms of species C to the number of atoms of species D in terms of known or measurable quantities. The intensity ratio, I_C/I_D, is obtained by measurements of the lightning spectrum. A discussion of the factors considered in making this type of measurement is given by Prueitt (1963). Statistical weights and energy levels for the atoms considered in this paper have been compiled by Moore (1949). The Einstein emission coefficients for the transitions involved and the references for these coefficients are given in Section 3.

The temperature appearing on the right hand side of (7) will be set equal to 24,000K, since we have assumed that the spectral lines to be considered are primarily emitted near this temperature. Note that 24,000K is, within a standard error, the excitation temperature of the particular stroke under consideration and is probably within 10 per cent of the peak temperature.

Sufficient data are therefore available so that N_C/N_D can be calculated. Gilmore (1955) lists the various particles per atom (the word "atom," used in this context, refers to neutral atoms, atomic ions, and atoms comprising molecules) in dry air at 24,000K for values of ρ/ρ_0 from 10^{-6} to 10 where $\rho_0 = 1.29 \times 10^{-3}$ gm cm^{-3}, the mass density of air at STP, and ρ is the mass density of the air under consideration. Thus, from Gilmore's data, graphs of N_C/N_D vs. ρ/ρ_0 can be constructed. These graphs, in conjunction with the values of N_C/N_D calculated from (7), are sufficient to determine ρ/ρ_0 for the lightning stroke under consideration. Once ρ/ρ_0 has been determined, Gilmore's tables can be used to find the quantities of all the constituent particles comprising the lightning channel at 24,000K and the partial pressures of those constituents.

3. Application of the theory

The following spectral lines were chosen for relative intensity measurements: 4935A(NI), 4368A(OI), 3995A(NII), and 4447A(NII). The statistical weights and energy levels of NI, OI, and NII used in the calculations are given in Table 1. If we make use of these values, the partition functions

$$B = \sum_i g_i \exp[-\mathcal{E}_i/kT] \quad (8)$$

of the atoms under consideration can be calculated for

Table 1. Statistical weights and energy levels for NI, OI, and NII Data adapted from Moore (1949).

NI		OI		NII	
g	$\mathcal{E}_i$ in cm^{-1}	g	$\mathcal{E}_i$ in cm^{-1}	g	$\mathcal{E}_i$ in cm^{-1}
$g_1=4$	0	$g_1=5$	0	$g_1=1$	0
$g_2=6$	19,223	$g_2=3$	159	$g_2=3$	49
$g_3=4$	19,231	$g_3=1$	227	$g_3=5$	131
$g_4=6$	28,840	$g_4=5$	15,838	$g_4=5$	15,316
$g_5=12$	83,319	$g_5=1$	33,792	$g_5=1$	32,687
		$g_6=5$	73,768	$g_6=5$	47,168
				$g_7=7$	92,238

Table 2. Einstein coefficients, statistical weights, and absorption oscillator strengths for spectral lines under consideration.

Lines	f	g'	g	A in sec^{-1}
4935A (NI)	—	2	—	2.24×10^6 [1]
3995A (NII)	0.42 [2]	3	5	1.05×10^8
4447A (NII)	0.275 [2]	5	5	5.57×10^7
4368A (OI)	0.641×10^{-2} [3]	3	9	7.36×10^5

[1] Richter (1961).
[2] Aller and Jugaku (1959).
[3] Jurgens (1954).

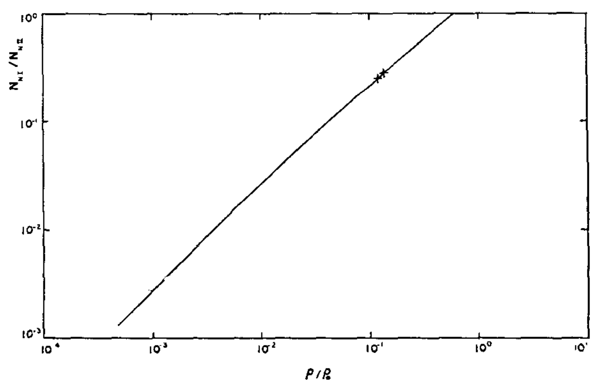

Fig. 1. A plot of the ratio of NI atoms to NII atoms vs. ρ/ρ_0 at 24,000K. Data adapted from Gilmore (1955). Cross marks indicate values of N_{NI}/N_{NII} calculated from equation (7).

