

On the Duration of Nonseasonal Temperature Oscillations¹

GUNNAR I. RODEN

Scripps Institution of Oceanography, La Jolla, Calif.

(Manuscript received 8 June 1964, in revised form 23 June 1964)

ABSTRACT

The duration of nonseasonal sea and air temperature oscillations above and below arbitrarily fixed levels is investigated. There is close agreement between the observed mean durations and those expected from theory for a Gaussian random variable. Mean durations of sea temperature anomalies from long term monthly means above and below the 0C level vary between 3 and 5 months, those of air temperature between 2.5 and 3 months. There is almost no dependence upon latitude. The probability distributions are log-normal for durations not exceeding 2 or 3 times the mean duration. The largest observed durations above and below the 0C level are between 1 and 2 years while those above the +1.5C or below the -1.5C levels are between 1 and 2 months.

1. Introduction

The duration of nonseasonal temperature oscillations above and below arbitrarily fixed levels is of importance to both meteorologists and oceanographers, particularly those interested in the interaction between the sea and the atmosphere. Yet, until recently, a rational approach to the problem was made difficult by the lack of a suitably developed theory for obtaining numerical estimates. The break-through came first in communications research. Towards the end of World War II, Rice (1944, 1945) presented an approximate theory for obtaining the distribution of intervals between successive zeros of a stationary, random and Gaussian time series. Subsequent investigators have applied this theory to problems ranging from ocean waves (Longuet-Higgins, 1957a; Ehrenfeld *et al.*, 1958), to irregularities in the ionosphere (Longuet-Higgins, 1957b). Not all the problems of finding the statistical distribution of intervals between successive zeros of a random function have been solved however, and it is of interest to know whether an analysis of the duration of sea and air temperature oscillations can shed some additional light on the problem.

In the following we shall regard the temperature oscillations as belonging to a stationary, random, Gaussian and ergodic process. This implies that oscillations are adequately described by the second order correlation functions and their derivatives and that, by analyzing a finite record length, we are indeed justified to draw conclusions as to the general nature of the temperature fluctuations.

2. Data

The following investigation is based on sea and air temperature records kept by the U. S. Coast and Geodetic Survey (1956, 1962) and the U. S. Weather Bureau (1949-1963), respectively. Sea temperature records for Balboa, Panama Canal Zone, for 1909-1963 were kindly furnished by the Panama Canal Company (unpublished). The locations of the tide gage and meteorological stations are shown in Fig. 1. Monthly mean sea temperatures are obtained from daily observations around 0800. Monthly mean air temperatures represent half the sum of the daily minimum and maximum readings. The accuracy of the monthly means is estimated to be about 0.1C. In all cases the linear trend was removed from the records before proceeding with the analysis.

3. The mean duration of nonseasonal temperature oscillations

Let $\theta(t)$ be a stationary random variable with zero mean and let $\theta'(t)$ and $\theta''(t)$ denote the first and second derivatives of $\theta(t)$. Then, provided the temperature and its derivatives possess a jointly normal distribution, the mean duration of the temperature above an arbitrarily fixed level I (or below $-I$) is given by Rice (1945, 1958), and Longuet-Higgins (1958, 1962a) as:

$$\overline{DUR}(\theta) = \pi \left(\frac{\varphi_0}{\varphi_0'} \right)^{\frac{1}{2}} \cdot \exp(I^2/2\varphi_0) \cdot (1 - P(I/\varphi_0^{\frac{1}{2}})) \quad (1)$$

where \overline{DUR} refers to the mean duration, and φ_0 and φ_0' are the variances of $\theta(t)$ and $\theta'(t)$, respectively. The function $P(x)$ refers to the probability integral:

$$P(x) = \frac{2}{(2\pi)^{\frac{1}{2}}} \int_0^x e^{-t^2/2} dt. \quad (2)$$

¹ Contribution from the Scripps Institution of Oceanography, University of California San Diego.

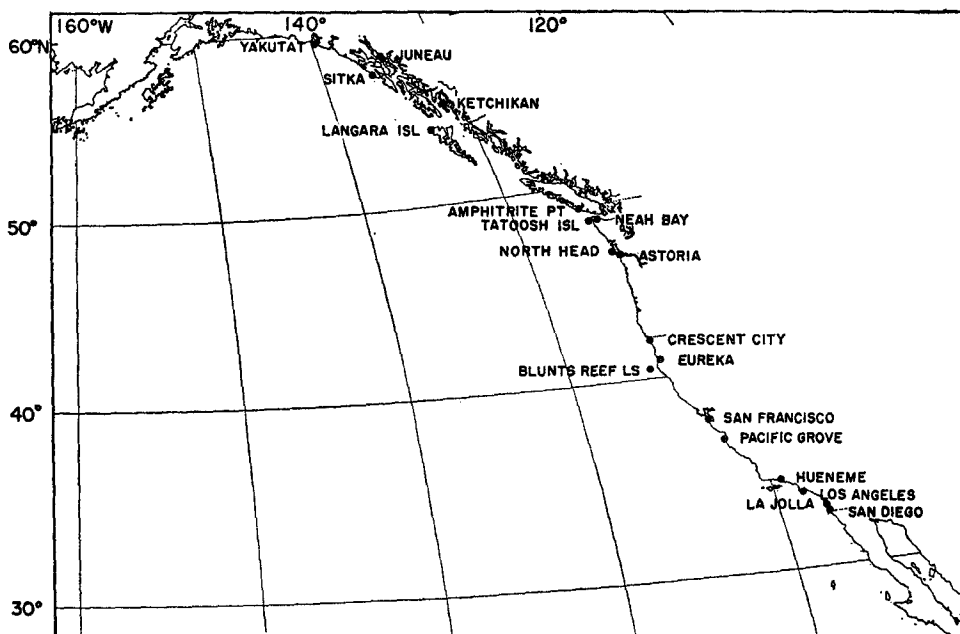


FIG. 1. Locations of the tide gage and meteorological stations employed.

