Characteristics of Quasi-Horizontal Mesoscale Eddies

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1. Introduction

Even in stably stratified air, there is no resistance to the formation of eddies on isentropic surfaces. It is therefore likely that wind shear in such surfaces will produce eddies which may have considerable importance for lateral spread of contaminants.

2. Theory

According to Bannon (1949), any wind shear produces turbulence; according to Arakawa (1942), turbulence is created only with cyclonic shear greater than half the Coriolis parameter; according to Dugstad (unpublished) turbulence is created by any horizontal shear, cyclonic or anticyclonic, provided it exceeds the Coriolis parameter. This paper will suggest a fourth and again different criterion for the formation of isentropic eddies, and test it in a qualitative manner.

The basis for the discussion is the eddy energy equation. It can be derived by multiplying each component of motion by the corresponding eddy velocity component, adding the three equations, and averaging over a large volume. The result can be written:

$$\frac{dE}{dt}=-\overline{u_i'u_j'}\frac{\partial\overline{u_i}}{\partial x_j}+\overline{u_i'F_i'}-D. \qquad (1)$$

Here E is the mean eddy kinetic energy, u_i denotes the various velocity components in the directions x_i, a bar over a quantity expresses an average, and a prime the deviation from such an average. In general, the the primed quantities will refer to "eddy" quantities. F_i stands for the various forces, and D for the molecular dissipation of turbulent energy. A repeated index in any term implies summation over that index.

In the following discussion, we will identify x_1, x_2 and x_3 with the two isentropic coordinates, x and y, and the coordinate, z, at right angles. x will be directed along the mean flow. The velocity components in the x, y and z directions will be denoted by u, v and w.

The first expression on the right of Eq. (1), sometimes called the eddy production function, consists of 9 terms. Two of these give rise to the familiar effect of vertical wind shear on turbulence. We shall first consider the term $\overline{u'v'}\,\frac{\partial\bar{u}}{\partial y}$. In order to evaluate this quantity we consider a parcel initially in geostrophic equilibrium, displaced a distance L in the y-direction. For this parcel, u' will be given by: $L\left(\frac{du}{dy}-\frac{\partial u}{\partial y}\right)=L\left(f-\frac{\partial u}{\partial y}\right)$, provided that the eddies do not disturb the pressure field. Now, let an exchange coefficient A be defined by: $A=\rho\overline{L'v'}$, the term finally becomes: $-\frac{A}{\rho}\left(f-\frac{\partial\bar{u}}{\partial y}\right)\frac{\partial\bar{u}}{\partial y}$ where ρ is the density.

Terms like $\overline{u'w'}\,\frac{\partial\bar{u}}{\partial z}$ and $\overline{v'w'}\,\frac{\partial\bar{v}}{\partial z}$ have been considered by other investigators and certainly play an important role in regions of large vertical wind shear, for example in the Jet Stream Front and produce the familiar small-scale turbulence (CAT), as shown, for example, by Colson (1963). Quantities like $\overline{u'w'}(\partial\bar{w}/\partial x)$, $\overline{v'w'}(\partial\bar{w}/\partial y)$, and $\overline{w'^2}(\partial\bar{w}/\partial z)$ are probably negligible for the eddy scales here considered since $\bar{w}$ varies slowly in the horizontal, and $\overline{w'^2}$ is small, except in regions of CAT.

The terms $\overline{u'^2}(\partial\bar{u}/\partial x)$ and $\overline{v'^2}$ $(\partial\bar{v}/\partial y)$ can be combined into $(\partial\bar{u}/\partial x)$ $(u'^2-v'^2)$ since for large-scale motion, $\frac{\partial\bar{u}}{\partial x}\sim-\frac{\partial\bar{v}}{\partial y}$. This term vanishes in horizontally isotropic turbulence, but may be of some importance in actual turbulence. More study of this point is desirable.

Finally, the quantity $\overline{u'v'}(\partial\bar{v}/\partial x)$ can be written as $\overline{u'v'}(\bar{u}/R)$ where R is the radius of a curvature of a stream line, positive if curved cyclonically. The general effects of this term are complex; in particular it cancels the term $\overline{u'v'}$ $(\partial\bar{u}/\partial y)$ in solid rotation.