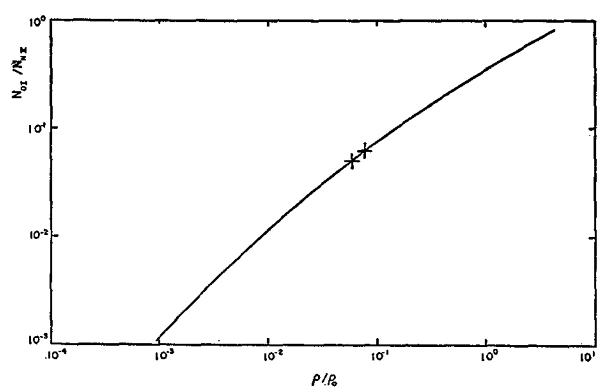

Fig. 2. A plot of the ratio of OI atoms to NII atoms vs. ρ/ρ_0 at 24,000K. Data adapted from Gilmore (1955). Cross marks indicate values of N_{OI}/N_{NII} calculated from equation (7).

$T = 24{,}000$K. The results of these calculations are

$$\text{for NI,}\quad B = 8.33$$
$$\text{for OI,}\quad B = 11.1$$
$$\text{for NII,}\quad B = 11.4.$$

The Einstein coefficients used are given in Table 2. In some cases the coefficients were calculated from absorption oscillator strengths, f, by the relationship

$$A = \frac{g'}{g}\frac{8\pi^2|e|^2\nu^2}{mc^3}f, \qquad (9)$$

where g' is the statistical weight of the upper level, g is the statistical weight of the lower level, c is the velocity of light, m the electron mass, and $|e|$ the magnitude of the electronic charge. The final data needed in the calculation of N_C/N_D from (7) are the upper energy levels of the spectral lines considered,

$$4935\text{A(NI)}:\quad 106{,}478.6\ \text{cm}^{-1}$$
$$4368\text{A(OI)}:\quad 99{,}680.4\ \text{cm}^{-1}$$
$$3995\text{A(NII)}:\quad 147{,}212.9\ \text{cm}^{-1}$$
$$4447\text{A(NII)}:\quad 187{,}092.2\ \text{cm}^{-1}$$

and the measured intensity ratios,

$$I_{4935\text{A (NI)}}/I_{4447\text{A (NII)}} = 0.62,\ I_{4368\text{A (OI)}}/I_{4447\text{A (NII)}} = 0.23$$
$$I_{4935\text{A (NI)}}/I_{3995\text{A (NII)}} = 0.15,\ I_{4368\text{A (OI)}}/I_{3995\text{A (NII)}} = 0.066.$$

In Fig. 1 a graph of $N_{\text{NI}}/N_{\text{NII}}$ vs. ρ/ρ_0 at 24,000K adapted from the data of Gilmore (1955) is shown. In Fig. 2 a graph of $N_{\text{OI}}/N_{\text{NII}}$ vs. ρ/ρ_0 at 24,000K adapted from the data of Gilmore (1955) is shown. The cross marks on these two graphs denote the values of N_C/N_D calculated from (7). Thus ρ/ρ_0 can be found for the sets of spectral lines compared. These results are presented in Table 3. The average density in the lightning stroke at 24,000K is

$$(\rho/\rho_0)_{\text{avg}} = 0.1. \qquad (10)$$

With $\rho/\rho_0 = 0.1$, the distribution of molecular, atomic, and ionic species in the lightning stroke at 24,000K can be determined from Gilmore's calculations. This information is given in Table 4. The total pressure of air at $\rho/\rho_0 = 0.1$ and 24,000K is given by Gilmore as 32.0 atmospheres. Using this value for total pressure and the data presented in Table 4, we find the electron pressure to be about 14 atmospheres and the pressure due to heavy air particles (atoms, ions, and molecules) to be about 18 atmospheres. An electron pressure of 14 atmospheres at 24,000K corresponds to an electron density of approximately 4.3×10^{18} cm^{-3} (the ideal gas law is assumed valid).