TABLE 1. Observed mean durations (months) of nonseasonal sea surface temperature oscillations (anomalies from long term monthly means).

Station	Above deg C							Below deg C						
	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
Panama, C. Z.														
Balboa	33.9	20.0	7.3	3.0	1.7	1.3	1.2	1.3	1.5	1.6	3.0	6.9	16.9	34.0
California														
La Jolla	26.9	10.3	5.2	3.1	2.6	2.4	2.2	1.4	1.7	2.6	3.3	6.6	13.7	29.1
Los Angeles	18.9	10.2	4.9	3.4	2.6	2.2	1.8	1.5	1.8	2.3	3.8	6.1	12.8	20.5
Hueneme	15.6	9.3	4.2	3.4	2.8	2.3	1.9	1.4	1.9	2.2	4.0	6.4	11.6	17.6
Pacific Grove	48.5	11.7	5.3	3.5	2.4	1.9	1.5	1.1	1.5	2.4	3.9	7.4	11.7	25.0
San Francisco	40.0	12.9	6.2	3.3	2.5	1.8	1.3	1.1	1.6	2.3	3.5	7.6	11.5	24.6
Blunts Reef LS	36.8	10.6	4.4	3.0	2.1	1.5	1.3	1.2	1.4	2.3	3.4	5.5	10.0	24.0
Crescent City	13.3	8.1	6.2	3.3	2.1	1.5	1.3	1.1	2.1	2.8	2.9	4.2	8.7	22.6
Oregon														
Astoria	17.0	9.6	5.2	3.0	2.3	2.0	1.8	1.3	1.5	2.1	3.1	6.3	11.4	19.9
Washington														
Neah Bay	22.6	10.5	6.1	3.1	2.3	2.0	1.7	1.4	2.0	2.7	3.1	5.9	10.0	20.7
British Columbia														
Amphitrite Point	36.4	11.0	6.8	3.7	3.0	2.2	1.4	1.3	1.6	3.0	4.5	7.6	13.4	36.2
Langara Island	51.8	11.0	6.7	4.3	2.8	1.8	1.3	1.0	1.3	2.3	4.5	7.8	15.8	42.7
Alaska														
Ketchikan	43.6	12.5	6.2	3.9	2.3	1.5	1.0	1.1	1.5	2.1	3.9	7.0	19.0	53.6
Sitka	33.1	18.0	9.6	4.7	2.7	1.8	1.0	1.1	2.0	3.0	4.5	7.3	15.4	59.3
Juneau	26.8	11.7	5.8	3.4	2.2	1.5	1.0	1.2	1.5	2.1	3.4	5.7	12.4	27.0
Yakutat	22.6	15.4	6.0	3.3	2.9	1.9	1.5	1.4	2.2	2.5	3.6	6.8	11.3	20.5

Similarly, the mean duration of the temperature gradient θ' above an arbitrarily selected level J (or below $-J$) is given by the above mentioned authors as:

$$\overline{DUR}(\theta') = \pi \left(\frac{\varphi_0'}{\varphi_0''} \right)^{\frac{1}{2}} \cdot \exp(J^2/2\varphi_0') \cdot (1 - P(J/\varphi_0'^{\frac{1}{2}})), \quad (3)$$

where φ_0'' is the variance of $\theta''(t)$ and the other symbols retain their previous meaning.

If $I=0$ and $J=0$, equations (1) and (3) take the simple form:

$$\overline{DUR}(\theta) = \pi \left(\frac{\varphi_0}{\varphi_0'} \right)^{\frac{1}{2}}, \quad (4)$$

and

$$\overline{DUR}(\theta') = \pi \left(\frac{\varphi_0'}{\varphi_0''} \right)^{\frac{1}{2}}. \quad (5)$$

If $I \gg -\varphi_0^{\frac{1}{2}}$ and $J \ll -(\varphi_0')^{\frac{1}{2}}$

$$\overline{DUR}(\theta) \rightarrow 2\pi (\varphi_0/\varphi_0')^{\frac{1}{2}} \exp(I^2/2\varphi_0); \quad (6)$$

$$\overline{DUR}(\theta') \rightarrow 2\pi (\varphi_0'/\varphi_0'')^{\frac{1}{2}} \exp(J^2/2\varphi_0').$$

If $I \gg \varphi_0^{\frac{1}{2}}$ and $J \gg (\varphi_0')^{\frac{1}{2}}$

$$\overline{DUR}(\theta) \rightarrow \frac{\varphi_0}{I} (2\pi/\varphi_0')^{\frac{1}{2}}; \quad (7)$$

$$\overline{DUR}(\theta') \rightarrow \frac{\varphi_0'}{J} (2\pi/\varphi_0'')^{\frac{1}{2}}.$$

