Energy Spectra of Meso-Scale Turbulence Along and Across the Jet Stream[1]

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ABSTRACT

The wind velocities measured by an aircraft flying parallel and perpendicular to jet streams (Project Jet Stream, 1956–1957) have been analyzed; a smoothing technique has been used to separate the meso-scale turbulence from the mean flow. Eulerian auto-correlation coefficients and energy spectra are computed for the longitudinal and transversal components of the horizontal wind velocities. The distributions of the auto-correlation coefficients and the energy spectra appear to be similar for both the longitudinal and transversal components of the velocities, whereas the corrected meso-scale energy spectrum increases with decreasing wave number and is approximately proportional to k^{-2} in the range between 10^{-2} and 10^{-1} cycles km^{-1}.

An analysis is also made of the distribution of the Richardson number in a cross section perpendicular to the jet stream. A good relationship is found between the areas of turbulence and the regions of small Richardson number.

1. Introduction

Since the development of fast flying aircraft, there has been an increasing concern about clear-air turbulence. In recent years, considerable research has been done on clear-air turbulence, (Endlich and McLean, 1957; Reiter, 1961; Colson, 1962; Reiter, 1963). From these studies we have learned a great deal about correlations of turbulence location with atmospheric parameters, measured as closely as possible to the time of occurrence.

Clear-air turbulence is primarily of microscale. However, accurate wind measurements in regions of clear-air turbulence cannot easily be made. It is felt that detailed analyses of turbulence on a somewhat larger scale (meso-scale) would lead to a better understanding of the nature of the microscale clear-air turbulence. In this paper, wind and temperature measurements made by aircraft flying along and across the jet stream are analyzed.

Studies of energy spectra and correlation coefficients of wind velocities are important in the understanding of the mechanism of atmospheric turbulence and diffusion. In recent years, considerable research has been done on the Eulerian microscale energy spectra (Panofsky and Van der Hoven, 1955; Gifford, 1956; Hay and Pasquill, 1959; MacCready, 1962; and Pasquill, 1962). Lagrangian mesoscale energy spectra in the free atmosphere have also been studied (Angell and Pack, 1962; Mantis, 1963), as have Eulerian and Lagrangian macroscale turbulence in the free atmosphere (Ogura, 1958; Angell, 1960; Kao, 1963; Pinus, 1963; Kao and Bullock, 1964). However, very little study has been done on the structure of the Eulerian meso-scale turbulence across and along the jet stream. One purpose of this paper is to make such a study.

2. Analysis

During "Project Jet Stream" (Endlich and Rados, 1959), a number of flights were made at altitudes from 25,000 to 40,000 ft in a B-47 aircraft equipped with instruments for measuring winds, temperatures, pressure, turbulence, etc. For our present purpose, the most important instrument is the airborne wind-measuring equipment. The only method with sufficient accuracy in measuring the direction and speed of the horizontal component of the wind during flights of high-speed aircraft is based on a Doppler radar system in conjunction with a highly accurate airspeed indicator. The aircraft was equipped with a Doppler auto-navigator (APN-66); groundspeed is measured by the Doppler shift of a radar beam reflected from the earth's surface. Drift produced by winds is also measured using Doppler effects. Magnetic heading is measured by a gyrocompass and is corrected for magnetic variation to give true heading. In the APN-66, groundspeed, true airspeed, true heading, and drift angle are fed to the analogue computer which continuously computes the horizontal wind velocities. The APN-66 was also modified to facilitate flying directly with or against the wind. In this application of the auto-navigator, the aircraft follows a streamline by changing heading so that the drift angle remains zero. A panel containing indicators

