NOTES AND CORRESPONDENCE

Convective Vortex over a Horizontal Surface with Nonuniform Temperature

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This note supplements a recent study by the authors (1966) on convective motions caused by nonuniform heating or cooling of a horizontal surface without consideration of the earth's rotation. If one includes the coriolis terms fv and fu in the radial and tangential Navier-Stokes' momentum equations, one obtains a horizontal circulation in addition to the vertical motion. The constant f denotes the coriolis parameter and (u,v,w) are the velocity components in the cylindrical coordinate system (r,φ,z). The method of solution is based on the local boundary layer approximation of first order: $u(r,z)=rt_sU(z)$, $v(r,z)=rt_sV(z)$, $w(r,z)=t_sW(z)$, where $t_s(r)$ is the dimensionless surface temperature. This procedure, which follows closely that by the authors (1966), is omitted here and reference is made to the full report by the authors (1965).

The most striking feature of the numerical results is that the direction of the tangential velocity component $V(z)$ of the rotating fluid changes at a certain height resulting in a spiralling motion as sketched in Fig. 1. The rotational direction depends on the sign of the radial velocity u and is cyclonic for horizontal convergence and anticyclonic for horizontal divergence. Therefore, the local heating of a surface generates in the Northern Hemisphere a vortex which is counterclockwise in the lower part of the friction layer and clockwise in the upper part. For a cooled surface the situation is reversed.

Quantitatively, the horizontal circulation increases with the dimensionless coriolis parameter $f^*=fL^2/\nu$ [ν and L are the kinematic viscosity and the characteristic length defined in Lugt and Schwiderski (1966)] and is more pronounced for a heated surface than for a cooled area as shown in Fig. 2. The location where the rotational direction changes coincides almost with the transition point from radial inflow to radial outflow. For a cooled surface the vertical extent of the entire convection is smaller than for a heated one. The meridional circulation decreases in magnitude for increasing f^* (Figs. 3 and 4).

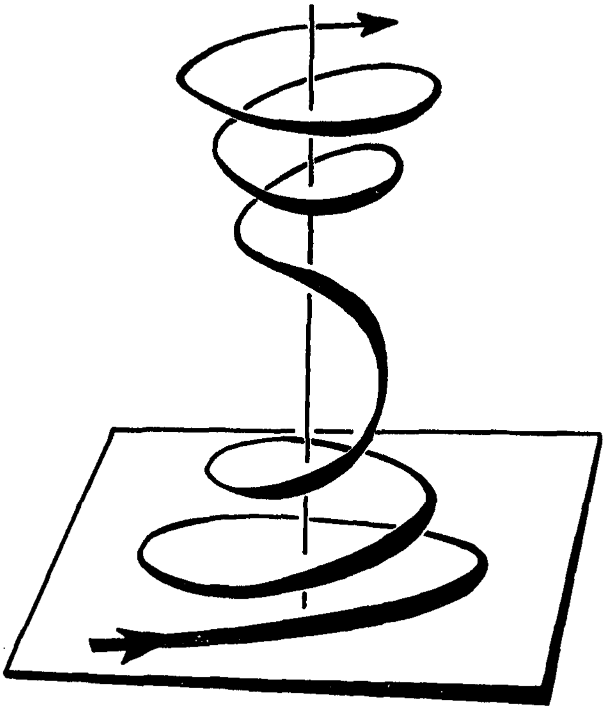

FIG. 1. Sketch of the path of a fluid particle in a convection flow caused by local heating of a surface in the Northern Hemisphere.

The property of convective vortices to change their rotational direction is confirmed by observed data in hurricanes. Riehl (1954, p. 300) gives for the directional change near the eye a value of about 9 km, which may