The latter asymptotic expression follows from

$$1 - P(x) \rightarrow \frac{2}{(2\pi)^{\frac{1}{2}}} \frac{e^{-x^2/2}}{x} \quad \text{for } x \gg 1. \quad (8)$$

The physical interpretation of the asymptotic expressions (6) and (7) is as follows: if the fixed levels I or J are placed sufficiently low, most of the values of the random variable θ or θ' will lie above these levels, and the mean duration becomes very large (infinite in the limit); on the other hand, if the fixed levels I or J are placed sufficiently high, most values of θ and θ' will lie below these levels and the mean duration above will become very short (zero in the limit).

It is of interest to note that equations (1) to (8) are completely determined by the autocorrelations (variances) of the random variable, and its derivatives.

As a rather trivial but useful example, assume that the temperature θ varies from its annual mean value as

$$\theta(t) = \theta_0(t) \cos(2\pi t/T + \epsilon), \quad (9)$$

where $\theta_0(t)$ is the amplitude, varying slowly with time (and having as its rms value $\bar{\theta}_0$), T is the period (here

taken as 12 months), t is time, and ϵ is a phase lag. What is the mean duration of the temperature above its annual mean value, and what is that of the positive temperature gradient? Knowing that the autocorrelations of temperature and its derivatives at lag zero, are, respectively:

$$\varphi_0 = \bar{\theta}_0^2/2; \quad \varphi_0' = \left(\frac{2\pi}{T} \right)^2 \varphi_0; \quad \varphi_0'' = \left(\frac{2\pi}{T} \right)^4 \varphi_0;$$

we find easily that $\overline{DUR}(\theta) = 6$ months, and $\overline{DUR}(\theta') = 6$ months, as it should be.

To estimate the mean duration of nonseasonal temperature oscillations, such as that of temperature anomalies from long term monthly means, is more difficult. Here, the answer cannot be anticipated as in the above example, but must instead be derived from the actual observations. As the theory applies only to stationary functions, it is important to eliminate any inherent trends from the records by the method of least squares or any other suitable means, before proceeding with the analysis. In the subsequent computations the temperature gradients were obtained from the finite-difference approximations

$$\theta(t) = \theta_{t+1} - \theta_t; \quad \theta''(t) = \theta_{t+2} - 2\theta_{t+1} + \theta_t, \quad (10)$$

and the variances were computed from the autocorrelations of $\theta(t)$, $\theta'(t)$ and $\theta''(t)$:

$$\begin{aligned} \varphi(\tau) &= \langle \theta(t)\theta(t+\tau) \rangle; & \varphi'(\tau) &= \langle \theta'(t)\theta'(t+\tau) \rangle; \\ \varphi''(\tau) &= \langle \theta''(t)\theta''(t+\tau) \rangle \end{aligned} \quad (11)$$

at $\tau=0$. The brackets $\langle \rangle$ refer to the mean over the entire record length.

a. Mean durations of nonseasonal sea surface temperature oscillations. The observed mean durations above and below a few selected levels are shown in Table 1 for several stations along the west coast of North America. The mean duration of the nonseasonal oscillations (henceforth referred to briefly as anomalies) above and below the 0C level varies generally between 3.0 and 4.5 months. There is surprisingly little change with latitude, indicating that the processes affecting the duration occur on a large scale. It is also seen that large anomalies, on the average do not last long. The mean duration of anomalies above +1C and below -1C is, for example, only of the order of 2 months.

The fact that the durations come out in months is not an accident; it is largely due to having used monthly means as the basic data. Had hourly means been used, the answer would have been in hours. For yearly means, the durations would have been in years. This is the effect of smoothing upon the persistence (Munk, 1960). We are here interested in time scales in the order of months or, more precisely, in the month-to-month duration of monthly means.

The agreement between the observed and calculated

mean durations is shown in Fig. 2. The dots refer to the observation, while the curves were obtained from the theoretical relationship expressed by equation (1). Note that the theoretical relationship is the same for the mean durations above and below fixed levels, which is a consequence of the assumption that the temperature and its derivatives possess a jointly normal distribution (Rice, 1945). The agreement between theory and observation is usually good. Deviations do, however, occur at the

far ends of the curves shown here. They may indicate departures from the Gaussian assumption upon which the theory rests. The decrease of the mean duration with increase of the magnitude of the temperature anomaly is rather rapid. Thus, at San Francisco, the mean duration above -1°C is of the order of 13 months, while that above $+1^{\circ}\text{C}$ is only 1.8 months.

b. Mean duration of nonseasonal sea temperature gradient oscillations. The gradients here referred to are those

