Analyses of Isotropic and Anisotropic Turbulent Dispersion of Particles in the Atmosphere[1]

S.-K. KAO AND B. L. NIEMANN
University of Utah

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ABSTRACT

Analyses of the isotropic and anisotropic diffusion of clusters of fluid particles in the atmosphere are made. Lin's theory of diffusion is subjected to an observational test. The autocorrelation coefficients of the vector acceleration are computed and the value of B_0 in Lin's one particle theory is found to be 3 and 5 cm² sec⁻³ for two series of experiments. The mean autocorrelation coefficients of the vector relative acceleration are computed and the value of B in Lin's isotropic diffusion theory is found to be 68 and 28 $cm^2\ sec^{-3}$ for two series of experiments. The horizontal eddy diffusivities for the isotropic diffusion process are found to range from 0.4×10^3 to 6.4×10^3 $cm^2\ sec^{-1}$. The mean cross-covariance coefficient of the relative acceleration is computed and the value of B_{ij} in Lin's anisotropic diffusion theory is found to be 30 $cm^2\ sec^{-3}$. The values of the diffusion tensor for the anisotropic diffusion process are found to range from 0.3×10^3 to 1.8×10^3 $cm^2\ sec^{-1}$.

1. Introduction

Since the publication of Taylor's pioneering work on the statistical theory of turbulence (1921, 1935), Richardson's law of turbulent diffusion (1926), and Kolmogoroff's similarity theory of turbulence (1941), many advances in the study of atmospheric turbulence and diffusion have been made. Comprehensive reviews of the advances in microscale turbulence and diffusion may be found in the two recent monographs (Lumley and Panofsky, 1964; Pasquill, 1962).

Lin (1960a,b) has recently made an extensive study of the process of anisotropic diffusion which may serve as a basic framework for the analysis of atmospheric diffusion where the horizontal coefficient of diffusion may be many times larger than that in the vertical. Furthermore, Lin's theory helps to understand the mechanism for Richardson's law, and its relationship to Kolmogoroff's theory.

Analysis of the motion of a cluster of fluid particles in the Lagrangian system is fundamental to the understanding of turbulent diffusion in the atmosphere. Such an analysis requires information about the trajectories of many fluid particles, which is available from a recent experiment (Edinger, 1953) for the microscale in the atmosphere. The purposes of this paper are to make such an analysis and to subject Lin's theory of diffusion by continuous movements to an observational test.

2. Data

The data used in this study are obtained from two experiments (Edinger, 1953) designed to observe directly the trajectories of many individual particles diffusing near the 500-ft level in the atmosphere. The particle motions at this level should result from atmospheric turbulence that has a dynamic rather than a surface frictional origin.

The tracer used in these experiments was a soap bubble approximately six centimeters in diameter. The soap bubbles were emitted from a small bubble generator suspended one hundred feet below a large captive balloon. A high-magnification motion picture camera located on the ground recorded the positions of the bubbles every two-tenths of a second as they drifted through the field of view vertically above. The field of view was a square, 42 feet on each side. Since only one camera was used, only the horizontal projections of the bubble trajectories were obtained.

The site of the two experiments was War Eagle Field in the Mojave Desert of California. The surface of War Eagle Field was flat and overgrown with brush that averaged two to three feet in height. Throughout these experiments, active convection typical of midday in the desert indicated a slightly superadiabatic lapse rate. Edinger characterized Run 02 as an example of small-scale motions and Run 03 as an example of comparatively large-scale motions involved in the diffusion of the bubbles. In the two experiments, the bubbles were emitted from the bubble generator in groups, with vacant intervals between adjacent groups. Two of the largest groups with the longest bubble trajectories were selected in both Run 02 and 03. A summary of the information concerning the two experiments is presented in Table 1.

In Table 1, the numbers in the parentheses indicate the number of bubbles.

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TABLE 1. Summary of experiments.