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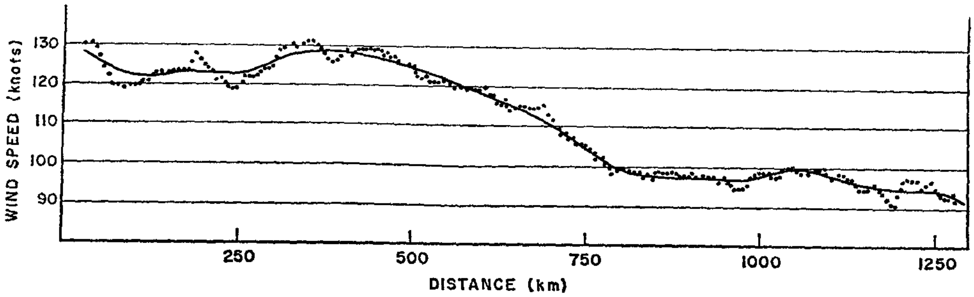

FIG. 1. The longitudinal velocity profile for Flight No. 22, parallel to the jet stream.

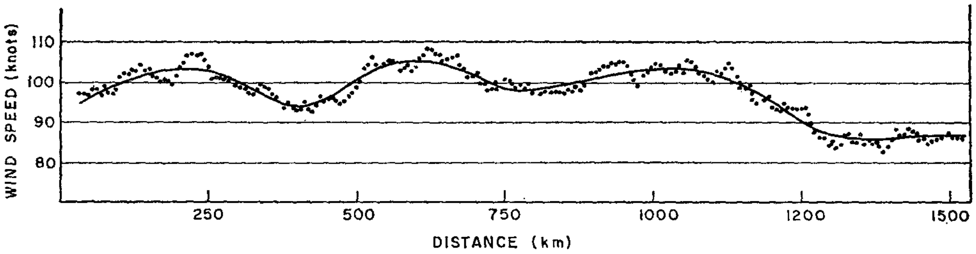

FIG. 2. The longitudinal velocity profile for Flight No. 20, perpendicular to the jet stream.

of the various meteorological and aeronautical parameters was photographed to give the record of data. Accuracies of the data are given in the Appendix.

In order to show the characteristics of meso-scale turbulence in the jet stream, flights were selected which were either parallel or perpendicular to the mean flow. In the analysis, the wind velocities were resolved into two components parallel and perpendicular to the mean flow, the longitudinal velocity (V_s) and the transversal velocity (V_n). (s and n will be denoted collectively by i throughout this paper.) Wind velocities parallel and perpendicular to the mean flow are represented by $V_s(s)$, $V_n(s)$ and $V_s(n)$, $V_n(n)$, respectively, where s and n in the brackets (denoted collectively by j throughout this paper) indicate distance parallel and perpendicular to the mean flow, respectively.

Time constants in the Doppler system are adjusted so that the indicated winds are averages over the previous 30–45 seconds. Let this average of the velocity be denoted by:

$$V_i(t;\tau)=\frac{1}{\tau}\int_{t-\tau/2}^{t+\tau/2} V_i(t)dt,$$

where τ represents the average time-interval. Figs. 1 and 2 show sample profiles of the longitudinal component of the horizontal wind velocity, parallel and perpendicular to the jet stream, each point representing a measurement of the V_s. It is to be noted that, in these profiles, waves of both meso-scale and large-scale exist, and for the convenience of computing the energy spectra of the meso-scale turbulence, it is necessary to filter out the long waves. To do so, the velocity profiles were smoothed by taking a running average of the velocity over a certain time-interval T. Let this running mean of the velocity be denoted by:

$$V_i(t;\tau,T)=\frac{1}{T}\int_{t-T/2}^{t+T/2} V_i(t;\tau)dt.$$

The solid curve in Figs. 1 and 2 represents the computed running mean of the velocity profile. Further, let the velocity fluctuations from the running mean be denoted by:

$$V_i'(t;\tau,T)=V_i(t;\tau)-V_i(t;\tau,T).$$

These velocity fluctuations will now be defined as the "meso-scale" turbulent velocities.