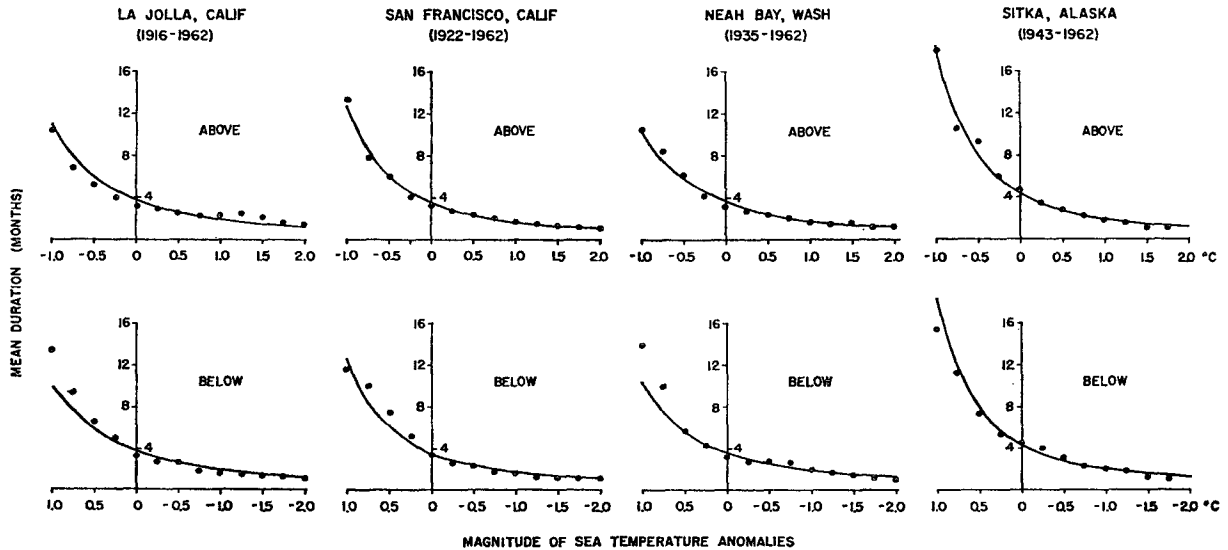


FIG. 2. Agreement between observed and calculated mean durations of sea temperature anomalies. Dots indicated observed values, and curves were calculated from equation (1).

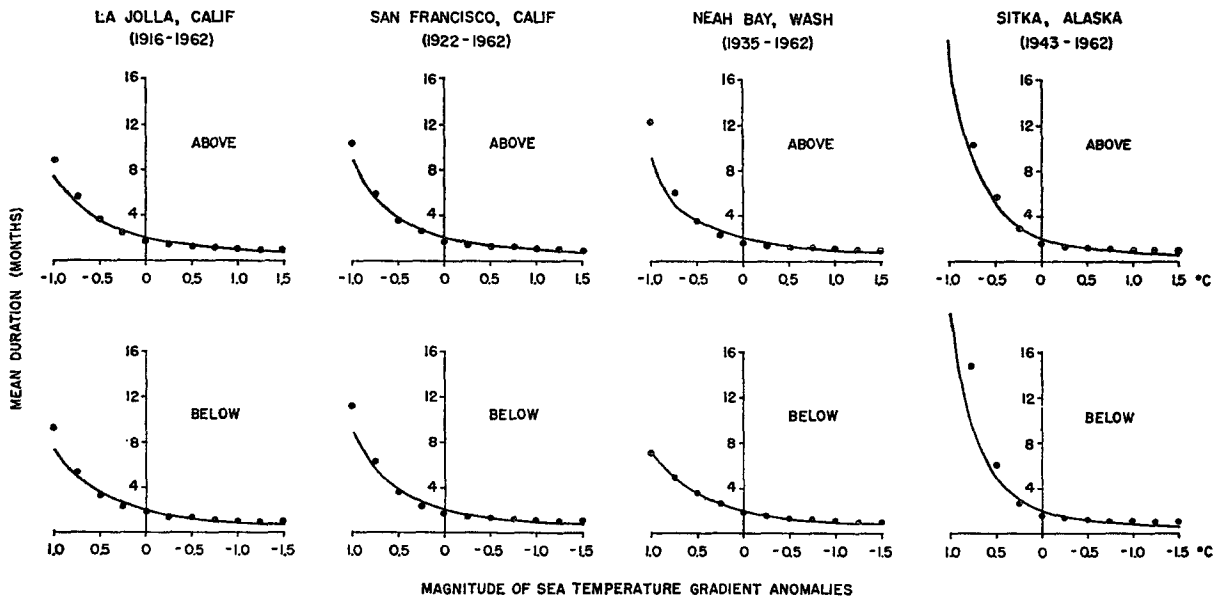


FIG. 3. Agreement between observed and calculated mean durations of sea temperature gradient anomalies. Dots indicate observed values, and curves were calculated from equation (3).