Run	Time of run	Height of bubble release	Duration of run	First cluster of bubbles	Second cluster of bubbles
02	11:10 a.m.	866 ft	26.2 sec	No. 52–62 (10) (No. 53 missing)	No. 64–72 (9)
03	11:27 a.m.	568 ft	35.0 sec	No. 59–68 (10)	No. 69–77 (9)

An important consideration of observational error in these experiments is the accuracy with which the horizontal components of the bubble's positions were determined in space. Edinger estimates that the bubble's positions in space were determined with an uncertainy of two to three centimeters.

Since the horizontal velocities and accelerations computed every two-tenths of a second are very sensitive to the error which is possible in the coordinate positions, it was found necessary to smooth the coordinate positions by a "running average" over a one-second interval. This "running average" has an effect of smoothing out the random errors in the bubble's coordinate positions.

3. One particle analysis: Vector acceleration correlation

The analysis of turbulence and diffusion is concerned primarily with the statistical characteristics of the turbulent motion. To obtain the statistical properties of the turbulent flow, an averaging procedure with respect to a large number of particles is usually made. To make such an averaging procedure, the field of turbulent motion relative to the center of mass of the particles will be assumed to be statistically homogeneous and stationary. Mean values of the turbulent quantities will be regarded as stochastic averages, i.e., as averages over a large number of independent experiments or trials under similar conditions. The vector mean and vector turbulent velocity in this study are defined respectively:

$$\bar{\mathbf{v}}(t;T)=\frac{1}{T}\int_{t-T/2}^{t+T/2}\mathbf{v}(t)dt \tag{1}$$

$$\mathbf{v}'(t;T)=\mathbf{v}(t)-\bar{\mathbf{v}}(t;T), \tag{2}$$

where $T=3$ seconds was chosen.

Considering the motion of a single particle released in a stationary homogeneous field of turbulent motion, Lin (1960a) has applied a similar approach as Taylor (1921) to determine the velocity correlation of this same particle at an interval of time τ, apart. Since the velocity $\mathbf{v}(t)$ may be obtained by the integration of the acceleration $\mathbf{a}(t)$, we have:

$$\mathbf{v}(t+\tau)-\mathbf{v}(t)=\int_{t}^{t+\tau}\mathbf{a}(t)dt \tag{3}$$

and

$$\mathbf{v}'(t+\tau)-\mathbf{v}'(t)=\int_{t}^{t+\tau}\mathbf{a}(t)dt \tag{4}$$

using Eq. (2) for $\mathbf{v}'(t+\tau)$ an $\mathbf{v}'(t)$. Here we assume that $\bar{\mathbf{v}}(t+\tau;T)=\bar{\mathbf{v}}(t;T)$ for appropriately small values of τ. From Eq. (4), Lin has shown by straightforward derivation that

$$\overline{|\mathbf{v}'(t+\tau)-\mathbf{v}'(t)|^2}=2B_0\tau \quad \text{for} \quad \tau\gg\tau_1, \tag{5}$$

where

$$B_0=\overline{|\mathbf{a}|^2}\int_0^{\tau_1}R_{\mathbf{a}}(\tau)d\tau \tag{6}$$

and

$$R_{\mathbf{a}}(\tau)=\frac{\overline{\mathbf{a}(t+\tau)\cdot\mathbf{a}(t)}}{\overline{|\mathbf{a}|^2}}. \tag{7}$$

Eq. (5) has been written down by dimensional arguments, based on Kolmogoroff's similarity theory of turbulence (1941), with B_0 replaced by ϵ (the average rate of dissipation of turbulent energy per unit mass of fluid), and with an indefinite numerical constant. In Lin's theory, the Lagrangian parameter B_0 is of the same dimensions as the rate of energy dissipation ϵ, but is defined by a different physical process in Eq. (6).