The term $\overline{u_i'F_i'}$ measures the eddy energy produced by the work of eddy forces. In particular, the buoyancy force can add or subtract energy from the fluid depending on whether the stratification is stable or unstable. The effect of horizontal forces is less clear. However, under no circumstances can any work be done by the Coriolis force which is always at right angles to the motion. The work done by the pressure gradient force is quite uncertain. If the eddies do not disturb the pressure field, F' is zero, $\overline{u_i'F_i'}$ vanishes. Since this condition may nearly be satisfied, the simplest procedure is to disregard the term $\overline{u_i'F_i'}$ in the study of the production of horizontal eddies.

For the purpose of studying the effect of horizontal wind shear on eddies, we may then write:

$$\frac{dE}{dt}=\frac{A}{\rho}\left(\frac{\partial \bar{u}}{\partial y}-f\right)\frac{\partial \bar{u}}{\partial y}-D. \qquad (2)$$

The equation states that any cyclonic wind shear will set up isentropic eddies, but anticyclonic wind shear only when it exceeds the Coriolis parameter. Since this is rare, one might expect relatively little isentropic turbulence on the anticyclonic side. Another hypothesis seems reasonable on the basis of Eq. (2) namely that turbulent energy should increase with increasing wind shear.

3. Test of the hypothesis

The consequences of the theory were tested on the results of isobaric flights by B-47 aircraft in connection with the U. S. Air Force Project Jet Stream between 25,000 and 40,000 ft. As part of the instrumentation, the planes carried Doppler radar so that detailed isobaric wind patterns could be studied. Clearly, there are situations where isobaric and isentropic wind patterns are different; however, a qualitative test of the hypotheses should be possible by neglecting the difference. The uncertainty of the wind speeds is given by Endlich and Rados (1959) as 3 knots and direction as 1°. As will be seen, these errors are small compared to the velocities in the eddies studied.

All flights (consisting of one or more traverses) were screened and only those meeting the following criteria were used in this study:

1. Traverse described a cross section nearly perpendicular to the jet stream;
2. Traverse maintained constant or near-constant altitude;
3. Continuous traverse extended at least 100 nautical miles either to the cyclonic or anticyclonic side of the maximum wind encountered in that traverse.

After screening, 62 individual traverses were available, of which 33 were on the cyclonic side and 29 were on the anticyclonic side of the jet maximum. Only the wind speed components of these traverses parallel to the mean wind were examined in detail.

The length of the traverses averaged 218 miles, with observations every 2 miles.

The observed field of wind speed was resolved into mean and turbulent components in the following way: first, parabolae were fitted to the traverses, one on each