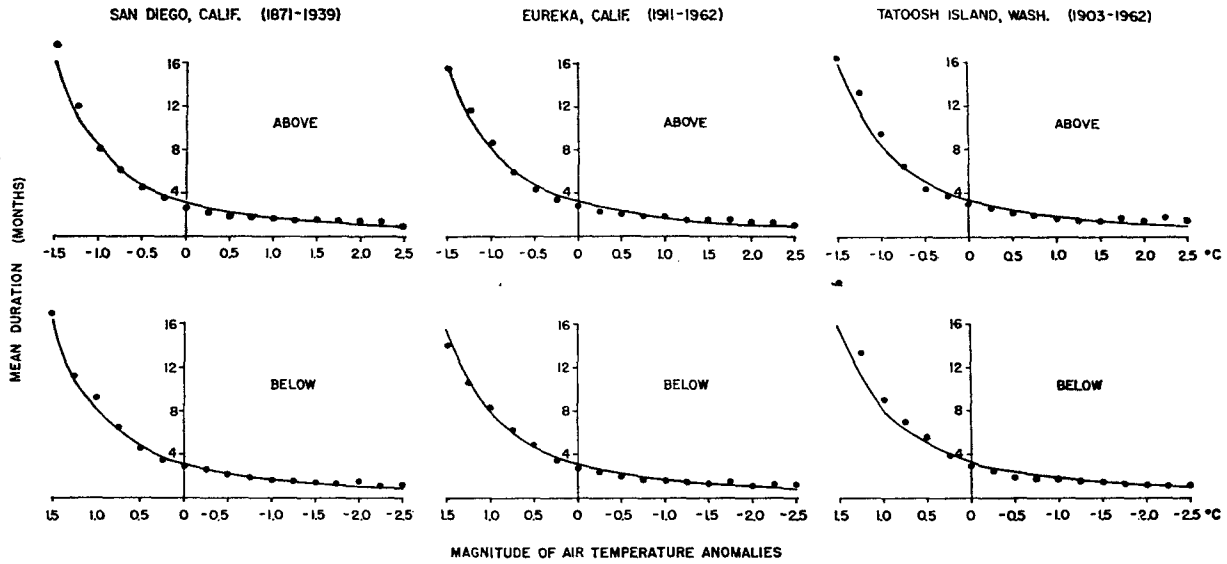


FIG. 4. Agreement between observed and calculated mean durations of air temperature anomalies. Dots indicated observed values, and curves were calculated from equation (1).

TABLE 2. Observed mean durations (months) of nonseasonal air temperature oscillations (anomalies from long term monthly means).

Station	Above deg C							Below deg C						
	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
California														
San Diego	17.7	8.0	4.5	2.8	2.0	1.7	1.6	1.4	1.7	2.1	2.8	4.6	9.2	16.9
San Francisco	12.8	6.8	4.2	2.7	2.0	1.5	1.4	1.2	1.6	2.1	2.6	4.0	7.7	15.6
Eureka	15.5	8.5	4.3	2.8	2.1	1.8	1.5	1.4	1.6	2.0	2.9	4.9	8.3	14.1
Washington														
North Head	12.0	8.1	4.1	2.8	2.4	1.8	1.7	1.4	1.7	1.9	2.8	4.8	8.1	13.3
Tatoosh Island	16.3	9.4	4.3	3.0	2.2	1.7	1.7	1.3	1.7	1.9	3.0	5.6	9.0	20.8

of time ($C \text{ month}^{-1}$). The agreement between theory and observation is shown in Fig. 3. The curves were obtained from equation (3). As in the previous case, the observations conform rather closely to the theoretical relationship. The mean duration of gradient anomalies above and below $0C \text{ mo}^{-1}$ varies from 1.5 to 2.0 months. Anomalies above $+1C \text{ mo}^{-1}$ and below $-1C \text{ mo}^{-1}$ last only one month on the average. The physical interpretation is that large positive and negative anomalies of the temperature gradient last merely a short time.

c. Mean duration of nonseasonal air temperature oscillations. The observed mean durations above and below selected levels are summarized in Table 2 for coastal first order meteorological stations in California and Washington.² The mean durations of air temperature

anomalies above and below the $0C$ level vary between 2.6 and 3.0 months. There is almost no dependence upon latitude, which seems to indicate that physical processes affecting the duration occur on a large scale.

The relation between the observed mean durations and those calculated from (1) are shown in Fig. 4. The agreement between theory and observation is good, as in the previous cases. The duration of large anomalies, such as those above $+2C$ and below $-2C$ is one month, on the average. On the whole, the mean durations of air temperature anomalies seem to be of the same order of magnitude, or slightly less, as those of the sea surface temperature in the vicinity of the meteorological station.

4. The probability distribution of the duration of nonseasonal temperature oscillations

The mathematical calculation of the probability distribution presents considerable difficulties. No closed form expressions are known to exist for the distribution

² For Tatoosh Island and North Head, Wash., the entire record was used. For Eureka, Calif., the record for the homogeneous period 1911-1962 was considered. For San Francisco and San Diego, Calif., the periods before the rapid growth of the cities were selected, extending from 1871-1935, and from 1871-1939, respectively.