Eq. (5) implies that τ, while large compared with τ_1, must be appropriately small in a certain sense. This is exactly the kind of requirement found in Kolmogoroff's theory. Another requirement of Kolmogoroff's theory for Eq. (5) to be valid, is an extermely large Reynold's number of turbulence. In this analysis, we are able to examine whether this requirement is necessary for Lin's derivation to be valid.

Expanding (5) and assuming that the mean level of turbulent energy is constant, $\overline{|\mathbf{v}'|^2}(t+\tau)=\overline{|\mathbf{v}'|^2}(t)$, for sufficiently large τ, it can be shown that:

$$R_{\mathbf{v}}(\tau)=\frac{\overline{\mathbf{v}'(t)\cdot\mathbf{v}'(t+\tau)}}{\overline{|\mathbf{v}'|^2}}=1-\frac{B_0\tau}{\overline{|\mathbf{v}'|^2}} \tag{8}$$

or

$$B_0=\frac{\overline{|\mathbf{v}'|^2}}{\tau}[1-R_{\mathbf{v}}(\tau)]. \tag{9}$$

Thus, the value of B_0 can be determined in two ways,

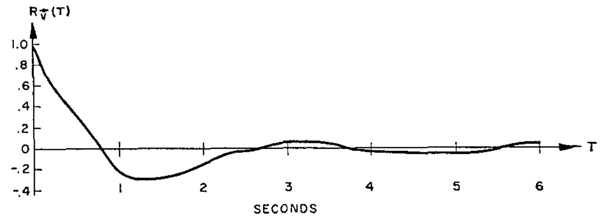

FIG. 1. Auto-correlation of the vector velocity for Run 02.

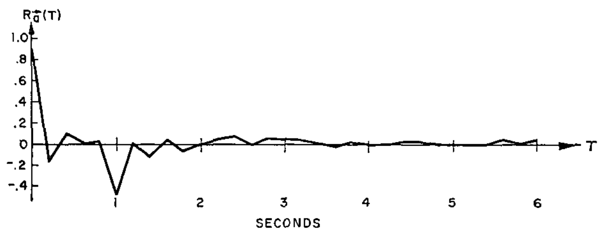

FIG. 2. Auto-correlation of the vector acceleration for Run 02.

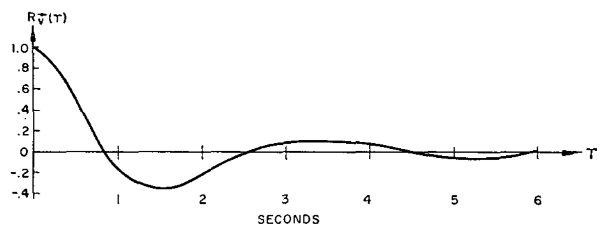

FIG. 3. Auto-correlation of the vector velocity for Run 03.

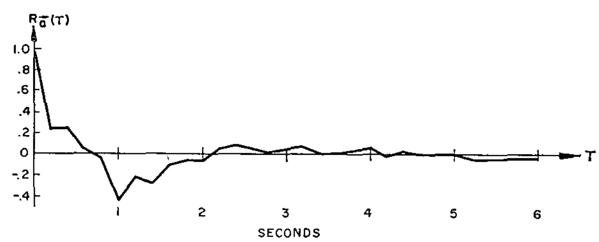

FIG. 4. Auto-correlation of the vector acceleration for Run 03.

by Eqs. (6) and (9), and the theory can be subjected to an observational test.

The autocorrelation coefficients of the vector acceleration and vector velocity were computed according to Eqs. (7) and (8), respectively, for the groups of bubbles mentioned previously. The mean autocorrelation coefficients of the vector velocity and vector acceleration for Run 02 and Run 03 are shown in Figs. 1, 2, 3 and 4. The observational results are presented in Table 2 for comparison. It may be noted that the values B_0, as determined from Eqs. (6) and (9), agree quite closely and are of the same order of magnitude for Run 02 and Run 03.