The method used in arriving at the average-interval T was to plot the components of velocity and to determine the linear regression lines for various sections of the data; T was then determined by the average of the wave intervals. The longitudinal and transversal components of the velocity parallel to the flow had an average wavelength of 21 observation-intervals, whereas those of the velocity measured perpendicular to the flow had an average wavelength of 19 observation-intervals. The average wavelength of the long waves was found to be of the order of 600 kilometers, which is just out of the spectrum range of meso-scale motion and into that of large scale atmospheric motion.

To show the meso-scale structure of the atmosphere, analyses of Flight No. 19 are shown in Figs. 3–6. Take-off and landing for the flight were at 1509 and 2149 GCT on 16 February 1957. The jet stream, with maximum wind speeds of 161 knots from 270 degrees, lay over Lexing-

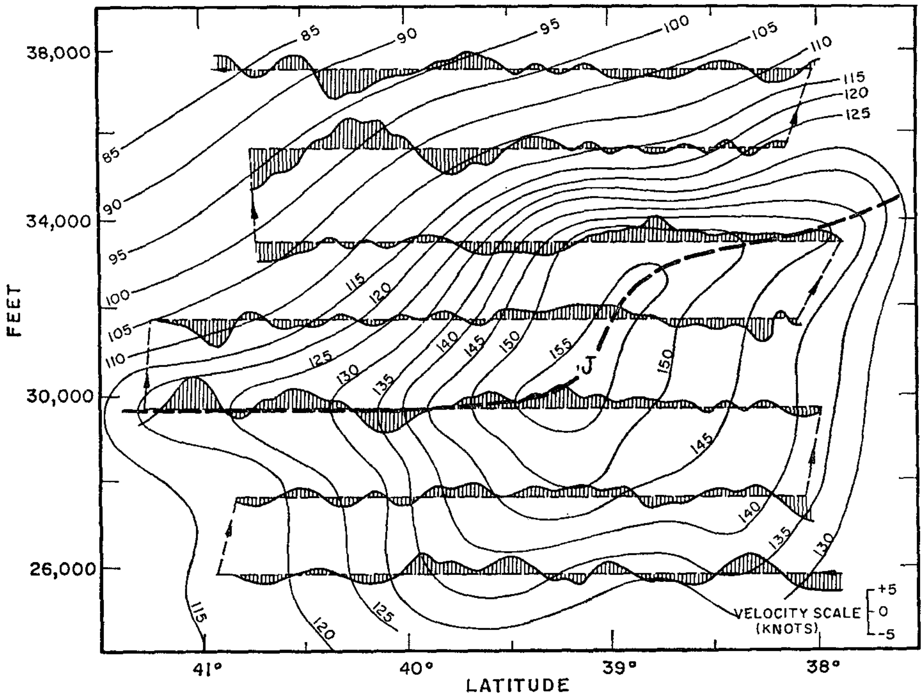

Fig. 3. Cross section of jet stream for Flight No. 19, 16 February 1957. Solid lines are isotachs of the wind field (knots). The heavy dashed line marks the position of the jet core. The "meso-scale" fluctuations of the longitudinal component of the horizontal wind velocity are indicated by the hatched areas (amplitudes in knots have been entered with respect to the flight route as base line). Dashed lines give the route of the flight.