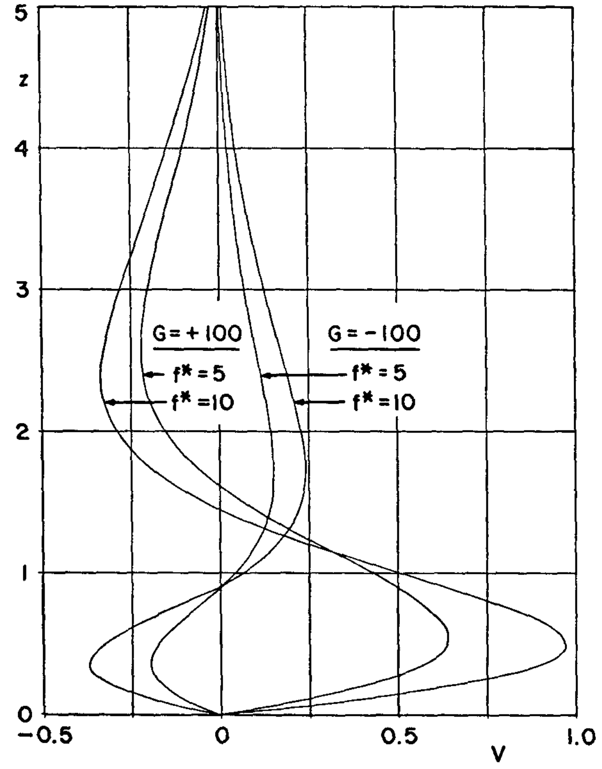

FIG. 2. The dimensionless tangential velocity V vs. the dimensionless height z for $P_r=0.7$, $G=\pm 100$, and for different f^*.

vary with the strength of the hurricane. The rotational change has also been calculated for motions in Bénard cells (Chandrasekhar 1961, p. 111).

If the coriolis force is absent ($f^*=0$), the deviation angle χ between the spiral motion and the circular flow at the surface is 90°. Otherwise, χ diminishes with increasing f^* but does not decrease below $\approx 75°$ for the examples computed (Table 1). Such a strong radial flow near the surface has also been found in the boundary layer of a potential vortex (Schwiderski and Lugt, 1964) and has been observed in water tank experi-

TABLE I. Nusselt numbers N and deviation angles χ for various G, P_r, and f^*.

		N			χ	
P_r	G	$f^*=0$	$f^*=5$	$f^*=10$	$f^*=5$	$f^*=10$
0.7	+100	1.817	1.824	1.842	81.0°	74.5°
	+10	1.985	1.986	1.987	82.9°	77.5°
	−10	2.014	2.014	2.013	83.3°	77.9°
	−100	2.121	2.118	2.111	84.5°	79.4°
	−1000	2.600		2.581		84.0°
	−2000	2.857		2.837		85.5°
	−5000	3.286		3.268		87.0°
7	+10	1.812	1.821	1.841	82.5°	76.4°
	−10	2.123	2.119	2.111	83.9°	78.5°

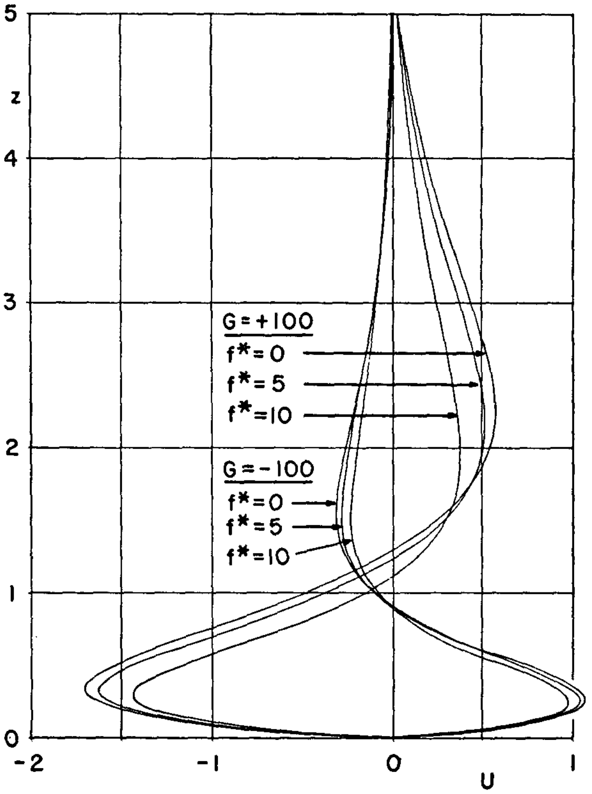

FIG. 3. The dimensionless radial velocity U vs. the dimensionless height z for $P_r=0.7$, $G=\pm 100$, and for different f^*.