The value of B_0 is relatively easy to determine from (9) since $R_{\mathbf{v}}(\tau) \doteq 0$ for appropriately large values of τ. But the value of B_0, as determined from (6), is more

TABLE 2. Values associated with the one particle analysis.

Run	$\overline{\|\mathbf{v}'\|^2}$ ($cm^2\ sec^{-2}$)	$\overline{\|\mathbf{a}\|^2}$ ($cm^2\ sec^{-4}$)	B_0 $cm^2\ sec^{-3}$ Eq. (6)	B_0 $cm^2\ sec^{-3}$ Eq. (9)	τ (sec)
02	24.70	347.7	2.60	2.47	$\tau = 10.0$ $\tau_1 = 6.0$
03	51.29	372.1	5.40	5.13	$\tau = 10.0$ $\tau_1 = 6.0$

difficult to evaluate accurately. When the correlation of the vector acceleration displays some regularity as in the case of Run 03 in Fig. 4 for larger Reynolds number than Run 02, the integral of $R_{\mathbf{a}}(\tau)$ is possible and meaningful. However, if the correlation of the vector acceleration displays a very random type of acceleration, the integral of $R_{\mathbf{a}}(\tau)$ is difficult and the meaning doubtful. In Fig. 2, the correlation of the vector acceleration for Run 02 displays this type of random acceleration for smaller Reynold's number than Run 03. This suggests there is some evidence that Kolmogoroff's requirement of a very large Reynold's number is necessary for Eq. (6) to be valid. However, it should also be pointed out that Lin's one particle analysis is three dimensional and only the two dimensional case has been considered in this analysis.

It is interesting to note that Obukhov (1959) has assumed the parameter B_0 in Eq. (5) to be proportional to the rate of dissipation of turbulent energy ϵ, with a constant coefficient of the order of unity. Since this assumption is certainly plausible from the point of view of Kolmogoroff's theory, the values of B_0 determined in this one particle analysis study should be close to the value of ϵ. Brunt (1939) has estimated that rate at which the kinetic energy of the earth's atmosphere is dissipated by turbulence is of the order of 5 cm^2sec^{-3} in the lowest 10 km. This value agrees quite well with the order of the values obtained in this analysis.

4. Two particle analysis: Isotropic diffusion process

Lin (1960a,b), in making an extensive study of the process of anisotropic diffusion, has shown that Richardson's law of turbulent diffusion is a special case of his anisotropic theory when the field of turbulent motion is assumed to be isotropic in spatial orientation. In making his analysis, Lin has considered the motion of a pair of particles simultaneously released in a turbulent wind. Such a pair of particles meander together with the wind according to the large-scale motions, and drift apart due to the small-scale motions. Since Lin's theory deals with the effect of these small-scale motions on diffusion, the data from Run 02 are used in evaluating Lin's theoretical relationships. The observational results for the isotropic diffusion theory are presented and discussed in this section.

By considering the ensemble average of a number of pairs of particles taken at the same time, Lin has shown

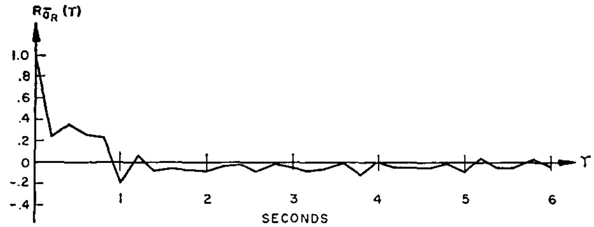

FIG. 5. Mean auto-correlation of the vector relative acceleration for bubbles No. 52–62.