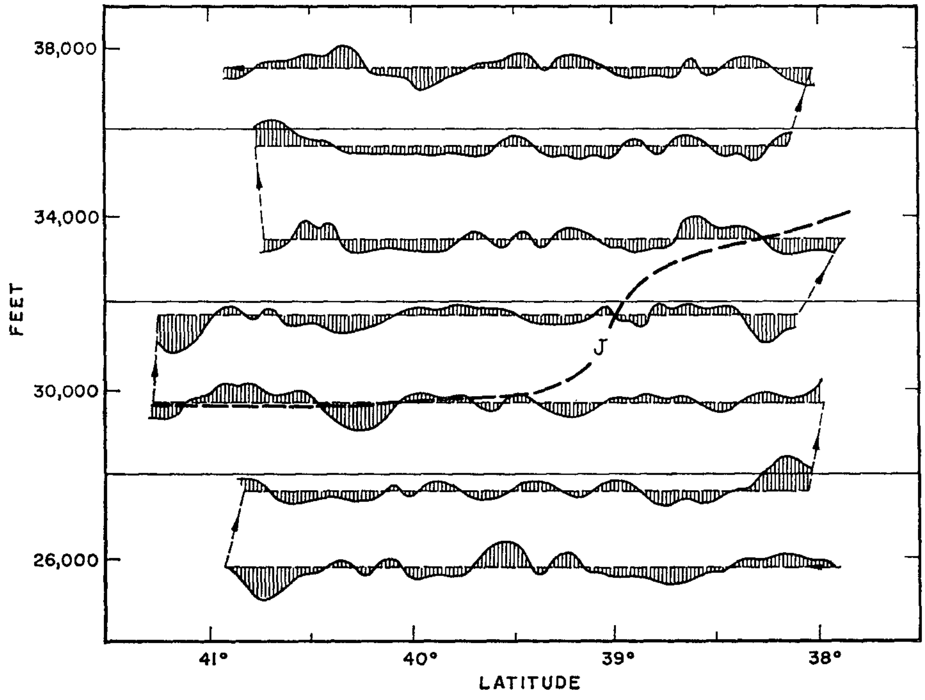

Fig. 4. "Meso-scale" fluctuations of the transversal component of the horizontal wind velocity. "Meso-scale" fluctuations, position of jet core and route of the flight are indicated as in Fig. 3.

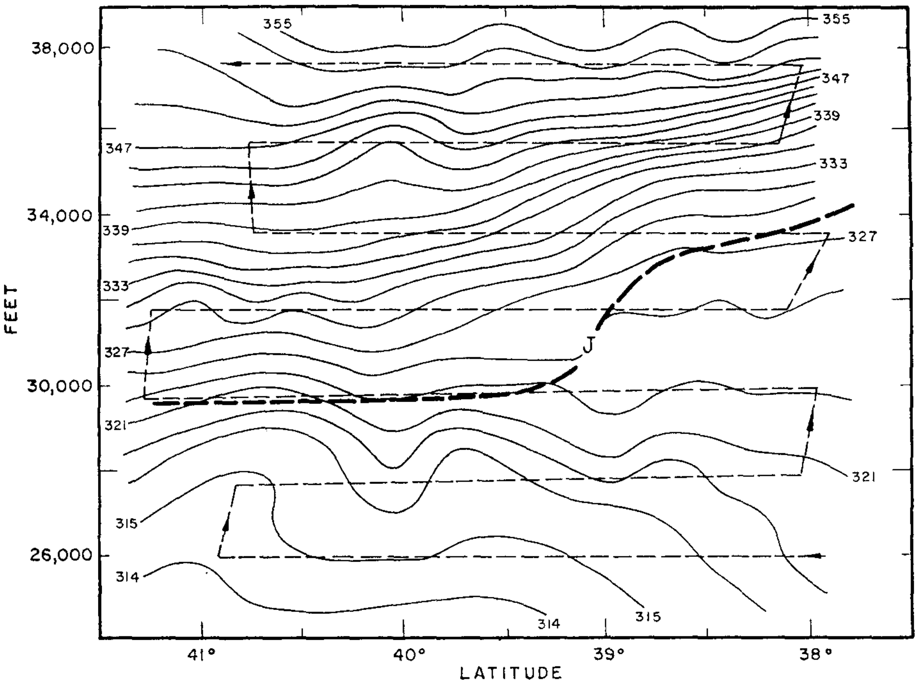

FIG. 5. The distribution of potential temperature (deg K) for Flight No. 19.