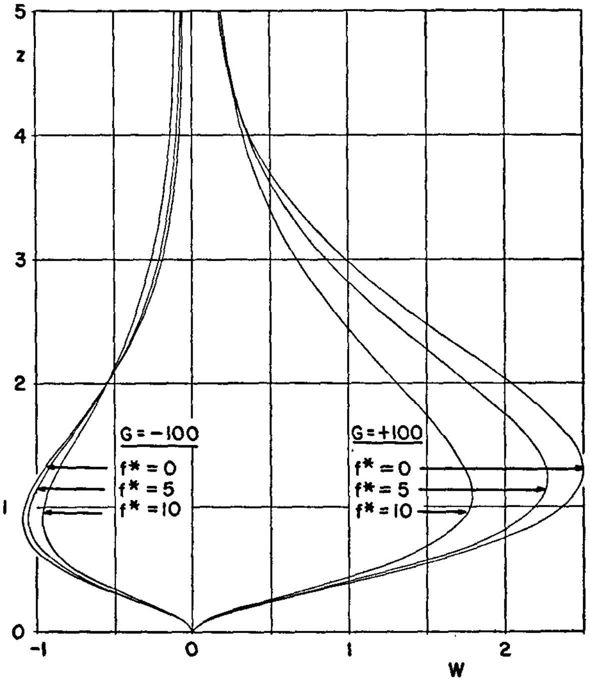

FIG. 4. The dimensionless axial velocity W vs. the dimensionless height z for $P_r=0.7$, $G=\pm 100$, and for different f^*.

ments[1] and in tornadoes (Brooks, 1951). Table 1 also shows some values of the Nusselt number N for the Prandtl numbers $P_r=0.7$ and 7.

Adjustable boundary layer approximations of first order lead to nonexistence of solutions if the similarity character of the flow quantities is not preserved. This happens in wake type motions from a certain critical value on. In Lugt and Schwiderski (1966) the critical Rayleigh number for $f^*=0$, $P_r=0.7$, has been calculated to be $R_{\text{crit}}\approx 120$. For $f^*\neq 0$ an example is $f^*_{\text{crit}}\approx 3.6$ for $R=140$. Thus, the tendency toward instability decreases with the coriolis parameter.

Physically, the spatial instability may be interpreted as a separation phenomenon in the same way as this has been done in the Bödewadt problem (Schwiderski and Lugt, 1964). Beyond the critical Rayleigh number (or in Bödewadt's case the critical Reynolds number), the flow separates from the surface before it reaches the center line. This explanation, which reflects the typical behavior of wake type flows, is supported by the tea-cup experiment that has been demonstrated by recently obtained photographs (Schwiderski and Lugt, 1965). Below the critical Reynolds number the tea leaves move toward the axis of rotation and heap up at the wake center, while beyond the critical Reynolds number the tea leaves settle distinctly on a ring around the wake center. The inner wall of this clearly visible torus is remarkably steep, which gives it much resemblance to the cloud dome around the eye of a hurricane. Hence, it appears that the eye of a hurricane must be considered as a similar sort of flow separation.

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[1] See the Sink Vortex, National Committee for Fluid Mechanics Film, FM-70, Educational Services, Inc., Watertown, Mass., 1964.

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Computation of Droplet Growth from Giant Salt Particles in Ascending Air below Cloud Base

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1. Introduction

Cloud droplet distributions for convective clouds are conventionally calculated for conditions above cloud base where the broad condensation nuclei population is involved in determining the droplet size distribution. Below cloud base, however, condensation on a small number of giant salt nuclei does not depend on the large numbers of adjacent small condensation nuclei, since these latter can only initiate droplets exceeding the critical size when subjected to the supersaturations found above cloud base. Thus, the growth of a droplet on a giant salt nucleus can be calculated as though there were no other nuclei involved.

This note examines the quantitative aspect of the growth rate of large salt particles as they rise in the up-current to the base of a convective cloud. The need for the computations arose when examining the possibility that large droplets (25–60 μ diameter), observed with a continuous particle collector at the base of some cumulus clouds in Arizona, could have originated from giant salt particles. The observations and details of these calculations are given in a project report by MacCready *et al.* (1965). If the droplet rise through the increasing humidity to cloud base were very slow, the droplet diameter would continually approximate the equilibrium diameter where the ambient water vapor pressure is just balanced by the vapor pressure of the water-salt solution (at the large sizes considered here, the surface curvature effect on vapor pressure becomes negligible). If the droplet rise were exceedingly fast, the particle diameter would lag behind the equilibrium diameter at the various heights and humidities. The equilibrium diameter is compared to the instantaneous