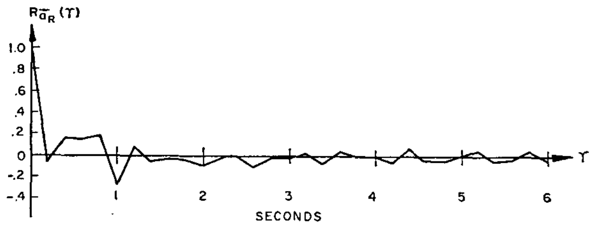

FIG. 6. Mean auto-correlation of the vector relative acceleration for bubbles No. 64–72.

that:

$$D(t)=(\tfrac{3}{2})^{\frac{2}{3}}B^{\frac{1}{3}}\overline{\mathbf{x}_R{}^2}(t)^{\frac{2}{3}} \tag{10}$$

where

$$B=\overline{\mathbf{a}_R{}^2}\int_0^{\tau 1} R_{\mathbf{a}R}(\tau)d\tau \tag{11}$$

and

$$R_{\mathbf{a}R}(\tau)=\frac{\overline{\mathbf{a}_R(t+\tau)\cdot\mathbf{a}_R(t)}}{\overline{\mathbf{a}_R{}^2}}. \tag{12}$$

In these equations, $D(t)$ is the horizontal eddy diffusivity, $R(\tau)$ is the autocorrelation of the vector relative acceleration, $\mathbf{a}_R$, between a pair of particles, and $\overline{\mathbf{x}_R{}^2}(t)$ is the mean square isotropic diffusion. Eq. (10) is Richardson's law of diffusion (1926) with the numerical coefficient B determined by a definite physical process in Eq. (11).

A requirement of Lin's isotropic theory is that τ and τ_1 can be chosen such that $\int_\tau^{\tau_1} R(\tau)d\tau$ is negligible compared to $\int_0^{\tau_1} R(\tau)d\tau$. Here τ is large compared with τ_1, but not so large that $\int_\tau^{\tau_1} R(\tau)d\tau$ is no longer negligible compared with $\int_0^{\tau_1} R(\tau)d\tau$.

The correlations of the vector relative acceleration computed for a maximum time lag of 12 seconds were found to satisfy this requirement. The mean correlation of the vector relative acceleration for 5 pairs of bubble trajectories in the group No. 52-62 is shown in Fig. 5. The mean correlation of the vector relative acceleration for 4 pairs of bubble trajectoriees in the group No. 64-72 is shown in Fig. 6. In these figures, only 6 seconds of the lag range is shown since the correlation functions converged rapidly and remained so for 12 seconds.

Another requirement of Lin's isotropic theory is that for each pair of particles the initial separation $\mathbf{x}_R(0)$

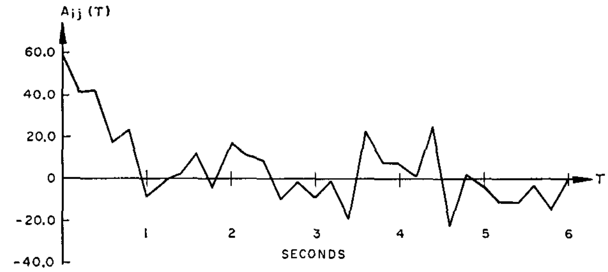

FIG. 7. Mean cross-covariance of the relative acceleration for bubbles No. 52–62.

and the initial relative velocity $\mathbf{v}_R(0)$ are zero or negligibly small compared with their values at the time under consideration. Since the bubbles in these experiments were released from a point source in groups with vacant intervals between groups, this aforementioned requirement should be fulfilled. In this study, τ is the time interval after the group of bubbles comes into the field of view instead of the time interval after the initial release of the bubbles.

From Lin's theory, it is also possible to find the following relationship for the isotropic diffusion process:

$$\overline{\mathbf{v}_R{}^2}(t)=12^{\frac{1}{3}}B^{\frac{2}{3}}\overline{\mathbf{x}_R{}^2}(t)^{\frac{1}{3}} \tag{13}$$

which predicts a linear relationship between the mean square relative velocity and the mean square isotropic diffusion raised to the one-third power. Thus, the value of B can be determined in two ways, Eqs (11) and (13), and the isotropic diffusion theory can be subjected to an observational test.