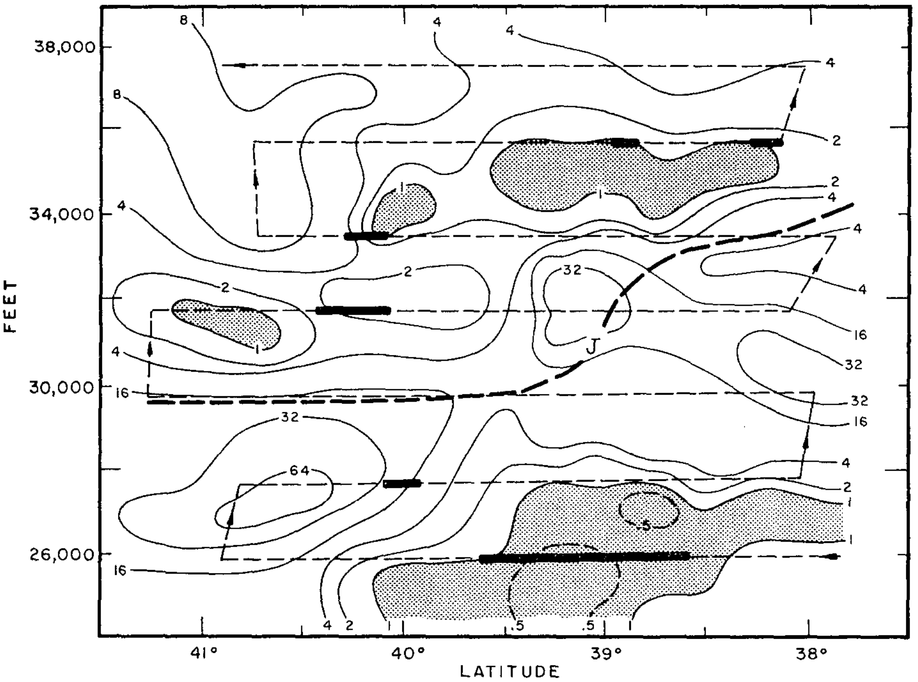

FIG. 6. The distribution of Richardson's number, Ri, for the cross section shown in Fig. 3. Areas with Ri<1 are marked by shading. Heavy solid lines indicate zones with clear-air turbulence which occurred as measured by the VGH instrument.

ton, Ky., at 32,000 ft. The flight consisted of traverses (perpendicular to the winds) along the trough line of a broad upper low, at altitudes ranging from 26,000 to 38,000 ft, thus covering the region from below the maximum wind into the lower stratosphere.

The meso-scale structure of the wind field is shown in Fig. 3. The analysis is based upon data observed at 34-second intervals along the traverses. The solid lines are isotachs of the wind field (knots); the heavy dashed line marks the position of the jet core, and the dashed line indicates the route of the flight. The hatched areas give the meso-scale fluctuations of the longitudinal component of the velocity. The transversal component of velocity is shown in Fig. 4. Fig. 5 shows an analysis of potential temperature obtained from the vortex thermometer of the aircraft. Fig. 6 shows the distribution of Richardson number, Ri. The heavy lines indicate zones with clear-air turbulence as measured by the VGH instrument (Endlich and Rados, 1959). Subjective turbulence reports, made by the crew, have not been entered into this diagram, because they remained below the threshold values of the VGH instrument. Areas with Ri less than one are shaded. From Fig. 6, it can readily be seen that the association of low Ri with clear-air turbulence is good.