In order to get a better feeling for the results presented in the previous paragraph, calculations of the stroke pressure due to heavy air particles and of the electron density can be made as follows. At STP approximately 99 per cent of air is composed of O_2 and N_2. At 24,000K and $\rho/\rho_0 = 0.1$, approximately 99 per cent of the heavy air particles are atomic and ionic oxygen and nitrogen. Thus the high temperature air has approximately half the average mass per heavy air particle as air at STP. Since the mass density of the high temperature air is down by an order of magnitude from STP, the heavy air particle density must be down by about half an order of magnitude from STP. At STP the air pressure is 1 atmosphere. If we make use of the ideal gas law, the value for the stroke pressure due to heavy air particles at $\frac{1}{5}$ normal heavy air particle density and 24,000K can be calculated. The result of this calculation is a value of about 18 atmospheres for the pressure due to heavy air particles, a result agreeing with that previously presented. In order to verify the value of electron density presented in the preceding paragraph, we will use the Saha equation,

$$n_e = \frac{N_{\text{NI}}}{N_{\text{NII}}}\frac{2}{h^3}[2\pi mkT]^{\frac{3}{2}}\frac{B_{\text{NII}}}{B_{\text{NI}}}\exp[-\chi/kT], \qquad (11)$$

TABLE 3. The values of ρ/ρ_0 found from the comparison of four pairs of spectral lines.

Lines considered	N_C/N_D	ρ/ρ_0
4935A–3995A	$N_{\text{NI}}/N_{\text{NII}} = 0.28$	0.14
4935A–4447A	$N_{\text{NI}}/N_{\text{NII}} = 0.25$	0.12
4368A–3995A	$N_{\text{OI}}/N_{\text{NII}} = 0.063$	0.078
4368A–4447A	$N_{\text{OI}}/N_{\text{NII}} = 0.052$	0.060

TABLE 4. The distribution of particles in the lightning stroke at 24,000K with $\rho/\rho_0 = 0.1$. Data adapted from Gilmore (1955). The air comprising the lightning stroke is assumed to be dry. According to Gilmore, "the values for all the molecules are subject to large errors."

Particle	Number of particles per air atom
electron	8.1×10^{-1}
NI	1.4×10^{-1}
NII	6.4×10^{-1}
NIII	8.9×10^{-4}
NIV	7.4×10^{-11}
OI	5.0×10^{-2}
OII	1.6×10^{-1}
OIII	7.9×10^{-5}
OIV	5.4×10^{-13}
O^-	1.3×10^{-5}
CI	1.8×10^{-5}
CII	1.4×10^{-4}
NeI	1.2×10^{-5}
NeII	8.3×10^{-6}
AI	4.4×10^{-4}
AII	4.2×10^{-3}
AIII	5.2×10^{-5}
CO	9.6×10^{-11}
CO^+	7.9×10^{-10}
N_2	7.4×10^{-7}
N_2^+	7.2×10^{-6}
NO	3.3×10^{-7}
NO^+	5.8×10^{-6}
O_2^-	3.4×10^{-11}
O_2	5.1×10^{-8}
O_2^+	5.3×10^{-7}

to calculate the electron density (χ is the ionization potential of nitrogen, 14.5 ev). If $N_{NI}/N_{NII}=0.28$ (from Table 3), then $n_e=6.8\times10^{18}$ cm^{-3}. If $N_{NI}/N_{NII}=0.25$ (from Table 3), then $n_e=6.1\times10^{18}$ cm^{-3}. The agreement between this determination of electron density and the value 4.3×10^{18} cm^{-3} derived from Gilmore's tables should be considered very good considering the various sources of error involved. It should be noted that the values of electron density calculated from equation (11) are independent of the nitrogen partition functions since N_{NI}/N_{NII} was determined using the calculated value of B_{NII}/B_{NI} [see equation (11)].

4. Validity of assumptions

A number of assumptions have been made in the preceeding analysis. The validity of these assumptions will now be considered: 1) It has been assumed that the lightning stroke is optically thin. A discussion of the validity of this assumption is given by Prueitt (1963). 2) It has been assumed that thermodynamic equilibrium is achieved within the stroke in a time short compared to the time in which the parameters of the stroke change. A discussion of the validity of this assumption is given by Prueitt (1963) and by Mandel'shtam (1959). 3) It has been assumed that the temperature and particle densities in the stroke at a given time are approximately constant across the cross-section of the stroke. Mandel'shtam (1959) has found this to be approximately the case for high temperature laboratory sparks in air.

4) It has been assumed that the thermodynamic properties of dry air and of moist air do not differ appreciably. Since the maximum amount of water vapor that could be present in the air prior to the lightning stroke is only several per cent of the total particle density, the stroke will contain less than several per cent of hydrogen atoms and ions at 24,000K. It is physically reasonable to expect that the presence of such small amounts of hydrogen will not change greatly the thermodynamic properties of air from those calculated by Gilmore (1955).