The mean square relative velocity and the mean square isotropic diffusion raised to the one-third power were computed for an ensemble average of 45 pair combinations for Bubbles No. 52-62 and an ensemble average of 36 pair combinations for Bubbles No. 64-72. In both cases, this represents the maximum number of pair combinations possible. In this study, the values of the mean square isotropic diffusion raised to the one-third power and of the mean square relative velocity were computed for two-tenths of a second intervals. The mean square isotropic diffusion raised to the one-third power was found to have a smoothly increasing characteristic with time, whereas the mean square relative velocity was found to have an irregular, oscillatory characteristic with time. This characteristic of the mean square relative velocity is to be expected for a random diffusion process. It is important to note that the characteristic of the relative displacements, not relative velocities, is important in diffusion processes. Thus the value of B was difficult to determine accurately from Eq. (13); however, the estimated value agrees in order of magnitude with the value of B determined from Eq. (11). The observational results of the isotropic diffusion process are presented in Table 3.

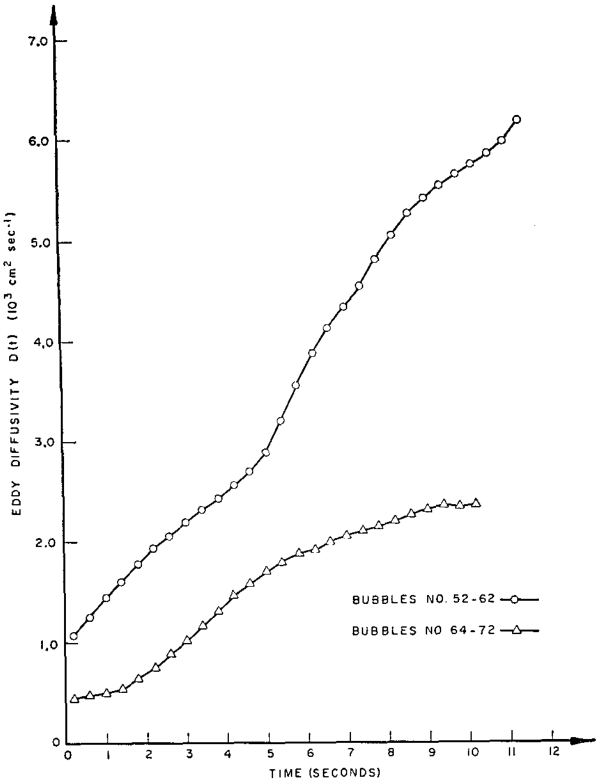

Fig. 8. Eddy diffusivity of the isotropic dispersion.

Table 3. Values associated with the isotropic diffusion process.

Run 02	$\overline{a_R^2}$ (cm² sec⁻⁴)	B cm² sec⁻³ Eq. (11)	B cm² sec⁻³ Eq. (13)	τ (sec)
Bubbles No. 52–62	682.2	68.2	60	$\tau=12.0$ $\tau_1=6.0$
Bubbles No. 64–72	472.8	28.8	30	$\tau=12.0$ $\tau_1=6.0$

The values of the horizontal eddy diffusivity were determined from Eq. (10) using the value of B from (11). Fig. 8 shows the horizontal eddy diffusivity as a function of time for the two groups of bubbles. Here the values are shown for every four-tenths of a second. This figure shows the well defined time variation of the horizontal eddy diffusivity predicted by the theory. It may be noted that the range of eddy diffusivity for Bubbles No. 52-62 is much larger than the range for Bubbles No. 64-72. The horizontal eddy diffusivities for the isotropic diffusion process range from 0.4×10^3 to 6.4×10^3 cm² sec⁻¹.