3. Velocity correlations

The Eulerian auto-correlation coefficients of the longitudinal and transversal velocities measured by aircraft flying parallel and perpendicular to the mean flow may be expressed by:

$$\mathrm{Ri}(j)=\frac{\overline{V_i'(j_0)V_i'(j_0+j)}}{\overline{V_i'^2(j)}}.$$

In order to arrive at the mean Eulerian space coefficients, use was made of the following space-time transformations:

$$s=[\bar{V}_{gs}-\bar{V}_s(s)]\tau \quad n=[\bar{V}_{gn}-\bar{V}_n(n)]\tau,$$

where $\bar{V}_{gs}$ and $\bar{V}_{gn}$ are the mean groundspeed of the aircraft, respectively, parallel and perpendicular to the jet stream. $\bar{V}_s(s)$ and $\bar{V}_n(n)$ are the mean wind speeds, respectively, parallel and perpendicular to the jet stream. τ is the time interval of observations. The values of $[\bar{V}_{gs}-\bar{V}_s(s)]$ and $[\bar{V}_{gn}-\bar{V}_n(n)]$ were found to be 228.5 and 212.1 m sec^{-1}, respectively. Fig. 7 shows the Eulerian space auto-correlograms $R_s(s)$, $R_n(s)$, $R_s(n)$ and $R_n(n)$. The values of the mean and variance of the velocity deviations are presented in Table 1.

It may be noted that the variance of the horizontal velocities derived from the data of Project Jet Stream shows that, in the case of flights parallel to the flow, the variance of the transversal velocity is almost twice that of the longitudinal velocity. In the case of the flights perpendicular to the flow, the variance of the longitudinal and transversal components are of the same magnitude.

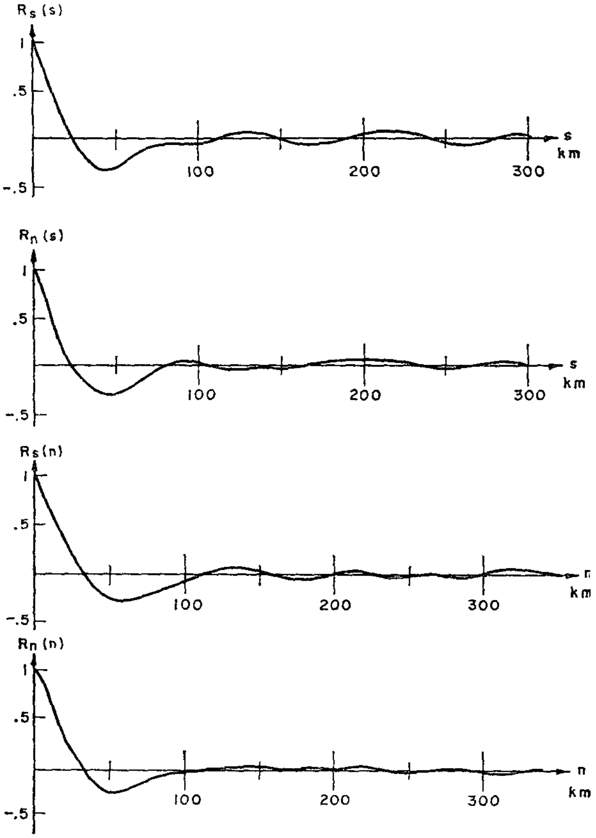

FIG. 7. Mean Eulerian auto-correlation coefficients of the longitudinal and transversal velocity components for flights parallel and perpendicular to the jet stream.

TABLE 1. Means and variances of velocity components.

	Mean velocity (m sec^{-1})	Variance of velocity (m^2 sec^{-2})
$V_s(s)$	59.0	1.00
$V_n(s)$	6.1	1.61
$V_s(n)$	62.3	2.06
$V_n(n)$	6.9	2.03

The values of $R_s(s)$, $R_n(s)$, $R_s(n)$ and $R_n(n)$ were computed for a maximum distance-lag of 652 km (100 intervals of observation), but, due to the rapid damping of the amplitude, only a range of 300 km of the distance-lag is presented in Fig. 7. In the analysis parallel to the jet stream, the horizontal wind velocities were extracted from Flights 14, 15, 16, 22 and 23 (Endlich and Rados, 1959). Flights 15 and 23 were flown with the jet stream, whereas Flights 14, 16 and 22 were flown against the jet stream. In the analysis perpendicular to the jet stream, data were extracted from Flights 13, 17, 19, 20, 25 and 27.