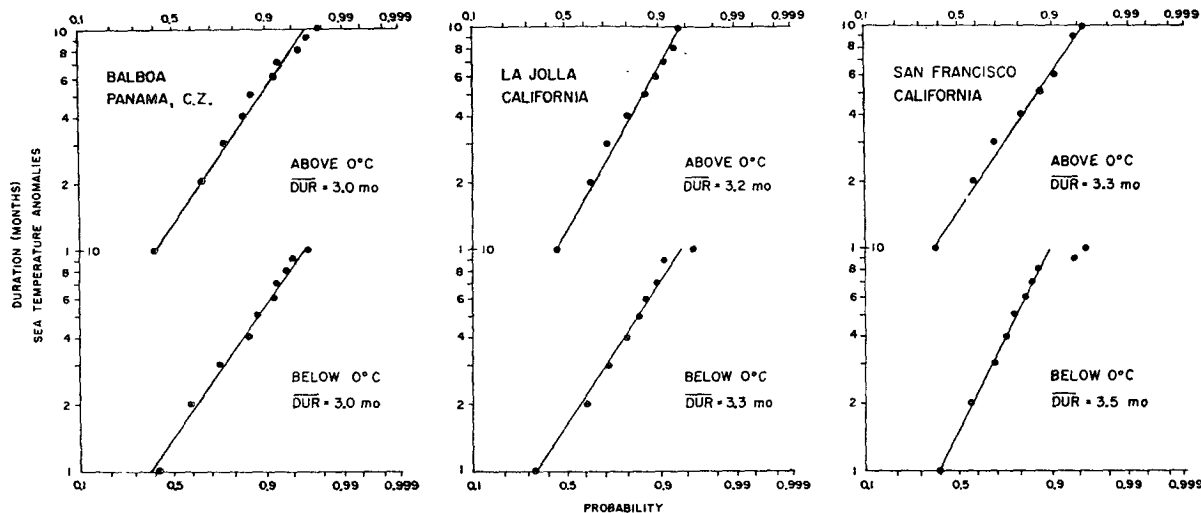


FIG. 5. Probability of the duration of nonseasonal sea temperature oscillations. Ordinate shows duration on a logarithmic scale, and abscissa indicates probability.

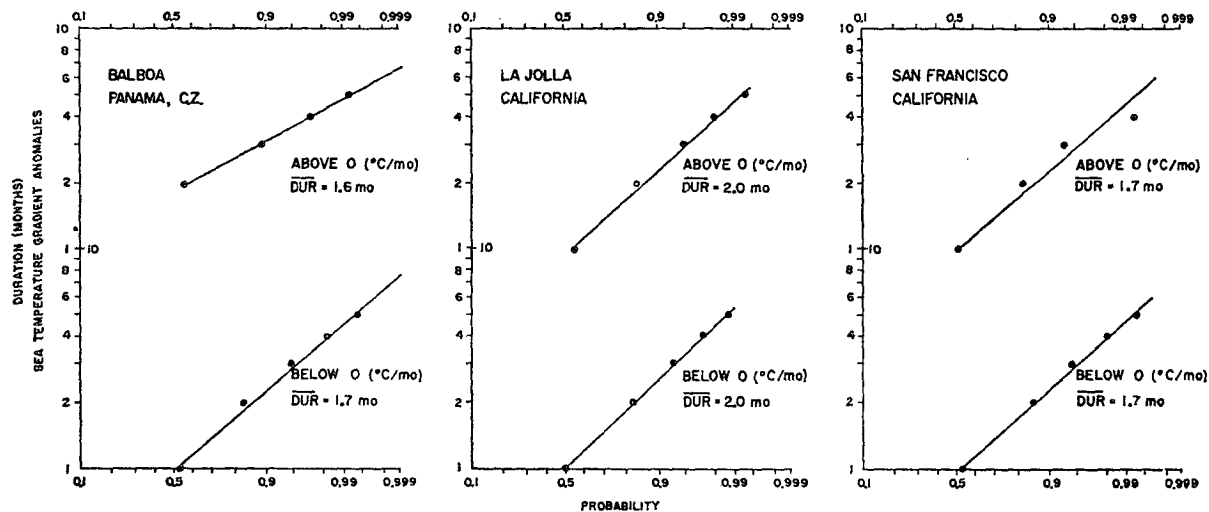


FIG. 6. Probability of the duration of the gradient of nonseasonal sea temperature oscillations. Ordinate shows duration on a logarithmic scale, and abscissa indicates probability.

of the duration. In a few cases, asymptotic forms have been derived by a method of successive approximation (Longuet-Higgins, 1958). Most of the results are valid only for stationary, Gaussian random variables having a narrow power spectrum. More recently, Longuet-Higgins (1962b) derived exact upper and lower bounds to the probability functions of random variables with prescribed power spectra, not necessarily narrow. He was able to prove rigorously, that for a Gaussian process the complete distribution function cannot be of the type of a simple negative power law, as has been claimed previously by Favreau *et al.* (1956), on the basis of incomplete data. It is, however, generally believed that for *very* large durations, the probability decreases asymptotically as a negative exponential (Kuznetsov *et al.*, 1954; Rice, 1958).

In the present case, the analysis is based on discrete data, rather than on continuous functions, and the power spectra of the nonseasonal temperature oscillations are wide rather than narrow (Roden, 1960, 1962). Moreover, the durations that are of interest here are neither very small nor very large. This makes a comparison with the hitherto developed theoretical approximations difficult. The following discussion is therefore based on empirical findings entirely.

a. The probability of the duration of nonseasonal sea temperature oscillations. The cumulative probability functions for the durations above and below the 0°C level are shown in Fig. 5. The ordinate shows the duration on a logarithmic scale, and the abscissa indicates probability. The dots refer to the observations, and the straight line has been fitted to them by eye. The out-