5. Stationary anisotropic diffusion process

The observational results for Lin's (1960b) anisotropic diffusion theory are presented and discussed in this section.

Again considering the ensemble average of a number of pairs of particles taken at the same time, it can be shown that:

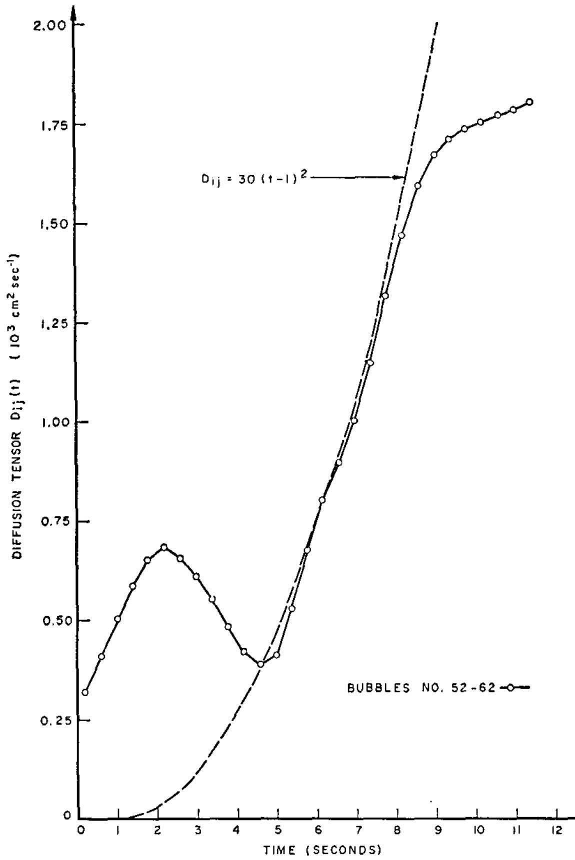

Fig. 9. Diffusion tensor for the stationary anisotropic dispersion.

$$D_{ij}(t)=(\tfrac{3}{2})^{\frac{2}{3}}B_{ij}^{\frac{1}{3}}\overline{x_i(t)x_j(t)}^{\frac{2}{3}}, \tag{14}$$

where

$$B_{ij}=\int_0^{\tau_1}A_{ij}(\tau)d\tau \tag{15}$$

and

$$A_{ij}(\tau)=\tfrac{1}{2}[\overline{a_i(t)a_j(t-\tau)}+\overline{a_j(t)a_i(t-\tau)}]. \tag{16}$$

For a pair of particles $x_i(t)=x_{i1}(t)-x_{i2}(t)$, $(d/dt)x_i(t)=v_i(t)$, and $(d/dt)v_i(t)=a_i(t)$. Here, i and j are the longitudinal and transversal components, respectively. In these equations $D_{ij}(t)$ is the diffusion tensor, $A_{ij}(\tau)$ is the cross-covariance coefficient of the relative acceleration, and $\overline{x_i(t)x_j(t)}$ is the mean square anisotropic diffusion. Equation (14) is again Richardson's law of diffusion with the numerical coefficient B_{ij} determined by a definite physical process in Eq. (15).

A requirement of Lin's anisotropic theory is that τ, τ_1, and τ_2 can be chosen such that $\int_0^{\tau_1}A_{ij}(\tau)d\tau \gg\int_{\tau_1}^{\tau_2}A_{ij}(\tau)d\tau$ for $\tau_1<\tau<\tau_2$, where τ_2 is chosen as large as permissible and τ_1 is chosen as small as permissible within the desired accuracy of the analysis. This method is one of asymptotic approximation. The cross-covariance coefficients of the relative acceleration were computed for five pairs of bubble trajectories from Bubbles No. 52-62 and for four pairs of bubble trajectories from Bubbles No. 64-72. The mean cross-covariance coefficient for Bubbles No. 52-62 is shown in Fig. 7. This cross-covariance coefficient, which is shown for a lag range of only six seconds, has the characteristic which satisfies this requirement of the theory for 12 seconds. The cross-covariance coefficient for Bubbles No. 64-72 did not satisfy this requirement.