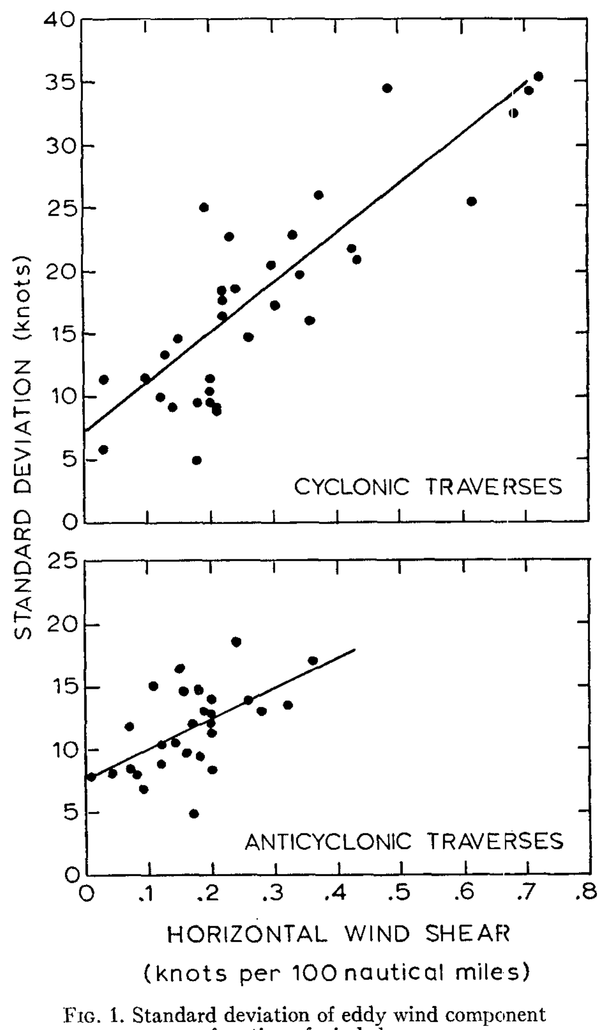

FIG. 1. Standard deviation of eddy wind component as function of wind shear.

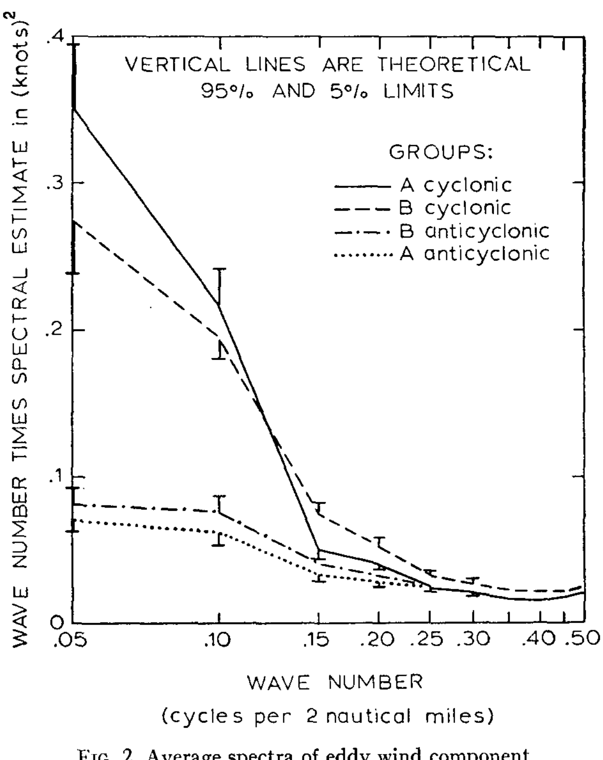

FIG. 2. Average spectra of eddy wind component parallel to mean wind.

side of the jet maxima; these defined the mean flow, and deviations defined the mesoscale turbulence.

The variances of the turbulent deviations of wind components parallel to the mean flow were then used as indicators of eddy energy. Altogether, the eddy energy on the cyclonic side exceeded that on the anticyclonic side by 75%.

Fig. 1 shows the relation between standard deviation of turbulent speed fluctuations and the isobaric wind shear over 100 miles. Apparently, the relation is good with a linear correlation coefficient of 0.85 on the cyclonic side.

Spectra of the fluctuations were also obtained by standard methods. They showed 85% of the eddy energy was contained in a band of wavelengths between 20 and 40 miles, consistent with the conclusions by Mantis (1963), that there is little energy in wavelengths less than 10 miles. Average spectra are shown in Fig. 2 for two groups each on the cyclonic and anticyclonic side—the groups being chosen at random.

In summary, then, it can be stated that in a stable atmosphere, mesoscale quasi-horizontal (probably isentropic) eddies exist, principally in cyclonic shear, the energy of which increases with increasing shear.

Taking a typical eddy velocity here as 10 m sec^{-1} and "eddy scale" of order $\frac{1}{6}$ of a wavelength, we can estimate an isentropic diffusion coefficient for an average wind shear in the neighborhood of jet streams as of order 10^5 m^2 sec^{-1}. Since the shears in this study are larger than those ordinarily encountered, average diffusion coefficients are probably considerably smaller.

It is suggested that similar studies be made in areas without pronounced jet streams, and that isentropic wind shear be estimated in future experiments. It is suggested that isentropic mesoscale eddies are a very general meteorological phenomenon, as indicated for example by the meandering of smoke plumes in stable air, and by the lateral spread on condensation trails, and that they are practically important for diffusion and for the stability of missiles.

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