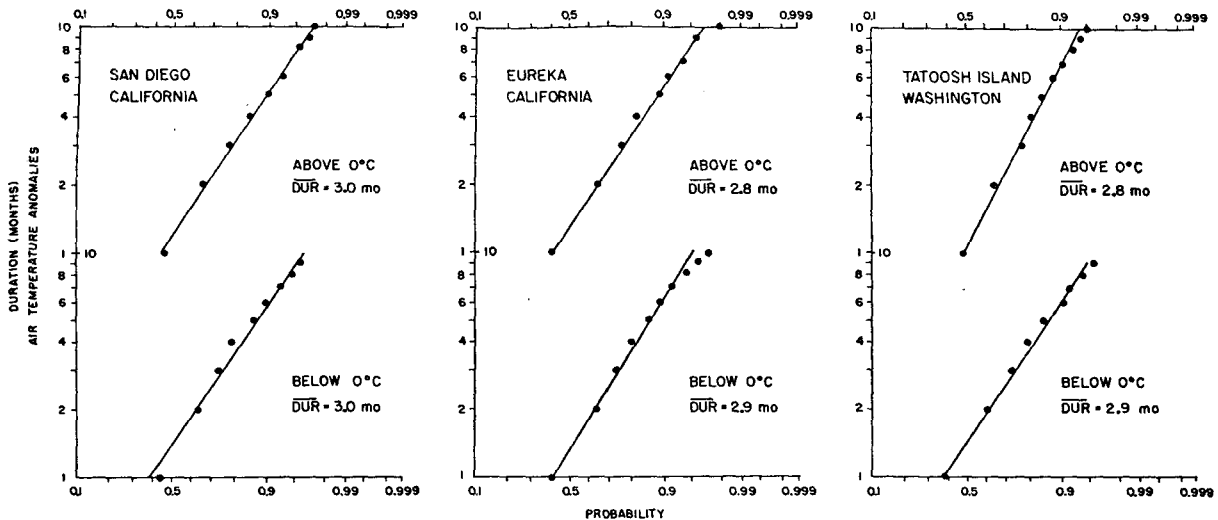


FIG. 7. Probability of the duration of nonseasonal air temperature oscillations. Ordinate shows duration on a logarithmic scale, and abscissa indicates probability.

standing feature is that for durations up to 2 or 3 times the mean duration, the cumulative probability function is approximately log-normal. This relation does not necessarily hold for durations in excess of 2 or 3 times the mean durations, as a careful inspection of the figure will indicate. To illustrate the probabilities involved, consider the duration of positive sea temperature anomalies at La Jolla, Calif.; here the probability that the anomaly will last 3 months or more is approximately 0.25, 7 months or more, 0.1, and 11 months or more, about 0.05.

b. The probability of the duration of the gradient of nonseasonal sea temperature oscillations. The cumulative probability functions are shown in Fig. 6. They are approximately of a log-normal type. Durations above 6 months have not been observed for the cases shown here. The probability that a gradient anomaly will last for a long time is rather small. At La Jolla, Calif., for example, the probability is only 0.02 that such an anomaly will last 4 months or more.

c. The probability of duration of nonseasonal air temperature oscillations. The relevant cumulative probability functions are shown in Fig. 7. They are approximately log-normal for durations not exceeding 2 or 3 times the mean duration. For the stations considered here, the following probabilities are characteristic for the duration of positive and negative air temperature anomalies (above and below the 0°C level): The probability that the anomalies will last for 3 months or more is about 0.25, for 6 months or more, about 0.10, and for 9 months or more, roughly 0.04.

5. Extreme durations of nonseasonal temperature oscillations

The largest observed durations of sea and air temperature anomalies above and below selected levels are sum-

marized in Tables 3 and 4, respectively. It is seen that the largest durations above and below the 0°C level are of the order of 1 to 2 years. Those above the +1°C level and below the -1°C vary mostly between 4 and 10 months, and those above the +1.5°C level and below the -1.5°C level have characteristic magnitudes between 1 and 2 months. The physical implication of this is that even the extreme durations of large positive and negative sea and air temperature anomalies are short.

6. Conclusions

The following conclusions can be drawn from an analysis of the duration of nonseasonal sea and air temperature oscillation (anomalies from long term monthly means):

- 1) There is close agreement between the observed mean durations and those expected from theory for a stationary Gaussian random variable.
- 2) the mean duration of sea temperature anomalies above and below the 0°C level varies between 3 and 5 months. Anomalies above +1°C and below -1°C last between 1.5 and 2.0 months.
- 3) the mean duration of anomalies of the sea temperature gradient above and below the 0°C mo^{-1} level varies between 1.5 and 2.0 months.
- 4) the mean duration of air temperature anomalies above and below the 0°C level falls between 2.5 and 3.0 months. Anomalies above +1°C and below -1°C last between 1.5 and 2.0 months.
- 5) The probability distributions for the duration of sea and air temperature anomalies are log-normal for durations not exceeding 2 or 3 times the mean duration. For longer durations this relationship does not necessarily hold.

TABLE 3. Observed extreme durations (months) of nonseasonal sea surface temperature oscillations (anomalies from long term monthly means).