Another requirement of Lin's anisotropic theory is that for each pair of particles the initial separation $x_i(0)$ and the initial relative velocity $v_i(0)$ are zero. As discussed in the previous section, this aforementioned requirement should be fulfilled.

From Lin's anisotropic theory, it is also possible to find the following relationship for the anisotropic diffusion process:

$$\overline{v_i(t)v_j(t)} = 12^{\frac{1}{3}} B_{ij}^{\frac{2}{3}} \overline{x_i(t)x_j(t)}^{\frac{1}{3}} \tag{17}$$

which predicts a linear relationship between $\overline{v_i(t)v_j(t)}$ and $\overline{x_i(t)x_j(t)}^{\frac{1}{3}}$. Thus, the value of B_{ij} can be determined in two ways, Eqs. (15) and (17), and the anisotropic diffusion theory can be subjected to an observational test.

The values of $\overline{v_i(t)v_j(t)}$ and $\overline{x_i(t)x_j(t)}^{\frac{1}{3}}$ were computed for an ensemble average of 45 pair combinations for Bubbles No. 52-62 and an ensemble of 36 pair combinations for Bubbles No. 64-72. This again represents the maximum number of pair combinations possible in both groups of bubbles. The $\overline{x_i(t)x_j(t)}^{\frac{1}{3}}$ for Bubbles No 64-72 did not show a generally increasing characteristic with time. The $\overline{x_i(t)x_j(t)}^{\frac{1}{3}}$ for Bubbles No. 52-62 was found to have a generally increasing characteristic with time, whereas the $\overline{v_i(t)v_j(t)}$ was found to have an irregular, oscillatory characteristic with time. Here again this characteristic of $\overline{v_i(t)v_j(t)}$ is to be expected for a random diffusion process. The value of B_{ij} was difficult to determine accurately from Eq. (17), but the estimated value was found to agree in order of magnitude with the value of B_{ij} from Eq. (15). The observational results of the anisotropic diffusion theory are presented in Table 4.

The values of the diffusion tensor were determined from Eq. (14) using the value of B_{ij} from (15). Fig. 9 shows the horizontal eddy diffusivity as a function of time for Bubbles No. 52-62. The values are shown for every four-tenths of a second. This figure generally shows a well defined time variation of the diffusion tensor. The values of the diffusion tensor for the anisotropic diffusion process range from 0.3×10^3 to 1.8×10^3 cm² sec⁻¹.

Because of uncertainty in the initial conditions, it might be reasonable to consider a hypothetical origin of time, and compare the dispersion tensor in Fig. 9 with the formula

$$D_{ij}(t) = 30(t-1)^2. \tag{18}$$

This means that we regard the theory as inapplicable for $t<5$ sec., but that from then on the behavior is similar to that in the idealized model beginning at rest at $t=1$ sec.

The curve of $D_{ij}=30(t-1)^2$ is shown in Fig. 9 for comparison with the actual dispersion tensor. It is seen that there is good agreement with a substantial range of the data. However, it may be noted that the values of D_{ij} begin to decline below the theoretical values at the end of the curve. This decline can probably be explained by the fact that the experimental uncertainties in measuring the horizontal positions of the trajectories are greater near the edge of the field of view of the camera.

TABLE 4. Values associated with the stationary anisotropic diffusion process.

Run 02	$(\overline{a_j^2}\overline{a_i^2})^{\frac{1}{2}}$ (cm² sec⁻⁴)	B_{ij} cm² sec⁻³ Eq. (15)	B_{ij} cm² sec⁻³ Eq. (17)	τ (sec)
Bubbles No. 52–62	336.1	30.4	27	$\tau_1=1.0$ $\tau_2=10.0$

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