5) The most questionable assumption made is the assumption that the NI, OI, and NII spectral lines considered in the analysis are primarily emitted at temperatures near 24,000K. A good case can be made for the validity of this assumption. The intensity of a spectral line emanating from a species of atoms is proportional to the number of those atoms in the appropriate excited state. We will show that the number of NI, OI, and NII atoms in the appropriate excited states increases with temperature up to about 24,000K, and thus, for reasonable lightning temperature versus time characteristics, that the observed spectral lines from these atoms are essentially emitted at temperatures near the peak temperature of the stroke. Equation (1) has been used to calculate the number of NI, OI, and NII atoms in the appropriate excited states at 12,000K, 18,000K, and 24,000K. The values of N in (1) are taken from the data of Gilmore (1955). The assumption is made that the stroke does not lose particles but expands to contain the same particles as the temperature rises and the stroke density decreases. In all cases the exponential term in (1) grows larger with increasing temperature. The number of NII atoms increases as the temperature approaches 24,000K; the number of NI atoms and of OI atoms decreases as the temperature approaches 24,000K. There are about a hundred times more NII atoms in the appropriate excited states at 24,000K than at 18,000K; about 10^6 times more at 24,000K than at 12,000K. There are about two times more NI atoms in the appropriate excited state at 24,000K than at 18,000K; about a hundred times more at 24,000K than at 12,000K. There are about two times more OI atoms in the appropriate excited state at 24,000K than at 18,000K; about a hundred times more at 24,000K than at 12,000K. Thus we can conclude that the measured spectral lines under consideration are primarily emitted at temperatures near 24,000K. The same type of reasoning is used by Uman (1964) to show that the excitation temperature (measured by Prueitt (1963)) for the stroke under consideration is within 10 per cent of the peak temperature of the stroke.

It is important to note that the results presented in Table 4 are relatively insensitive to the values used for the statistical weights, partition functions, and transition probabilities. As an example, assume that B_{0I} and B_{NI} have been underestimated by 50 per cent. If $(\rho/\rho_0)_{avg}$ is recalculated for values of both B_{0I} and B_{NI} 50 per cent larger than were used in the analysis, it is found that $(\rho/\rho_0)_{avg}\cong0.15$. For this new value of $(\rho/\rho_0)_{avg}$, the number of electrons per air atom is decreased by less than 5 per cent from its value of 0.81.

5. Summary

Values of N_{NI}/N_{NII} and N_{0I}/N_{II} have been determined for a single lightning stroke near peak temperature. An estimation of the density, pressure, and particle distribution in the stroke near peak temperature has been given. The thermodynamic properties of dry air as calculated by Gilmore (1955) have been used in conjunction with the measured properties of the spectrum of the stroke to determine the following properties of the stroke: At a temperature of 24,000K, a temperature near the peak temperature, the mass density of the stroke is about 0.1 of the mass density of air at STP; the pressure due to atoms, ions, and molecules in the stroke is about 18 atmospheres; the pressure due to electrons is about 14 atmospheres; the electron density is about 4.3×10^{18} electrons per cubic centimeter; and the number of electrons per air atom in the stroke is about 0.81. A listing of the particles per air atom for 25 species of particles present in the stroke at 24,000K has been presented. Two checks on the validity of Gilmore's data have been given. The validity of the assumptions used in the analysis has been discussed.

Acknowledgment. This research was performed in part under NSF grant GP-800 and in part under ONR contract Nonr 2173(06).

REFERENCES

Aller, Lawrence H., and Jun Jugaku, 1959: The atmospheres of the B stars. *Astrophys. J., Supplement IV*, 109–155.

Gilmore, F. R., 1955: Equilibrium composition and thermodynamic properties of air to 24,000K. Rand Corporation Research Memorandum Rm-1543 (Defense Documentation Center, AD-84052, unclassified).

Jurgens, G., 1954: Messung der Oszillatorenstärke einiger OI-Linien im wasserstabilisierten Lichtbogen. *Z. Physik*, **138**, 613–622.

Mandel'shtam, S., 1959: Excitation of the spectrum in a spark discharge. *Spectrochimica Acta*, **15**, 255–271.

Moore, Charlotte E., 1949: *Atomic energy levels.* Circular 467 of the National Bureau of Standards, Vol. I.

Prueitt, Melvin L., 1963: Excitation temperature of lightning. *J. geophys. Res.*, **68**, 803–811.

Richter, J., 1961: Uber Oszillatorenstärken von Multipletts des neutralen Stickstoffs. *Z. Astrophys.*, **51**, 177–186.

Uman, M. A., 1964: The peak temperature of lightning. *J. atmos. terr. Phys.*, **26**, 123–128.