Station	Above deg C							Below deg C						
	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
Panama, C. Z.														
Balboa	71	67	31	29	5	4	3	3	5	9	20	35	96	129
California														
La Jolla	104	51	22	17	16	11	6	4	6	14	19	31	60	158
Los Angeles	95	79	32	30	16	9	8	4	5	13	19	26	72	115
Hueneme	70	47	26	26	17	12	6	4	7	13	18	32	56	94
Pacific Grove	167	38	35	26	12	10	5	2	4	11	23	28	63	189
San Francisco	141	64	36	26	12	5	3	2	5	8	14	36	49	85
Blunts Reef LS	151	48	18	13	7	5	3	2	4	8	12	14	31	54
Crescent City	43	26	24	15	5	5	3	3	5	12	10	20	32	77
Oregon														
Astoria	74	58	26	25	12	7	6	3	5	10	14	29	44	67
Washington														
Neah Bay	86	37	31	12	10	7	4	3	5	8	16	28	35	60
British Columbia														
Amphitrite Point	72	40	34	17	14	10	3	3	5	16	26	40	79	186
Langara Island	142	42	35	20	8	5	3	2	4	12	15	53	103	187
Alaska														
Ketchikan	151	71	36	27	9	4	2	3	6	9	16	23	76	119
Sitka	82	73	41	22	13	9	1	2	8	10	28	47	73	176
Juneau	82	39	21	11	6	5	2	2	5	7	12	16	39	115
Yakutat	81	59	29	19	10	7	3	2	8	9	17	29	53	113

TABLE 4. Observed extreme durations (months) of nonseasonal air temperature oscillations (anomalies from long term monthly means).

Station	Above deg C							Below deg C						
	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
California														
San Diego	62	33	30	24	10	10	7	5	8	12	14	26	45	83
San Francisco	70	58	49	21	7	7	4	3	6	10	12	20	66	101
Eureka	94	66	46	11	10	6	5	6	6	11	19	24	29	67
Washington														
North Head	54	46	26	16	12	8	7	4	5	9	15	23	54	54
Tatoosh Island	71	49	23	21	10	7	6	3	8	11	15	29	56	108

- 6) the largest observed durations of sea and air temperature anomalies above and below the 0C level are of the order of 1 to 2 years, while those above the +1.5C and below the -1.5C levels are between 1 and 2 months.
- 7) there is close agreement between the durations of nonseasonal sea and air temperature oscillations.

Acknowledgments. The author has benefitted from many discussions with D. E. Cartwright and is also indebted to G. W. Groves, J. D. Isaacs, W. H. Munk, and J. L. Reid, Jr., for suggestions and advice. The computations were carried out by Terry Garate and by the UCSD Computer Center.

The investigation reported herein represents one of

the results conducted under the Marine Life Research Program, which is the Scripps Institution of Oceanography's part of the California Cooperative Oceanic Fisheries Investigations, a project sponsored by the Marine Life Research Committee of the State of California.

REFERENCES

Ehrenfeld, S., *et al.*, 1958: Theoretical and observed results for the zero and ordinate crossing problems. *Tech. Rep. N. Y. Coll. Eng.*, 76 pp.
 Favreau, R. R., *et al.*, 1956: Evaluation of complex statistical-functions by an analog computer. *Inst. Rad. Eng. Nat. Convention Rec.*, pt. 4, 31-37.
 Kuznetsov, P. L., *et al.*, 1954: On the duration of exceedences of a random function. *J. Tech. Phys. Moscow*, 24, 103-112.

- Longuet-Higgins, M. S., 1957a: The statistical analysis of a random moving surface. *Phil. Trans. Roy. Soc. London, A*, **249**, 321-387.
- , 1957b: A statistical distribution arising in the study of the ionosphere. *Proc. Phys. Soc. B*, **70**, 559-565.
- , 1958: On the intervals between successive zeros of a random function. *Proc. Roy. Soc. A*, **246**, 99-118.
- , 1962a: On the statistical geometry of a random surface. *Proc. Symp. Appl. Math.*, **13**, 105-143.
- , 1962b: Intervals between zeros of a random function. *Phil. Trans. Roy. Soc. London, A*, **254**, 557-559.
- Munk, W. H., 1960: Smoothing and persistence. *J. Meteor.*, **17**, 92-93.
- Rice, S. O., 1944: The mathematical analysis of random noise. *Bell Syst. Tech. J.*, **23**, 282-332.
- , 1945: The mathematical analysis of random noise. *Bell Syst. Tech. J.*, **24**, 46-156.
- , 1958: Distribution of the duration of fades in radio transmission. Gaussian noise model. *Bell Syst. Tech. J.*, **37**, 581-635.
- Roden, G. I., 1960: On the statistical prediction of ocean temperatures. *J. geophys. Res.*, **65**, 249-263.
- , 1962: On sea surface temperature, cloudiness and wind variations in the tropical Atlantic. *J. atmos. Sci.*, **19**, 66-80.
- U. S. Coast and Geodetic Survey, 1956: Surface water temperatures at tide stations. Pacific coast of North and South America and Pacific Ocean Islands. *Sp. Publ. 280*, Govt. Printing Office, Washington, D. C., 74 pp.
- , 1962: Surface water temperature and salinity. Pacific coast of North and South America and Pacific Ocean Islands. *Publ. 31-3*, Govt. Printing Office, Washington, D. C., 74 pp.
- U. S. Weather Bureau, 1949-1963: Local climatological data with comparative data for San Diego, San Francisco, Eureka, Calif., and North Head and Tatoosh Island, Wash. Govt. Printing Office, Washington, D. C.