The computations of the overlapping means, deviations, auto-correlation and energy spectra were accomplished with an IBM 1620 at the University of Utah, and with an IBM 7094 at the Western Data Processing Center of the University of California at Los Angeles.

4. Energy spectra

The energy spectra of the longitudinal and transversal components of the horizontal wind velocity parallel and perpendicular to the flow were computed with the use of the following transformation:

$$E_{ij}(k)=\frac{4\overline{V_i'^2}\int_0^\infty R_i(j)\cos 2\pi kj\,dj}{\left(\dfrac{\sin\pi k(\bar{V}_{gj}-\bar{V}_j)\tau}{\pi k(\bar{V}_{gj}-\bar{V}_j)\tau}\right)^2\left[1-\left(\dfrac{\sin\pi k(\bar{V}_{gj}-\bar{V}_j)\tau}{\pi k(\bar{V}_{gj}-\bar{V}_j)\tau}\right)^2\right]},$$

where k denotes the wave number. The meso-scale energy spectra so computed are shown in Figs. 8 and 9. In these figures, a line for k^{-2} is shown for comparison with the spectral distribution for the meso-scale turbulence. The spectral distributions of the kinetic energies for the longitudinal and transversal velocities parallel to the jet stream are similar to those perpendicular to the jet streamline. The kinetic energy generally increases from high to low wave number and may be approximated by k^{-2}. Recent analyses of large-scale disturbances at the 300- and 500-mb levels in the atmosphere show that the energy spectra for hemispheric wave number larger than about 4 (corresponding to a wavelength of 90 degrees in longitude) are proportional to $k^{-7/3}$ (Syōno, Kasahara and Sekiguchi, 1955; Ogura, 1958). It is well known that the theory of isotropic turbulence gives $k^{-7/3}$ for the spectrum of pressure fluctuation, and $k^{-5/3}$ for that of velocity in the inertial range of isotropic turbulence (Ogura and Miyakoda, 1954). It may be noted that the energy spectrum for the meso-scale turbulence in the jet stream appears to be proportional to k^{-2} which is between the above mentioned two values.

It can be shown that the maximum of the error variance will be found at frequencies greater than one half the Nyquist frequency. Since the space interval between observations is 5.81 and 6.56 km for flights parallel and

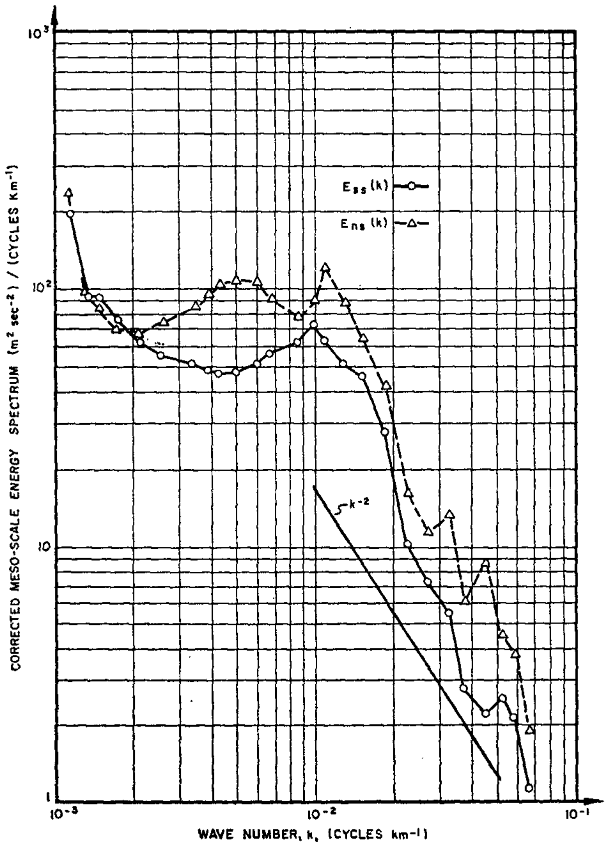

FIG. 8. Corrected meso-scale energy spectra of the longitudinal and transversal velocity components for flights parallel to the jet stream.

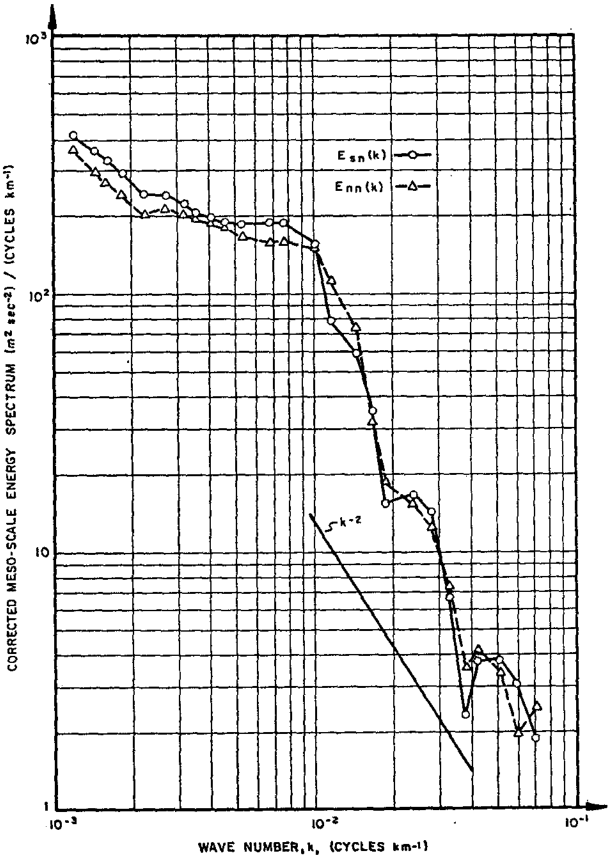

FIG. 9. Corrected meso-scale energy spectra of the longitudinal and transversal velocity components for flights perpendicular to the jet stream.

perpendicular to the jet stream, it may be estimated that these spectral estimates represent real velocity variance up to wave numbers 0.043 and 0.038 cycles km^{-1}.

5. Conclusions and discussion

Analyses of the wind velocity and temperature measured by aircraft flying parallel and perpendicular to jet streams show that a good relationship exists between the areas of turbulence and regions of small Richardson number. The distributions of the auto-correlation coefficients and the energy spectra appear to be similar for both the longitudinal and transversal components of the velocity, and the corrected mesoscale energy spectra are approximately proportional to k^{-2} in the range between 10^{-2} and 10^{-1} cycles km^{-1}.

A recent investigation in Russia (Pinus, 1963) has shown similar results in the energy spectra for large wave number. However, in our study an energy peak was found near 10^{-2} cycles km^{-1} for flight parallel to jet streams, whereas in Pinus's study no energy peak was found. This may be due to the fact that (1) Pinus did not separate the wind data according to flights parallel or perpendicular to jet streams, and (2) Pinus used the exponential approximation of the distribution of the auto-correlation coefficients for the computation of the energy spectra, whereas in this paper we used the actual velocity auto-correlation for the computation.

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APPENDIX

Accuracy of data

Based on a consideration of the calibration techniques, the accuracies of measuring the most important parameters are given below.

	Absolute accuracy	Resolution
Aircraft position	0.7 per cent of the distance from a known starting point	1 mile
Wind speed	3 knots	1 knot
Wind direction	Depends on wind speed. Typical values are 12° at 15 knots decreasing to 1° at 200 knots	1 deg
Vortex temperature	1C	0.1C
Stagnation temperature	1C	0.1C
Pressure altitude	200 ft	10 ft
Groundspeed	0.5 knots	1 knot
True airspeed	2.0 knots	1 knot